Imperfectly Enforceable Pollution Tax with Asymmetric Information

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Abstract

In this paper we model the interaction between a polluting firm that has to report its privately known abatement cost function to a regulator who seeks to implement an imperfectly enforceable Pigouvian tax to control aggregate emissions. Results with direct policy implications include the following. First, no matter the strictness of the level of enforcement of emissions, the firm will never report its true level of abatement costs. Second, this incentive to under-report is bounded with certainty, as in Bulckaen (1997), only when the regulator is able to enforce the level of emissions consistent with the firm's report of abatement costs, which is Bulckaen's case itself. If this is the case, the extent of under-reporting by the firm is also the same as in Bulckaen (1997). Otherwise, the incentive to under-report may be unbounded. Third, with imperfect enforcement of consistent emissions, the firm reports under-reports more than perfect enforcement. Fourth, the incentives to under-report will be unbounded with certainty, as in Kwerel (1977), only if the marginal expected penalty for not complying with the emissions level consistent with the report of abatement costs and the tax is lower than the tax itself. If this is the case, the regulator could apply consistent penalties, i.e.: penalties based on the level of emissions that minimizes total expected reported cost of compliance for the firm. But even in this case the incentive to under-report may be unbounded also.

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1. INTRODUCTION

It is a general claim in environmental economics that taxes on emissions are superior to uniform emission standards in terms of cost-effectiveness and efficiency. But this comparison is unfair if we do not consider that to set a tax the regulator needs to have at least some idea of the distribution of the marginal abatement cost (MAC) functions of the firms. The relative advantage of taxes as cost-effective instruments to control pollution over uniform emissions standards must be weighed against this fact, which may produce severe practical problems to achieve the desired aggregate level of emissions and which by the way is the usual reason behind the argument of the impossibility of allocating emissions standards among firms so as to mimic the tax result.

That taxes and standards pose similar informational burden on the regulator in the implementation phase has been stated many years ago by Weitzman (1974): "…it is neither easier nor harder to name the right prices than the right quantities because in principle exactly the *same* information is needed to correctly specify either" (p. 478). Nevertheless, no one has study the actual possibilities that a regulator has of obtaining true information about abatement costs from the firms when enforcement of the resulting tax is not perfect. The literatures of imperfect enforcement of emissions taxes on one side and asymmetry of information between the regulator and the firms concerning the latter's abatement costs on the other have evolved separately.

To my knowledge, the first to treat the case of an imperfectly enforceable pollution tax was Harford (1978). In this paper and in Harford (1987) he showed that the firm's actual level of emissions would be independent of the enforcement parameters. As in the case of perfect enforcement, the firm will equate MAC to the emission tax. Changes

in the fine or the probability of detection will affect the level of reported emissions, but not the level of actual emissions.[1] Recently, Sandmo (2002) took these results to emphasize that the cost-effectiveness property of taxes would continue to hold under imperfect compliance. All of these papers dealt with firms' incentives and properties of an emissions tax scheme in the context of self-reporting of emissions. The emissions tax was a parameter exogenously determined. The nature of the information asymmetry concerned only the firms' compliance status.

On the other hand, in the literature that deals with taxing pollution when there is asymmetric information concerning firms' abatement costs (AC) imperfect compliance is either not an issue or perfect enforcement of emissions is assumed. The literature could be divided into two sets of papers. First, a set of papers in which the regulator uses a probability distribution of the firms' abatement costs in order to maximize expected net benefits from pollution control. Second, a set of papers in which the regulator attempts to uncover the firms' abatement costs by asking them to report these costs.

A first example of the first set of papers is one already mentioned: Weitzman (1974). In this classic paper the author compares the relative advantages of implementing emission standards vs. emission taxes in terms of expected efficiency. Another example is Roberts and Spence (1976), who show that a mixed scheme composed of transferable permits complemented by subsidies and penalties (effluent charges) for levels of emissions below and above the number of permits held could reduce expected social costs as compared with pure licenses or pure effluent charges schemes. The authors ruled out the possibility that the regulator could iteratively adjust calculations. Finally,

[1] The same result can be observed in Linder and McBride (1984), who studied the incentives of decentralized enforcement agencies in the context of a similar model. The result does not hold when the probability of being inspected is a function of the relative amount of under-reporting, instead of the absolute amount, as it can be seen in Martin (1984).

Jebjerg and Lando (1997) derived the optimal tax scheme when the regulator is also interested in securing the firm survival.

The second set of papers differ from the first set because they assumed that the regulator asks the firms to report their abatement costs functions to set the optimal tax, instead of using subjective or known probabilities of their distribution. In other words, the regulator attempts to uncover or obtain some information on the abatement costs. The first work in this literature is Kwerel (1977). His concern was the incentives of firms to misreport abatement costs, and therefore the ability of the regulator to attain the optimum level of emissions under such a mechanism. In this context Kwerel showed that firms would have an unbounded incentive to under-report abatement costs under a pure effluent charge scheme and an unbounded incentive to over-report its abatement costs under a pure licensing scheme. He went on to show that under a mixed scheme of licenses and subsidies for levels of pollution below the number of licenses held by the firm, firms will report truthfully and the social optimum would be attained.

