To Tell the Truth: Imperfect Information and Optimal Pollution Control

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1. INTRODUCTION

In a world of perfect information, optimal regulation of an isolated economic variable would be relatively straightforward. Unfortunately, we do not live in such a world. Regulatory authorities typically find that the information which they need during the planning phase is known only by those who are to be regulated. In this situation a serious incentive problem may arise. Unless a system can be designed which makes the objectives of individual agents coincide with the regulator's objectives, self-interested agents will systematically deceive the regulatory authority when asked to reveal their information. One of the most interesting examples of a regulatory activity where such an incentive problem may arise is pollution control.

A necessary condition for designing an optimal pollution control plan is knowledge of both the damages resulting from pollution and the costs of reducing pollution. This paper will focus on the policy implications of an asymmetry between the regulatory authority and pollutors concerning information about clean-up costs. We examine the incentives of firms to deceive the regulatory authority when confronted with two standard pollution control policies, and then propose a new scheme which will induce cost-minimizing firms to reveal the true costs of cleaning up pollution.

2. ASSUMPTIONS AND NOTATION

In order to focus on the implications of an asymmetry in information about clean-up costs, we will make several simplifying assumptions. We assume that there is only one form of pollution, and that all waste discharged has the same impact on the environment no matter which firm discharges it. Ignoring the numerous difficulties involved in estimating damages, we assume that the government can assign a dollar value to the expected damages resulting from any aggregate level of pollution. These expected damages are denoted by $D(X)$, where X is the total amount of pollution discharged. It is assumed that $D'(X)>0$ and that $D''(X)>0$.

There are n firms, and $C_j(X_j)$ describes the cost of clean-up for firm j, where X_j is its output of pollution. Let $\bar{X}_j$ be the amount of pollution produced by the jth firm in the absence of government controls, i.e. $C_j(\bar{X}_j) = 0$. It is assumed that the cost of clean-up increases as the amount discharged is reduced. The absolute value of marginal clean-up costs is also assumed to increase as pollution is reduced. That is, $C_j'(X_j)<0$ and $C_j''(X_j)>0$ for all j, for $X_j<\bar{X}_j$. We assume that firms know their clean-up cost functions.[1]

The *aggregate clean-up cost function*, $C(X)$, gives the minimum total cost of achieving pollution level X, i.e.

$$C(X) \equiv \min_{\{X_j\}} \textstyle\sum_{j=1}^{j=n} C_j(X_j), \text{ subject to } \sum_{j=1}^{j=n} X_j = X.$$

Assuming that an interior solution exists, this expression is minimized when:

$$C_i'(X_i) = C_j'(X_j) \quad \text{for all } i \text{ and } j.$$

At the minimum, $C'(X) = C_j'(X_j)$ for all j. Since $C_j''(X_j) > 0$ for all j, it follows that $C''(X) > 0$.

The government's objective is to minimize the sum of clean-up costs and expected damages from pollution, $D(\sum_j X_j) + \sum_j C_j(X_j)$. The optimal X_j's must therefore satisfy the following two conditions:

$$D'(\textstyle\sum_j X_j) + C'(\textstyle\sum_j X_j) = 0$$

$$\textstyle\sum_{j=1}^{j=n} C_j(X_j) = C(\textstyle\sum_j X_j).$$

We assume that the government knows nothing about the aggregate clean-up cost function, $C(X)$. A central theme of this paper is to propose a method of inducing firms to reveal the portion of this function relevant for determining the social optimum. Suppose the government regulators ask all firms to report their pollution control cost functions. The function reported by firm j is denoted by $\hat{C}_j(\cdot)$, for all j. Let

$$\hat{C}(X) \equiv \min_{\{X_j\}} \textstyle\sum_{j=1}^{j=n} \hat{C}_j(X_j), \text{ subject to } \textstyle\sum_{j=1}^{j=n} X_j = X.$$

3. PURE LICENSING AND PURE EFFLUENT CHARGE POLICIES

In this section we will examine the incentive of a firm in a competitive environment to distort information under two pollution control policies which are often proposed.

