

Equity and Efficiency vs. Freedom and Fairness: An Inherent Conflict*

YEW-KWANG NG**

I. INTRODUCTION

Economists are well acquainted with the conflict between efficiency and equity¹. The outright pursuit of Pareto efficiency may result in an unacceptably unequal distribution of income. The society may be willing to achieve a more equal distribution at the cost of imposing such inefficiencies as the excess burden of taxation². On the other hand, the conflict between (vertical) equity and efficiency on the one hand, and freedom and fairness (horizontal equity) on the other (which we may call the *E-F conflict* for brevity) has not been adequately discussed.

As far as I know, ATKINSON and STIGLITZ [1976, 1980] first pointed out that an optimal system of indirect taxes may involve different rates, if feasible, on individuals of identical tastes and endowments (horizontal inequity of unfairness). BALCER and SADKA [1982] derived sufficient conditions to rule out unfairness. While the conditions are strong for a model with labour-incentive, they are quite reasonable for a model with only education-incentive.

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** Reader in Economics, Monash University.

1. See, e.g. MEADE [1964], MIRRLFFES [1971], OKUN [1975], COOTER [1978], HARBERGER [1978].

2. The pursuit of equality by the use of preferential treatment between the rich and the poor (in addition to the progressive income tax system) such as the use of differential weights in cost-benefit analysis, and other distortions to the free function of the market economy are however inefficient methods to achieve equality as they have additional distortion effects apart from their disincentive effects (usually forgotten). See NG [1984] for the argument (which takes account of second and third best considerations) that a dollar should be treated as a dollar to whomsoever it goes.

In the present paper, by considering the problem of allocating the population into urban and rural residents, or to military and civilian services, or some other similar problems, we see that unfairness may be optimal under very general conditions even if both labour and education incentives are assumed absent. Moreover, in addition to fairness, freedom may also be violated, i.e. it may be optimal to have selective conscription and not to allow freedom of choice of residency. The elucidation of the *E-F conflict* may thus partly explain the prevalence of certain social policies such as conscription despite their violation of equity, efficiency, freedom, and fairness.

The conflict between optimality and fairness originally pointed out by ATKINSON and STIGLITZ may appear to many people as mainly of academic interest, as the administrative costs and the costs of violating fairness are likely to be overwhelming in comparison to the slight gain in efficiency in the tax system. However, the issue of rural-urban division for countries like China and India, the issue of conscription for most countries, etc. are important practical problems. Moreover, the introduction of such either-or choices increases the extent of the *E-F conflict*. (Pedagogically, this allows our elucidation of the conflict to be put in the very simple terms of Section II below, accessible to all economists.) The *E-F conflict* (which extends the ATKINSON-STIGLITZ conflict to freedom) we discuss here is thus of important and direct policy significance. This exemplifies the general rule that many intellectual contributions may appear practically useless at first but may be shown otherwise with further development and application.

I was drawn to the concept of *E-F conflict* whilst reflecting on my argument against the unfair and illiberal Chinese policy of segregating rural and urban residents. Those born to families of farmers are not allowed to move into towns and be registered as urban residents. I raised the issue of this objectionable practice during a discussion with a number of students from China. A party member, who is rather liberal on many other issues, defended the policy on the ground that, without segregation, rural residents would flood into the cities, creating an impossible situation. I replied that this is so only because of the inequitable policy (somewhat reversed in the past few years) of widening the gap between the living standards of rural and urban areas by wages and prices policies as well as the provision (or the lack) of social services. (One may add the lack of economic development that would provide a healthy absorption of the expanding labour force.)