Dasgupta, et. al (1980) conclude that the regulator could obtain a truthful report of abatement costs from the firms if it could tax each one differently according to Groves (1973) incentive mechanism. Such a solution is not of interest here since we are interested in a uniform emissions tax for all sources.

Spulber (1988) derived necessary and sufficient conditions for an optimal effluent charge system when welfare effects on the product market were taken into account, assuming a specific quadratic form of the firms' costs functions.

Bulckaen (1997) re-evaluated Kwerel's result about the firms' unbounded incentives to under-report abatement costs when confronted with an emission tax. Bulckaen argued that the firm's incentive to under-report its abatement costs is no longer unbounded when the regulator require the firm to emit "consistently" with its own reports.

"Consistently" means to emit according to the point at which the reported MAC curve (not the true MAC curve) equals the tax.

Neither of the previous papers considered imperfect enforcement of emissions. The only exception is Swierzbinski's (1994). His main concern was the development of an instrument for optimal pollution regulation. This instrument proved to be the following scheme: First, the firm reports what its level of emissions will be and pays a fee based on this report. Second, the regulator monitors the actual level of the firm's emissions with a certain probability. If the regulator monitors the firm and finds that the firm is not in compliance with what it had reported, it is fined. If it is found emitting consistently with its report, it is paid a rebate. The regulator maximizes expected net benefits of pollution control and chooses K different triplets composed by a tax, an inspection probability and a rebate, for each of the K types of firms, where "type" refers to abatement costs level. But Swierzbinzki incorporates a "masquerade" constraint that guarantees that a type k firm will not choose a triplet designed for a type j firm. It is needed because the regulator cannot observe the firm's type but it is offering different schemes to different types of firms. This problem is the main motivation of my work.

In this work we bring together the literatures of imperfect enforcement of emissions taxes and asymmetry of information between the regulator and the firms concerning the latter's abatement costs. To do so, we re-evaluate Bulckaen's result dropping the assumption of perfect enforcement of the "consistent" level of emissions. In this case the firm can not only under-report its MAC but emit more than its consistent level of emissions. Then the firm would decide on two variables to minimize overall costs: (1) the truthfulness of the abatement costs report, and (2) its true level of emissions (or, in other words, its compliance status). Nobody has addressed the question of what are the firm's incentives in this case.

More precisely we address the following questions: (1) Does the firm have incentives to under-report? If it does, are these incentives bounded or unbounded? Under what conditions? (2) If bounded, is the level of under-reporting of AC larger or smaller than in the case of perfect enforcement, as analyzed by Bulckaen? (3) Could the Regulator design a penalty scheme so as to achieve perfect compliance with the consistent level of emissions?

Consequently, we model the decision that a polluting firm faces when it has to report its privately known Abatement Cost (AC) function to a regulator who seeks to implement an imperfectly enforceable Pigouvian tax to control aggregate emissions. We do this conserving the same rationality assumptions of Kwerel (1977) and Bulckaen (1997) for both the regulator and the representative firm. The regulator is able to observe (or estimate) the aggregate marginal damage function but it is unable to observe the firms' individual marginal abatement cost (MAC) functions needed to calculate the proper tax. It therefore asks the firms to report their AC functions. After setting the tax consistent with the AC functions reported, the Regulator also tries to enforce it by conducting audits to monitor emissions and imposing penalties in case of non-compliance.

2. THE MODEL

Assume that there are N firms. Let x_j be firm j's level of emissions. $X = \Sigma_j x_j$ is the total level of emissions discharged by the N firms. The Regulator is able to estimate the aggregate damage function of pollution $D(X) = D(\Sigma_j x_j)$; $D'(X)>0$ and $D''(X)>0$.

Let $C_j(x_j)$ be the firm j's total abatement cost function. As usual, it is assumed that $C_j'<0$ and $C_j'' >0$. $C_j(x_j)$ is privately known by firm j. The regulator asks each firm j to

report its abatement costs. It also declares that the information will be used to determine the optimal emission tax t. Each firm j then faces the decision about what level of AC to report to the regulator, knowing that the regulator will use this information to determine the optimal pollution tax. To capture this decision (and dropping the j subscript since we am going to deal only with the decision of one firm), j's AC function can be written as $C(x,\theta)$, where θ is a variable that captures the truthfulness of the report. Let $C(x,\theta^0)$ be the real abatement cost function. Assume that $-C_{x\theta}(x_j,\theta)>0$ and $C_{\theta}(x_j,\theta)>0$. Finally, given the other MAC functions reported by the other firms, the regulator sets the tax so that $-C_X(X,\theta) = D'(X) = t$, where $-C_X(X,\theta)$ is the aggregate reported MAC function. Therefore, $t = t(\theta)$. Finally, call $\bar{x} = x[t(\theta),\theta]$ the "consistent" (as in Bulckaen) level of emissions of the representative firm, determined by $t(\theta) = -C_x\left[x(t(\theta),\theta);\theta\right]$.

The game between the firm and the regulator has three stages: (1) The firm reports it abatement costs (θ); (2) The regulator sets $t(\theta)$; and (3) The firm chooses x (the actual level of emissions) and enforcement is applied.