A. *Pure Licensing*

Suppose the government plans to issue a fixed number of transferable licences for pollution. Let L be the number of licences issued, and p be the market price of a licence. It is assumed that the market for licences is competitive.

Firm j seeks to minimize the sum of treatment costs plus licence fees subject to the constraint that emissions do not exceed its licence purchases, L_j, or in symbols:

$$\min_{\{X_j, L_j\}} C_j(X_j) + pL_j, \text{ subject to } X_j \leqq L_j.$$

If $X_j < L_j$, the firm could reduce its costs by buying fewer licences and holding X_j constant. Thus, an optimum requires $L_j = X_j$. The first-order condition for a cost minimum is: $C_j'(X_j) + p = 0$. Notice that aggregate clean-up costs are minimized since all firms face the same p.

Now we will derive the market demand for licences. Let L_j^d be the jth firm's demand for licences, and let $L^d = \sum_j L_j^d$. If the price of a licence is p, each firm chooses X_j so that $-C_j'(X_j) = p$ and $L_j^d = X_j$. From the definition of the aggregate cost function $C(X)$ it follows that the total demand for licences at a given p will be that value of X such that $-C'(X) = p$. Also, since $C''(X) > 0$, $dL^d/dp < 0$.

When L licences are issued, the aggregate level of pollution will be L, since in equilibrium $\sum_{j=1}^{j=n} X_j = L^d = L$. The socially optimal L, which minimizes the sum of damages and clean-up costs, is given by the first-order condition, $D'(L) + C'(L) = 0$.

Suppose the government regulators ask firms to report their pollution control cost functions, and firms believe that the government will set L such that

$$D'(L) + \hat{C}'(L) = 0.$$

Each firm j will desire to report a $\hat{C}_j(\cdot)$ which will minimize p, the equilibrium price of a licence, since its minimum total cost, $\min C_j(X_j) + pX_j$ subject to $X_j \geqq 0$, is an increasing function of p. But, since $dL^d/dp < 0$, the equilibrium price of a licence will fall as the number of licences issued is increased.[2] Thus each firm will always be better off by reporting a $\hat{C}_j(\cdot)$ which induces the government to issue a larger number of licences.

There is no incentive to limit the extent of deception.[3] Therefore, the government will gain no useful information by asking firms to report their costs of clean-up when firms believe that the information will be used to set L in a pure licensing scheme.

As one might expect, the form of deception will be to exaggerate the cost of clean-up. It can be shown in the pure licensing case that given any clean-up cost function reported by a firm, and any cost functions reported by other firms, the firm would have been at least as well off by reporting a clean-up cost function with greater marginal costs for all levels of pollution output.[4]

B. *Pure Effluent Charge*

Suppose the government regulators plan to set a charge of e per unit of pollution. Under an effluent charge of e, each firm minimizes the sum of its clean-up costs and effluent fees, $C_j(X_j)+eX_j$. It does this by choosing X_j such that $-C_j'(X_j)=e$. This implies that aggregate clean-up costs are minimized and that the total output of pollution is given by $-C'(X)=e$.

The socially optimal e, which minimizes the sum of damages and clean-up costs, is given by the first-order condition

$$D'[X(e)] = -C'[X(e)] = e.$$

Suppose the government asks firms to report their pollution control cost functions, and firms believe that the government will set e such that

$$D'[X(e)] = -\hat{C}'[X(e)] = e.$$

Each firm j will desire to report a $\hat{C}_j(\cdot)$ which will minimize e, since its minimum total cost is an increasing function of e. The true cost function, $C_j(\ )$, and the socially optimal e, are both irrelevant to the firm in determining $\hat{C}_j(\ )$. Thus, the government will learn no useful information by asking firms to report their pollution control costs when firms believe that the information will be used to set e in a pure effluent charge plan.