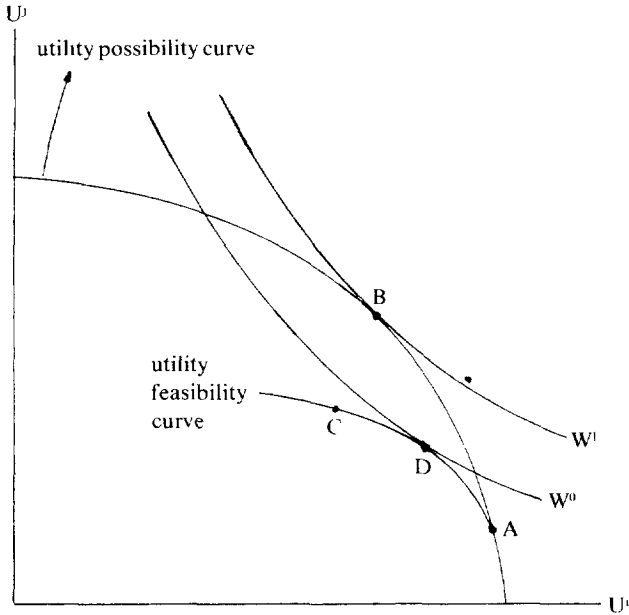
The next day, as I reflected, trying to reassure myself that my reply was correct, I became less confident. While it captures part of the truth, it may not be the whole truth. Even with the operation of the ideal policies with respect to prices, incomes, and social services, including achieving the equity condition of the equality of marginal social welfare significance of income between rural and urban residents, we cannot be sure that the free movement between urban and rural residents will not result in an undesirable degree of crowding or congestion in the urban area. It may be thought that the congestion can be reduced to any desired level by an appropriate taxation/subsidy scheme. However, such a scheme may lead to a level of urban income considered too low to satisfy the equity condition.

The condition for freedom and fairness is in terms of individual indifference (or the equality of *total* utilities) between urban and rural residency, while the condition for equity is in terms of the equality of *marginal* social welfare significance of income. The two conditions can be simultaneously achieved only by sheer coincidence. In fact, there is a presumption that the two will always conflict as most people prefer to live in the urban area even with the same income, and the opportunity of useful consumption is higher in cities, unless the urban area is allowed to be so overcrowded as to balance its advantages.

In a more developed economy, the distinction between urban and rural living is not so high, so that the conflict is negligible. The best policy may then be to opt for freedom and fairness. However, the conflict may be more acute for other problems. For example, the division between military and civilian services especially during wars. Compulsory military service for persons of specified ages is required in many countries. While economists are naturally inclined towards paying the soldier his hire (PARISH and WEISSER [1970]; see also FISHER [1969], ASH, UDIS and MCNOWN [1983]), a soldier's pay might have to be many times higher than the civilian average to attract a sufficient number of volunteers. While other factors are involved in opting for a conscript army, intuitive awareness of the *E-F conflict* may be a possible explanation.

The elucidation of the *E-F conflict* allows us to realize that social choices that are inconsistent with equity, efficiency, freedom, and fairness need not necessarily be sub-optimal. Since the four objectives cannot be simultaneously achieved, sacrificing a little of each may yet be the optimal policy. Needless to say, the *E-F conflict* does not justify all departures from equity, efficiency, freedom, and fairness.

Figure 1



II. THE E-F CONFLICT

The familiar conflict between equity and Pareto efficiency may be illustrated in *Figure 1*. If all the conditions for Pareto efficiency are satisfied, the economy is at a point on the utility possibility curve (*UPC*). Depending on the initial endowment, it may be at a point such as *A*. According to the specific welfare contours depicted, the (social) welfare maximum point is at *B* where the slope of the welfare contour ($-W_i/W_j$, where $W_i \equiv \partial W / \partial u^i$) equals the slope of the *UPC* ($-u_x^i/u_x^j$, where u_x^i is the marginal utility of income for person *i*). At the maximal welfare point, we thus have

$${}_iW_j = u_x^i / u_x^j, \text{ or } W_i u_x^j = W_j u_x^i$$

or the interpersonal equality of the marginal welfare of income (through personal marginal utility). This condition is the same as Condition (4) in the formal model of Section III.

It is normally infeasible to move from A to B. Due to disincentive effects, administrative costs, etc., the attempt to move to B will lead us to C instead. Rather than accepting this substantial loss in efficiency, society may settle on a compromise (between equity and efficiency) at the point D.

It is however not our objective to discuss the familiar conflict between equity and efficiency. We will thus highlight our central issue and simplify our discussion by abstracting from this familiar conflict. That is, we assume the absence of disincentive effects and administrative costs such that the welfare maximal point B is attainable. In a model of many individuals, the feasibility of attaining B also depends on the feasibility of personspecific lump-sum taxes/subsidies, which will be assumed. This last feasibility is not required if we assume that the marginal welfare of income ($W_i u'_x$) is the same for all persons of the same income. An appropriate system of (anonymous) income taxes would then be sufficient to attain the welfare maximal point.

Another innocuous simplifying assumption is that of identical individual taste. Conceivably, differences in taste might be such that a certain section of the population prefer to serve in the military and the rest prefer to be in the civilian sector in such a way that the division is efficient, with income levels satisfying the equality of the marginal welfare of income. Such a happy situation may approximately be the case in peace time but is unlikely to be so in war time when more soldiers are required unless patriotism dramatically lowers the supply curve. Since the introduction of differences in taste does not resolve the conflict, at least not generally, we will simplify the argument by ignoring taste differences.