In stage (3), θ and $t(\theta)$ have been chosen and assuming risk-neutrality the firm solves the following problem:

$$\min_x F(\theta) = C(x,\theta^0) + t(\theta) * \bar{x} + \pi\left[(t(\theta)+\phi)(x-\bar{x})\right]$$
$$s.t.\ x \geq \bar{x}$$

where ϕ is the constant penalty or fine per unit of emissions beyond the consistent level and π is the given probability of being inspected.

I assume that the penalty for not complying is not determined by the environmental regulator. It is previously set by legislators and the environmental regulator takes it as given. Furthermore, we assume a constant marginal penalty, as opposed to the frequently used increasing marginal penalties, because this assumption simplifies the exposition and does not have any implication for the results of the paper.

From the Kuhn - Tucker conditions of this problem, we know that the firm is going to comply with the consistent level of emissions ($x = \bar{x}$) if $-C_x\{\bar{x}, \theta^0\} \leq \pi[t(\theta) + \phi]$. Otherwise, if $-C_x\{\bar{x}, \theta^0\} > \pi[t(\theta) + \phi]$, then $x > \bar{x}$ and

$$C_x\{x, \theta^0\} + \pi[t(\theta) + \phi] = 0 \qquad \textbf{(1)}$$

I call $x^0 = x^0[t(\theta), \theta]$, the true level of emissions, the solution to this equation.

In stage 2, the regulator sets *t(θ)* and $\bar{x} = x[t(\theta), \theta]$ is implicitly defined. Therefore, in stage (1) the firm chooses θ (the truthfulness of the report) so as to minimize its expected costs:

$$\min_{\theta} F(\theta) = C\{x^0, \theta^0\} + t(\theta) * \bar{x} + \pi[t(\theta) + \phi](x^0 - \bar{x})$$

s.t.

$$(1) - C_x\{x^0, \theta^0\} \leq \pi[t(\theta) + \phi]$$

$$(2) C_x\{\bar{x}, \theta\} + t(\theta) = 0$$

Differentiating with respect to θ, substituting for the second constraint and re-arranging:

$$\frac{dF(\theta)}{d\theta} = C_x\{x^0, \theta^0\}\frac{dx^0}{d\theta} + \frac{dt(\theta)}{d\theta}\bar{x} - C_x\{\bar{x}, \theta\}\frac{d\bar{x}}{d\theta} + \pi\frac{dt(\theta)}{d\theta}(x^0 - \bar{x}) + \pi[t(\theta) + \phi]\left(\frac{dx^0}{d\theta} - \frac{d\bar{x}}{d\theta}\right)$$

In order to find out whether the firm has an incentive to under-report and if this is bounded or unbounded we need to sign this expression.

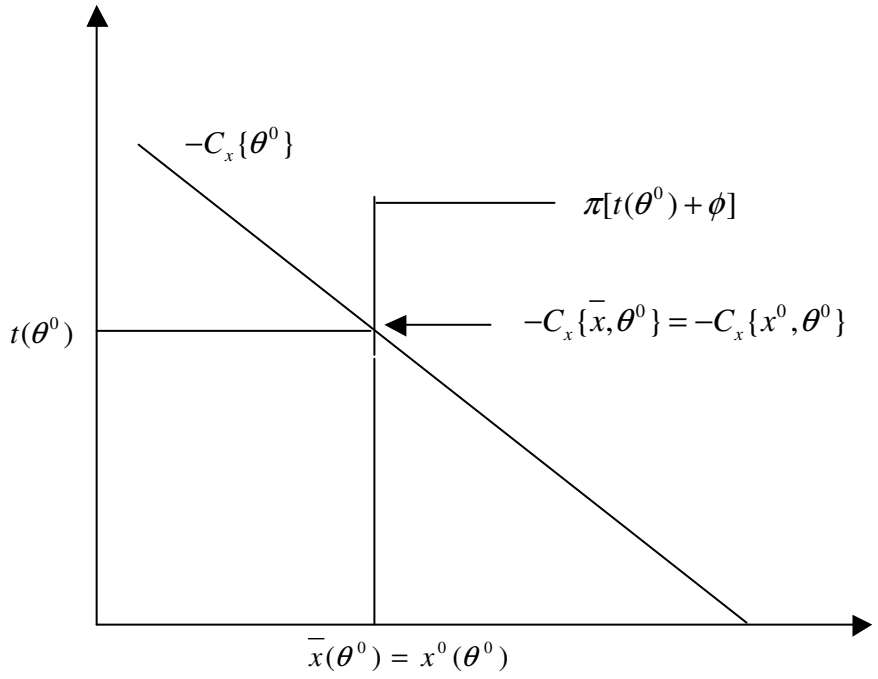
2.1. Incentives to Under-Report

To answer this first question (whether the firm has an incentive to under-report AC) we evaluate $\frac{dF(\theta)}{d\theta}$ at θ^0. To do it we distinguish different scenarios according to initial enforcement levels, given by the initial levels of the parameters (π, ϕ).