In contrast to the pure licensing case, here the form of deception will be to understate the cost of clean-up. If a firm anticipates a pure effluent charge, it can be shown that given any clean-up cost function reported by the firm, and any cost functions reported by other firms, the firm would have been at least as well off by reporting a clean-up cost function with lower marginal costs for all levels of pollution output.[5]

4. A MIXED EFFLUENT CHARGE-LICENCE PLAN

We turn now to a mixed pollution control plan.[6] It turns out that this scheme exactly balances the incentive to overstate costs under licensing with the incentive to understate costs under effluent charges, so that firms are induced to make socially optimal reports to the government. The plan has two parameters:

(i) L transferable licences are issued.

(ii) A subsidy of e per licence in excess of emissions is paid to firms holding such licences.

Assume the market for licences is competitive, and let p be the market price of licences. Given p, firm j seeks to minimize the sum of treatment costs plus licence fees minus rebates or

$$C_j(X_j)-e(L_j-X_j)+pL_j = C_j(X_j)+(p-e)L_j+eX_j$$

subject to the constraint that emissions do not exceed licences, i.e. $X_j \leqq L_j$.

The government asks all firms to report their pollution control cost functions after announcing that it will set the parameters L and e so that

$$D'(L) = -\hat{C}'(L) = e.$$

It is a remarkable fact that this simple scheme makes it in each firm's own interest to do what is socially desirable. This basic result of the paper is formalized in the following theorem.

Theorem. *Under the mixed effluent charge-licence plan, each firm's total costs are minimized when the government sets the socially optimal effluent subsidy and stock of licences.*

Before proving this theorem, we will discuss its implications. One striking implication which follows immediately is that under this scheme a cost-minimizing firm will reveal its true cost of cleaning up pollution, so long as it believes other firms are telling the truth. In more technical language, truth telling is a Nash equilibrium.

The social optimum has a strong stability property when the government uses the mixed effluent charge-licence system, in the sense that any rational firm always strives to force the government to reach the optimum. If a firm believes other firms are not telling the truth it will also not tell the truth. But it will lie only to compensate for the lies it thinks others are telling. Thus if a firm believes other firms are overstating their marginal costs it will understate its costs and vice-versa. If a firm actually knew the true cost functions of the deceitful firms and what they reported, then it would report a cost function which would offset their lies and induce the government to pick the optimal parameter values.[7]

Given that no firm can do better than when everyone tells the truth, it may be reasonable for each firm to assume that all other firms are telling the truth. Thus, a fair speculation might be that all firms would in fact tell the truth under the mixed plan.

Now we will prove the theorem.

Proof.[8] First we will derive the market demand for licences and an expression for the equilibrium price of licences. For the moment suppose the effluent subsidy is fixed at some arbitrary level.[9] Let L_j^d be the jth firm's demand for licences, and let $L^d = \sum_j L_j^d$. Denote by $\bar{L}$ the value of L such that $-C'(L) = e$ (see Figure 1).

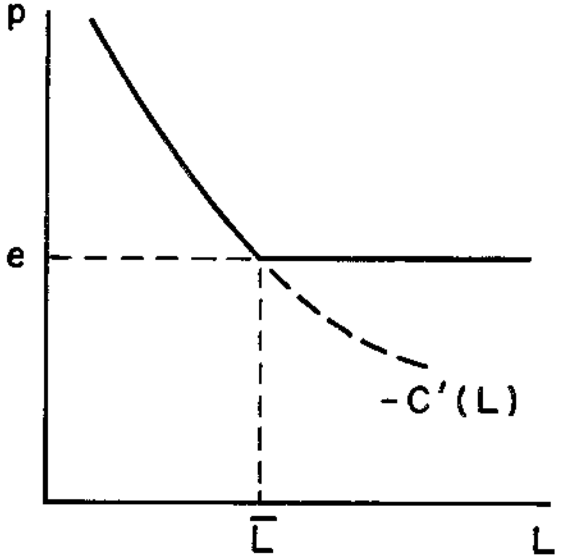

FIGURE 1

Market demand for licences, e given

Consider the demand for licences given different values of p:

(1) If $p > e$, a firm will choose $L_j = X_j$. The firm's minimum total costs are

$$TC_j(p) \equiv \min C_j(X_j) + pX_j \text{ subject to } X_j \geqq 0.$$

Each firm will choose X_j so that $-C_j'(X_j) = p$ and $L_j^d = X_j$. But this is identical to the pure licensing situation. Thus $-C'(L^d) = p$.