Case 1: $-C_x\{\bar{x}(\theta^0), \theta^0\} \leq \pi[t(\theta^0) + \phi]$

The case is illustrated in Figure 1. From the Kuhn-Tucker conditions of the emissions choice problem of the firm in stage 3 of the game, we know that in this case $x^0(\theta^0) = \bar{x}(\theta^0)$, the firm is going to comply with the tax.

Figure 1: The initial level of the enforcement is high



In this case,

$$\frac{dF(\theta^0)}{d\theta} = \frac{dt(\theta^0)}{d\theta} x^0(\theta^0) \qquad \textbf{(2)}$$

We know from Kwerel (1977) footonote (5) that

$$dt / d\theta = \left(C_{X\theta} D''\right) / \left(D'' + C_{XX}\right) > 0 \qquad \textbf{(3)}$$

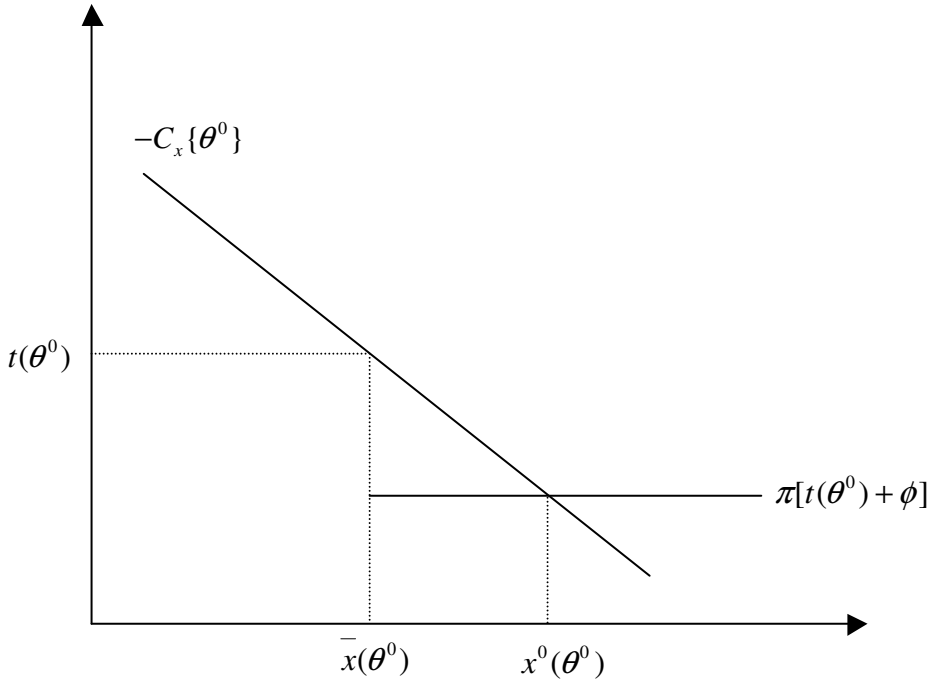
So the sign of equation (2) is positive. This means that the firm is not going to report truthfully its abatement costs even if the expected marginal penalty is high enough so as to make it comply with the consistent level of emissions when it reports truthfully.

Case 2: $-C_x\{\bar{x}(\theta^0), \theta^0\} > \pi[t(\theta^0) + \phi]$

The case is illustrated in Figure 2. Now the firm will choose $x^0(\theta) > \bar{x}(\theta)$. That is, even when telling the truth, the firm is going to violate the emissions level consistent with the tax according to

$-C_x\{x^0(\theta^0), \theta^0\} = \pi[t(\theta^0) + \phi]$.

Figure 2: The initial level of enforcement is low



In this case

$$\frac{dF(\theta^0)}{d\theta}=\frac{dt(\theta^0)}{d\theta}\bar{x}(\theta^0)+\pi\frac{dt(\theta^0)}{d\theta}(x^0-\bar{x})+\left[C_x\{x^0(\theta^0),\theta^0\}-C_x\{\bar{x}(\theta^0),\theta^0\}\right]\frac{d\bar{x}}{d\theta}$$

We already know that $\frac{dt(\theta)}{d\theta}\bar{x}>0$ and also that $\frac{d\bar{x}}{d\theta}=-\frac{C_{XX}}{C_{xx}^j}\left(\frac{C_{x\theta}}{D''+C_{XX}}\right)>0$ from Bulckaen (equation 7). Therefore, the sign of this expression depends on the sign of $C_x\{x^0(\theta^0),\theta^0\}-C_x\{\bar{x}(\theta^0),\theta^0\}$. By assumptions of Case 2,

$$C_x\{x^0(\theta^0),\theta^0\}-C_x\{\bar{x}(\theta^0),\theta^0\}=-C_x\{\bar{x}(\theta^0),\theta^0\}-\pi[t(\theta^0)+\phi]>0$$

This means that $\frac{dF(\theta^0)}{d\theta} > 0$. Also in this case the firm has an incentive to under-report abatement costs.

We have therefore a first result:

Result 1: *No matter the strictness of the level of enforcement of emissions the firm will never report its true level of AC.*

2.2. Bounded or Unbounded Incentives to Under-report?

To answer this question we also need to distinguish between different initial scenarios defined by enforcement levels. As before, let define

Case 1: $-C_x\{\bar{x}(\theta^0), \theta^0\} \leq \pi[t(\theta^0) + \phi]$.

It is important to note that $-C_{x\theta}\{\bar{x}(\theta), \theta^0\}\frac{d\bar{x}}{d\theta} < 0 \ \forall \theta$ and

$\frac{d\pi[t(\theta)+\phi]}{d\theta} = \pi\frac{dt(\theta)}{d\theta} > 0$ so this inequality cannot hold forever as long as θ decreases. Therefore we need to distinguish between two cases:

Case 1.a.: The level of the (exogenous) initial penalties are high enough so that $-C_x\{\bar{x}(\theta), \theta^0\} \leq \pi[t(\theta) + \phi]$ for all $\theta^* < \theta < \theta^0$, where θ^* is the level of θ that minimizes the total expected costs. If this is the case, the firm will set $x^0(\theta) = \bar{x}(\theta)$, and therefore we have

$$\frac{dF(\theta)}{d\theta} = \frac{dt(\theta)}{d\theta}x^0 + \left[C_x\{x^0, \theta^0\} - C_x\{x^0, \theta\}\right]\frac{dx^0}{d\theta}$$

We have just seen that $\frac{dt(\theta)}{d\theta}$ and $\frac{dx^0}{d\theta} = \frac{d\bar{x}}{d\theta}$ are positive, and it is easy to see that $\left[C_x\{x^0,\theta^0\} - C_x\{x^0,\theta\}\right] < 0$ for $\theta < \theta^0$. So when the firm under-reports ($\theta < \theta^0$), the term in brackets turns negative. Furthermore, $d\left[C_x\{x^0,\theta^0\} - C_x\{x^0,\theta\}\right]/d\theta = -C_{x\theta} > 0$. So ruling out the possibility that $d^2t(\theta)/d\theta^2 < 0$ (which is quite reasonable under the assumed tax-setting rule and the form of the damage function *D(X)*) is sufficient to assure that there will be a $\theta = \theta^*$ where the firm minimizes total expected costs *F (θ)*. At $\theta = \theta^*$,

$$\frac{dt(\theta^*)}{d\theta} x^0(\theta^*) = -\left[C_x\{x^0(\theta^*),\theta^0\} - C_x\{x^0(\theta^*),\theta\}\right]\frac{dx^0(\theta^*)}{d\theta}$$

The above expression tells us that the incentive to under-report is bounded. It is exactly equal to Bulckaen's equation (9). This is because Case 1.a. is in fact Bulckaen's case. In Bulckaen the regulator is always able to enforce the consistent level of emissions, which is what we have assumed in this case. Therefore, θ^* (the solution to Case 1.a) is the same as in Bulckaen.

We have therefore a second result, the answer to the third question addressed by this paper.

Result 2: *If the regulator wants to achieve perfect compliance with the consistent level of emissions it has to set the expected marginal penalty larger than the tax for every level of reported abatement costs.*

Case 1.b.: Now $-C_x\{\bar{x}(\theta^0),\theta^0\} \le \pi[t(\theta^0)+\phi]$, but the difference is not high enough to be maintained throughout the relevant range of θ. Therefore, at some $\theta < \theta^0$ we have that $-C_x\{\bar{x}(\theta),\theta^0\} > \pi[t(\theta)+\phi]$. Then $x^0(\theta) > \bar{x}(\theta)$ and $-C_x\{x^0(\theta),\theta^0\} = \pi[t(\theta)+\phi]$, as depicted in Figure 3. Consequently:

$$\frac{dF(\theta)}{d\theta} = \frac{dt(\theta)}{d\theta}\bar{x} + \pi\frac{dt(\theta)}{d\theta}(x^0 - \bar{x}) + \left[C_x\{x^0,\theta^0\} - C_x\{\bar{x},\theta\}\right]\frac{d\bar{x}}{d\theta}$$

The sign of the first two terms is positive. As for the sign of the term in brackets, we know that $\left(C_x\{x^0,\theta^0\} - C_x\{\bar{x},\theta\}\right) = t(\theta) - \pi[t(\theta)+\phi]$. From Case 1 condition ($t(\theta^0) \le \pi[t(\theta^0)+\phi]$), and because $\frac{dt(\theta)}{d\theta} > \frac{d\{\pi[t(\theta)+\phi]\}}{d\theta} = \pi\frac{dt(\theta)}{d\theta}$, then we can conclude that $\left[C_x\{x^0,\theta^0\} - C_x\{\bar{x},\theta\}\right] < 0$ for every $\theta < \theta^0$, and $\partial\left[C_x\{x^0,\theta^0\} - C_x\{\bar{x},\theta\}\right]/\partial\theta > 0$. Therefore, the incentive to under-report will be bounded if and only if the first and second terms of $dF/d\theta$ are decreasing with θ. To see if this is the case, let's call