(2) If $p = e$, the firm's minimum total costs are given by

$$TC_j(e) = \min C_j(X_j) + eX_j \text{ subject to } X_j \geqq 0.$$

Firms choose X_j to satisfy $-C_j'(X_j) = e$. This implies $-C'(\Sigma X_j) = e$ and by definition of $\bar{L}$, $\Sigma X_j = \bar{L}$. If $p = e$ a firm will be indifferent between all $L_j \geqq X_j$, thus L^d may take any value greater than or equal to $\bar{L}$.

(3) If $p < e$ a firm could always reduce total costs by buying more licences. Thus, L_j^d is infinite which implies that L^d is infinite, for $p < e$.

From (1), (2), (3) and $-C''(L^d) < 0$, it follows that the price at which demand for licences is equal to L is given by

$$p = -C'(L) > e \text{ for } L < \bar{L}$$
$$p = e \geqq -C'(L) \text{ for } L \geqq \bar{L}.$$

Thus, if L licences are issued, the equilibrium price of licences is given by

$$p = \max\{e, -C'(L)\}.$$

However, under the proposed mixed scheme, e is not set arbitrarily. Rather, it is related to L by $e = D'(L)$. So the equilibrium price of licences under the mixed plan is given by

(4) $p = \max\{D'(L), -C'(L)\}$.

Let L^* and e^* satisfy

$$D'(L^*) = -C'(L^*) = e^*.$$

Then the equilibrium price of licences must equal e^*. But from (2), if $p = e^*$, then $X = L^*$. Therefore X will satisfy $D'(X) + C'(X) = 0$. Also, since marginal costs of clean-up are equalized under the plan, $\sum_{j=1}^{j=n} C_j(X_j) = C(\sum_j X_j)$. Thus L^* and e^* are socially optimal.

Now from (4), $D'' > 0$, and $-C'' < 0$, it follows that $dp/dL < 0$ for $L < L^*$, and $dp/dL > 0$ for $L > L^*$. Thus p achieves a global minimum at L^*, e^* (see Figure 2).[10]

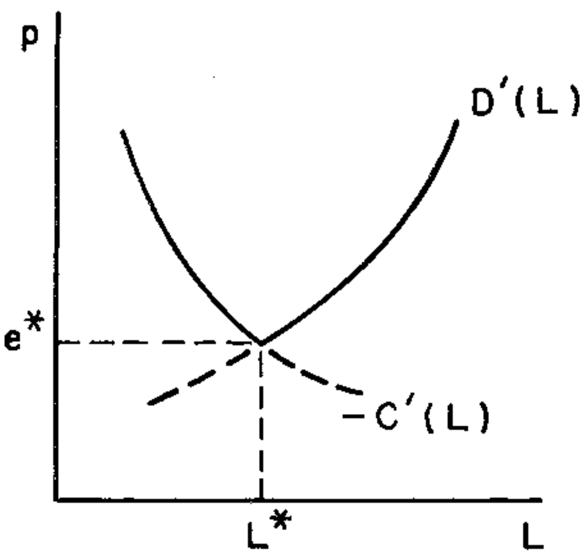

FIGURE 2

Equilibrium price of licences under the mixed plan, $p = \max\{D'(L), -C'(L)\}$

But, each firm's minimum total cost, $TC_j(p)$, is an increasing function of p.[11] Therefore, each firm's total costs are minimized at L^*, e^*.

5. CONCLUDING REMARKS

We will conclude the paper with an informal discussion of certain realistic considerations which might lead a regulatory agency to favour the proposed mixed scheme as a method of eliciting useful information from the units being regulated.