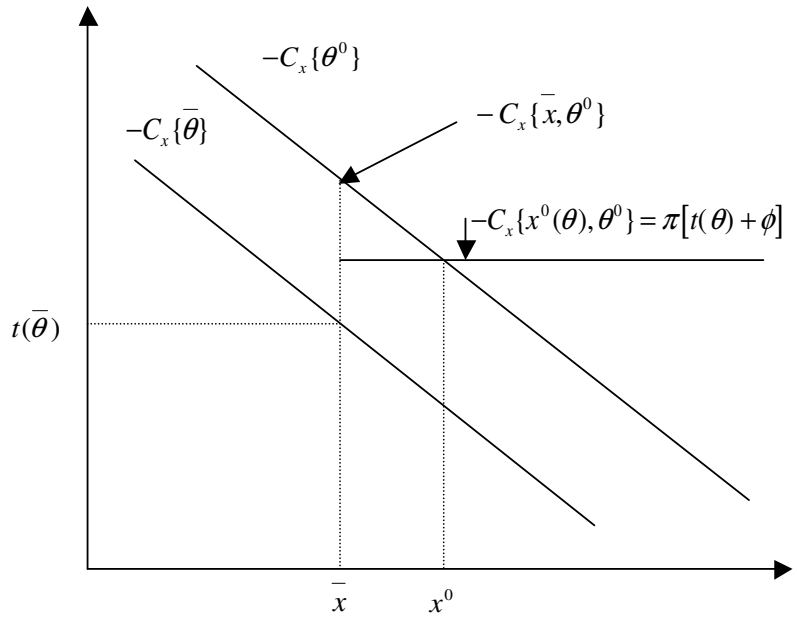
$$A(\theta) = \frac{dt(\theta)}{d\theta}\bar{x} + \pi\frac{dt(\theta)}{d\theta}(x^0 - \bar{x})$$

Then,

$$\frac{dA(\theta)}{d\theta} = \frac{d^2t(\theta)}{d\theta^2}\bar{x} + \frac{dt(\theta)}{d\theta}\frac{d\bar{x}}{d\theta} + \pi\left[\frac{d^2t(\theta)}{d\theta^2}\left(x^0 - \bar{x}\right) + \frac{dt(\theta)}{d\theta}\left(\frac{dx^0}{d\theta} - \frac{d\bar{x}}{d\theta}\right)\right]$$

Under the same assumptions of Case 1.a, all the terms of this expression are positive except for the last one, which is negative.[2] Therefore, the incentives of the firm to under-report abatement costs will also be bounded in this case if and only if $(x^0 - \bar{x})$ is not increasing "too much".[3]

Figure 3: Case 1.b.



<u>Case 2</u>: $-C_x\{\bar{x}(\theta^0),\theta^0\} > \pi[t(\theta^0)+\phi]$

In this case $-C_x\{x^0(\theta),\theta^0\} = \pi[t(\theta)+\phi] \forall\, \theta \le \theta^0$, and therefore $x^0(\theta) > \bar{x}(\theta) \forall\, \theta \le \theta^0$.

As in Case 1.b,

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[2] We already know that $d\bar{x}_j / d\theta > 0$. Deriving equation (1) with respect to θ and using (3), it can be easily seen that $dx_j^0 / d\theta = -\pi(dt/d\theta)/C_{xx}^j < 0$ for any firm *j*.

[3] Note that $\dfrac{dx_j^0}{d\theta} - \dfrac{d\bar{x}_j}{d\theta} = \dfrac{C_{x\theta}}{C_{xx}^j}\left(\dfrac{\pi D'' + C_{XX}}{D'' + C_{XX}}\right)$; the larger the probability of an inspection the larger the change in the level of a violation consistent with a change in the level of under-reporting for firm *j*.

$$\frac{dF(\theta)}{d\theta} = \frac{dt(\theta)}{d\theta}\bar{x} + \pi\frac{dt(\theta)}{d\theta}(x^0 - \bar{x}) + \left[C_x\{x^0, \theta^0\} - C_x\{\bar{x}, \theta\}\right]\frac{d\bar{x}}{d\theta} \quad \textbf{(5)}$$

and the sign of this expression depends again on $\left[C_x\{x^0, \theta^0\} - C_x\{\bar{x}, \theta\}\right]$.

From Case 2 conditions and $t(\theta) = -C_x\{\bar{x}[\theta], \theta\}$, we know that $-C_x(\bar{x}, \theta) > -C_x(x^0, \theta^0)$. So $\frac{dF(\theta^0)}{d\theta} > 0$. Nevertheless, we know that $t(\theta)$ decreases more rapidly that $\pi[t(\theta) + \phi]$, which means that $t(\theta)$ could reach $\pi[t(\theta) + \phi]$ if $t(\theta^0) - \pi[t(\theta^0) + \phi]$ is not large enough, and the third term of $\frac{dF(\theta)}{d\theta}$ may become negative as θ decreases. Therefore, one can say that the firm's incentives to under-report are unbounded if and only if (5) is positive for all $\theta < \theta^0$, which occurs when the marginal expected penalty is lower than the tax itself and sufficiently lower so as to make the marginal benefits of lying (a marginal decrease in tax payments) always larger the marginal costs of lying (an increase in the marginal penalty for not complying with the consistent level of emissions).

We have another result:

Result 3: *when the regulator is not able to enforce the consistent level of emissions for all levels of reported abatement costs, the incentive to under-report abatement costs may be unbounded.*

3. COMPARISON WITH THE CASE OF PERFECT ENFORCEMENT

In this section we compare the level of under-reporting in the cases of perfect and imperfect enforcement of the consistent level of emissions. First, evaluate $\frac{dF(\theta)}{d\theta}$ at $\bar{\theta}$, the optimal choice of θ in the case of perfect enforcement.

$$\frac{d\bar{F}(\bar{\theta})}{d\theta} = \frac{dt(\bar{\theta})}{d\theta}\bar{x} + \left(C_x\{\bar{x}, \theta^0\} - C_x\{\bar{x}, \bar{\theta}\}\right)\frac{d\bar{x}}{d\theta} = 0 \qquad \textbf{(6)}$$

While in the case of imperfect enforcement,

$$\frac{dF(\bar{\theta})}{d\theta} = \frac{dt(\bar{\theta})}{d\theta}\bar{x} + \pi\frac{dt(\bar{\theta})}{d\theta}(x^0 - \bar{x}) + \left(C_x\{x^0, \theta^0\} - C_x\{\bar{x}, \bar{\theta}\}\right)\frac{d\bar{x}}{d\theta} \qquad \textbf{(7)}$$

From (6)

$$\frac{dt(\bar{\theta})}{d\theta}\bar{x} - C_x\{\bar{x}, \bar{\theta}\}\frac{d\bar{x}}{d\theta} = -C_x\{\bar{x}, \theta^0\}\frac{d\bar{x}}{d\theta}$$

Then we can write (7) as

$$\frac{dF(\bar{\theta})}{d\theta} = \pi\frac{dt(\bar{\theta})}{d\theta}(x^0 - \bar{x}) + \left(C_x\{x^0, \theta^0\} - C_x\{\bar{x}, \theta^0\}\right)\frac{d\bar{x}}{d\theta}$$

We know that $-C_x\{\bar{x},\theta^0\} \geq -C_x\{x^0(\bar{\theta}),\theta^0\}$, therefore $\frac{dF(\bar{\theta})}{d\theta} > 0$ for $x^0(\bar{\theta}) > \bar{x}$, which occurs when $\pi[t(\bar{\theta})+\phi] < -C_x(\bar{x},\theta^0)$, and $\frac{dF(\bar{\theta})}{d\theta} = 0$ when $x^0\left(\bar{\theta}\right) = \bar{x}$ which occurs when $\pi[t(\bar{\theta})+\phi] \geq C_x(\bar{x},\theta^0)$. We are now able to formally state another result.

Result 4: *With imperfect enforcement of consistent emissions the firm always under-reports more than with perfect enforcement.*

4. CONSISTENT PENALTIES

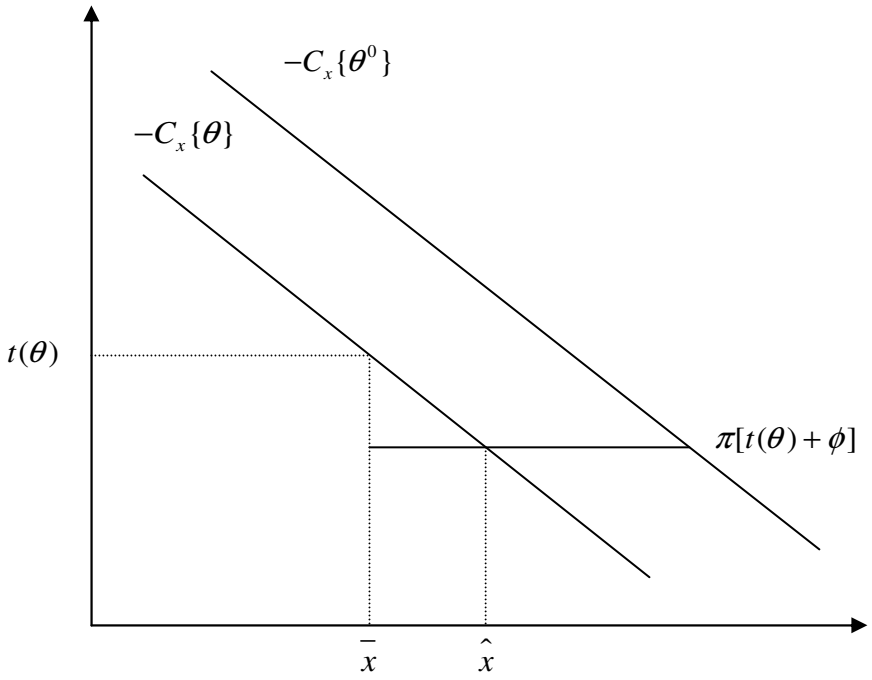
Imaging a situation as the one depicted in Figure 4 in which a firm is under-reporting. In this situation the regulator knows that, if the firm acts consistently with its reports, its emissions should be at $\hat{x}$ and not at $\bar{x}$, $\hat{x}\left(\theta\right)$ being the level of emissions at which

$$-C_x(\hat{x}\left(\theta\right),\theta) = \pi[t(\theta)+\phi]$$

Knowing this, the regulator could penalize the firm for being "consistently" violating the standard. In this case the objective function of the firm is:

$$F(\theta) = C\{x,\theta^0\} + t(\theta) * \bar{x}[t(\theta),\theta] + [t(\theta)+\phi] * \left[\hat{x}\left(t(\theta),\theta\right) - \bar{x}[t(\theta),\theta]\right] + \pi\{[t(\theta)+\phi](x - \hat{x}\left(t(\theta),\theta\right))\}$$