Central to our scheme is that firms know the price of licences will be determined by a competitive market. Thus it turns out that the market can be a powerful mechanism for forcing revelation of true costs. It also may be possible to use the market in another way to learn the true cost possibilities of regulated units. The government could attempt to close the information gap by varying the actual policy parameters and then observing the reaction of firms. However, there are adjustment costs every time a firm is faced with a new set of prices or a new quantity directive. These adjustment costs could be substantial for a large class of problems and thus significantly reduce the desirability of such a policy. This is likely to be the case for the particular regulatory problem we have examined because pollution reduction typically requires substantial investment in control equipment. Such investment may take years to plan and can be extremely costly to reverse once in place. Furthermore, if there are significant lags in adjustment, such iterative regulatory procedures may never converge to the optimal plan. We might also expect that frequent changes in the tax rate or number of licences issued would imply heavy administrative and enforcement costs for the regulatory authority. Thus, a central desirable feature of the mixed effluent charge-licence plan is its ability to hit the right point once-and-for-all.

In practice it is unlikely that a regulatory agency would seek information about hypothetical cost possibilities from every firm which it plans to regulate. While we have not explicitly modelled the fact that information collection is costly, such costs are in part the source of the information gap discussed in this paper. In view of these costs it seems that the government would often wish to select a small sample of firms from which to collect direct information. This would certainly be true in the polar case in which the government had prior information that all firms were identical, but did not know the cost function of any firm.

In the pre-implementation sampling phase the report of each firm might have a large influence on the price of licences. If the government asked just one, or a few firms (as it might wish to do in the polar case just mentioned), then each sampled firm's impact on the final outcome would be large indeed. The proposed mixed plan would be ideal for such situations. Under this plan, no matter how few firms were sampled, each firm would desire that the social optimum be achieved, and would act accordingly.[12]

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NOTES

1. The analysis is essentially unchanged if we interpret $C_j(X_j)$ as $E[C_j(X_j, \phi_j)]$, where ϕ_j is a random variable summarizing the firm's uncertainty about costs of clean-up, and $E[\cdot]$ is the mathematical expectation.

2. If $L<\Sigma_j\bar{X}_j$, the equilibrium price of a licence is given by $p = -C'(L)$. If $L = \Sigma_j\bar{X}_j$; where $\bar{X}_j$ is the amount of pollution produced by firm j in the absence of government controls, then the equilibrium price can take on any value $0<p \leqq -C'(\Sigma_j\bar{X}_j)$. This follows since if $0<p \leqq -C'(\Sigma\bar{X}_j)$ then $L^d = \Sigma\bar{X}_j$. If $L>\Sigma_j\bar{X}_j$, then the equilibrium price for licences is zero.

3. This is assuming no other sanctions for lying are imposed. However, checking up on firms, and fining them if they were lying is probably not an optimal way to induce accurate reporting, since estimating $C_j(\cdot)$ through direct inspection would probably be prohibitively expensive for the government.

4. This observation can be formalized by parameterizing an arbitrary cost function by firm j. Let θ be a parameter in $\hat{C}_j(X_j, \theta)$, the reported cost function of firm j, and let an increase in θ increase the

absolute value of marginal clean-up costs for all values of X_j, i.e. $\partial^2\hat{C}_j/\partial X_j\partial\theta < 0$. This implies that the reported aggregate cost function, $\hat{C}(X, \theta)$, is a function of θ and that $\hat{C}_{X\theta}(X, \theta) < 0$. We will show that if the government sets L such that $D'(L)+\hat{C}_X(L, \theta) = 0$, it will issue more licences when the firm reports that the costs of cleaning up pollution are higher. Differentiating this expression with respect to θ and rearranging terms we get $dL/d\theta = -\hat{C}_{X\theta}/(D''+\hat{C}_{XX}) > 0$.