Figure 4: Consistent penalties



The problem for the representative firm in the first stage of the game is

$$\min_{\theta} F(\theta) = C\{x^0(\theta), \theta^0\} + t(\theta) * \hat{x}[\theta] + \phi * \left[\hat{x}(\theta) - \bar{x}[\theta]\right] + \pi\{[t(\theta)+\phi](x^0(\theta) - \hat{x}(\theta))\}$$

s.t.

$$(1)\, C_x\{x^0(\theta), \theta^0\} + \pi[t(\theta)+\phi] = 0$$

$$(2)\, C_x\{\bar{x}(\theta), \theta\} + t(\theta) = 0$$

$$(3)\, C_x\left(\hat{x}(\theta), \theta\right) + \pi[t(\theta)+\phi] = 0$$

The necessary and sufficient condition for an interior solution to this problem, under the assumptions of the abatement cost function, is

$$C_x \frac{dx^0}{d\theta} + \frac{dt(\theta)}{d\theta}\hat{x} + t(\theta)\frac{d\hat{x}}{d\theta} + \phi\left[\frac{d\hat{x}}{d\theta} - \frac{d\bar{x}}{d\theta}\right] + \pi\frac{dt(\theta)}{d\theta}(x^0 - \hat{x}) + \pi[t(\theta)+\phi]\left(\frac{dx^0}{d\theta} - \frac{d\hat{x}}{d\theta}\right)$$

Substituting constraints (1) and (2) in the case of weak enforcement ($t(\theta) > \pi[t(\theta) + \phi]$), which is the only one that makes sense here,

$$\frac{dF(\theta)}{d\theta} = \frac{dt(\theta)}{d\theta}\hat{x} + \phi\left[\frac{d\hat{x}}{d\theta} - \frac{d\bar{x}}{d\theta}\right] + \pi\frac{dt(\theta)}{d\theta}(x^0 - \hat{x}) + \left(C_x\{x^0(\theta), \theta^0\} - C_x\{\bar{x}(\theta), \theta\}\right)\frac{d\hat{x}}{d\theta}$$

The sign of this expression depends on the sign of $\left[\frac{d\hat{x}}{d\theta} - \frac{d\bar{x}}{d\theta}\right]$. Differentiating constraint (3) and using previous results we obtain $\frac{d\hat{x}}{d\theta} > 0$ and

$$\frac{d\hat{x}}{d\theta} - \frac{d\bar{x}}{d\theta} = \frac{\pi D'' + C_{XX}}{C^j_{xx}}\left(\frac{C_{X\theta}}{D'' + C_{XX}}\right) - \frac{C_{X\theta}}{C^j_{xx}}$$

for any representative firm *j*. Since this expression cannot be signed, we can state the last result of the paper as the following:

Result 5: *Even when consistent penalties are applied, if enforcement is weak, the incentives to under-report may be unbounded.*

5. CONCLUSIONS

In this paper we model the decision that a polluting firm faces when it has to report its privately known Abatement Cost (AC) function to a regulator who seeks to implement an imperfectly enforceable Pigouvian tax to control aggregate emissions. We do this conserving the same rationality assumptions of Kwerel (1977) and Bulckaen

(1997) for both the regulator and the representative firm. After setting the tax consistent with the AC functions reported, the Regulator also tries to enforce it by conducting audits to monitor emissions and imposing penalties in case of non-compliance.

Results with direct policy implications include the following. First, no matter the strictness of the level of enforcement of emissions the firm will never report its true level of abatement costs. In other words, the firm always has an incentive to under-report no matter whether the enforcement of emissions is perfect or imperfect.

Second, this incentive to under-report is bounded with certainty, as in Bulckaen (1997), only when the regulator is able to enforce the level of emissions consistent with the firm's report of abatement costs, which is Bulckaen's case itself. If this is the case, the extent of under-reporting by the firm is also the same as in Bulckaen (1997). Otherwise, the incentive to under-report may be unbounded.

Third, with imperfect enforcement of consistent emissions the firm always under-reports more than with perfect enforcement.

Finally, the incentives to under-report will be unbounded with certainty, as in Kwerel (1977), only if the marginal expected penalty for not complying with the level of emissions consistent with the report of abatement costs and the tax is lower than the tax itself and sufficiently lower so that under-reporting has always positive marginal benefits for the firm, given the report of the other firms. Given that no rational regulator will set a marginal expected penalty below the tax, this case can only be explained by other factors, like severe enforcement budget constraint. If this is the case, the regulator could apply consistent penalties, i.e.: penalties based on the level of emissions that minimizes total expected reported cost of compliance for the firm. But even in this case the incentive to under-report may be unbounded also.

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