5. To demonstrate this, let θ be a parameter in $\hat{C}_j(X_j, \theta)$, the reported clean-up cost function of firm j, and let $\partial^2\hat{C}_j/\partial X_j\partial\theta < 0$. This implies that the reported aggregate cost function, $\hat{C}(X, \theta)$, is a function of θ and that $\hat{C}_{X\theta}(X, \theta) < 0$. The government is assumed to act as though total output of pollution is given by

$$-\hat{C}_X(X, \theta) = e. \qquad \text{...(F.1)}$$

Equation (F.1) implicitly defines X as a function of e and θ. Differentiating (F.1) with respect to e and θ.

$$\frac{\partial X}{\partial e} = \frac{-1}{\hat{C}_{XX}} < 0 \qquad \text{...(F.2)}$$

$$\frac{\partial X}{\partial \theta} = \frac{-\hat{C}_{X\theta}}{\hat{C}_{XX}} > 0. \qquad \text{...(F.3)}$$

We will show that if the government sets e to minimize $D(X)+\hat{C}(X, \theta)$, it will choose a lower e when the firm reports lower clean-up costs, i.e. a lower θ. Minimizing this expression with respect to e, we get the first-order condition

$$D'[X(e, \theta)]+\hat{C}_X[X(e, \theta), \theta] = 0. \qquad \text{...(F.4)}$$

Differentiating (F.4) with respect to θ, and rearranging terms, we have

$$\frac{de}{d\theta} = \frac{1}{\partial X/\partial e}\left[\frac{-\hat{C}_{X\theta}}{D''+\hat{C}_{XX}}\,\frac{-\partial X}{\partial \theta}\right]. \qquad \text{...(F.5)}$$

Substituting (F.2) and (F.3) into (F.5) gives

$$\frac{de}{d\theta} = \frac{-\hat{C}_{X\theta}D''}{D''+\hat{C}_{XX}} > 0.$$

6. A mixed pollution control plan with marketable licences and an effluent subsidy was originally suggested by Marc Roberts and Michael Spence in their paper, "Effluent Charges and Licences Under Uncertainty" [1]. However, the parameters of our mixed scheme are chosen differently than in their model. They assumed that firms know or can discover their clean-up cost functions, but the regulatory authority is uncertain about these costs. In their model the government is able to summarize its uncertainty about aggregate clean-up costs by a random variable. The government's problem is posed as choosing a plan which will minimize the expected total costs of pollution. In contrast, our paper poses the problem as discovering the true aggregate cost function and then achieving the optimal level of pollution.

7. It may not always be possible for a firm completely to offset the reports of other firms without reporting a $-\hat{C}_j(X_j)$ which is negative for some values of X_j.

8. I am indebted to Martin Weitzman for suggesting a simplification of my original proof.

9. We will assume $e \geqq -C'(\Sigma\bar{X}_j)$. If $e < -C'(\Sigma\bar{X}_j)$ and $e < p \leqq -C'(\Sigma\bar{X}_j)$, then $L^d = \Sigma\bar{X}_j$.

10. Note that under the mixed plan, no matter what firms report, the ex-post output never exceeds the optimal level, i.e. $X \leqq L^*$.

11. Differentiating $TC_j(p)$ with respect to p, and substituting in the first-order conditions for cost minimization, we get

$$\frac{d[TC_j(p)]}{dp} = C_j'(X_j)\frac{dX_j}{dp} + X_j + \frac{dX_j}{dp}p = X_j > 0.$$

12. This result is in sharp contrast with plans for forcing revelation of costs which rely on price taking behaviour during the planning phase. One such plan would work as follows. All firms would be asked to report their demand functions for licences. Then during the implementation phase they would be legally compelled to buy the number of licences specified by their reported demand functions at the price the government sets. If a firm considered negligible its influence on the price of licences set by the government, then it would report its true demand function (or equivalently, its true marginal cost function). Such a scheme might have substantial enforcement costs because it would rely entirely on the coercion of administrators, the police, and courts for enforcement as opposed to the impersonal enforcement of a market. Furthermore, I believe such a plan which forces all firms to report costs and then binds them to their reports would be politically infeasible.

REFERENCE

[1] Roberts, M. and Spence, M. "Effluent Charges and Licenses Under Uncertainty" (Stanford University, Institute for Mathematical Studies in the Social Sciences, Technical Report No. 146, October 1974).