Now let us discuss our concepts more precisely. An *allocation* specifies the amounts of all goods (including jobs and places of residence where relevant) consumed by each individual, and the amount of public goods in each locality. A (perfectly *efficient*) allocation (or *efficiency*), requires the non-existence of an alternative allocation that makes some individual better off without making any individual worse off. A (perfectly) *equitable* allocation (or *equity*) requires the equalization across all individuals of the marginal welfares of income (marginal utilities weighted by social welfare weights), continuity being assumed³. *Fairness*

3. The relaxation of continuity does not in general alter our conclusion.

of an allocation requires that similar individuals (of the same utility function) enjoy the same amount of total utility in the same situation. *Freedom* requires, among others, letting people choose their jobs and their places of residence at their own free will.

Note that the above are *necessary* conditions for *perfect* efficiency, equity, fairness, and freedom respectively. Thus, the violation of a necessary condition for say (perfect) efficiency does not necessarily mean complete inefficiency. But since our aim is only to establish the existence of *some* conflict, it does not matter. On the other hand, the necessity (not sufficiency) nature of our conditions leaves open for anyone who may wish to add additional conditions to the concepts. For example, one may wish to require *fairness* to involve treating all individuals (whether of the same utility function) in the same situation similarly. But since this implies the same total utility for similar individuals, our condition allows for more generality. Similarly, while different people may have somewhat different ideas of what freedom exactly means, most will agree that full freedom at least requires free choice of jobs and the place of residence.

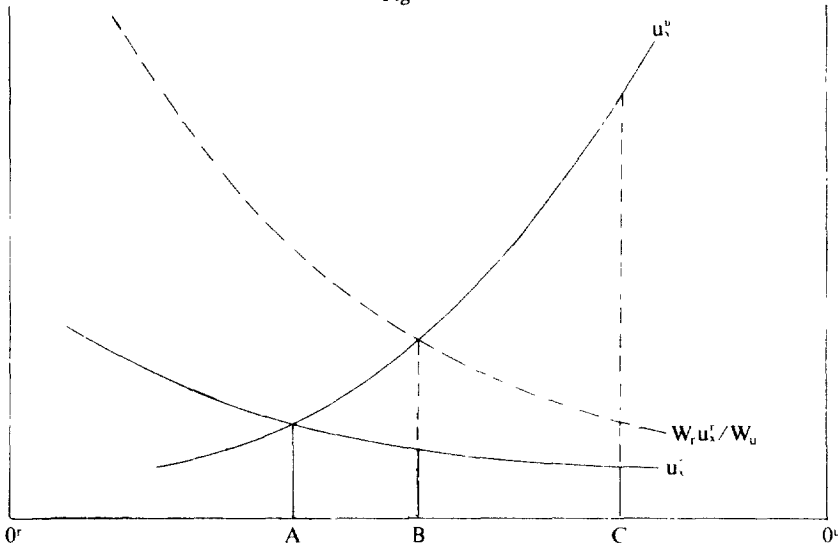
We are now ready to state our proposition.

Proposition: The achievement of efficiency and equity is in general inconsistent with freedom and fairness even if disincentive effects and administrative costs are absent (i.e. lump-sum taxes/subsidies are feasible).

Proof: The existence of the *E-F conflict* in one example satisfying the required conditions (absence of disincentive effects, administrative costs) is sufficient to establish the proposition. This (quite general) example is illustrated in *Figure 2*. Suppose that, at the optimal division of the population between urban and rural areas (taking account of all costs and benefits), the amount of income available to be allocated between a rural and an urban resident⁴ is represented by the distance $O'O^u$ in *Figure 2*. The income allotted to the rural resident is measured from O' and that to the urban resident is measured *leftward* from O^u . The curves u_r^r , u_x^u are the marginal utility of income curves for the rural and

4. If the number of urban residents is not equal to that of rural residents this amount of income is not defined until the desired distribution between urban and rural per capita income is determined. The simultaneous attainment of efficiency, distribution, and the urban-rural division is discussed in Section III.

Figure 2



the urban residents respectively⁵. Despite our assumption of identical taste, the two curves are not symmetrical since urban living is different from rural living. Typically, both the total and the marginal utilities are much lower for the rural resident due to the poor facilities there and the lack of useful opportunities to spend money.

The poor facilities may be but need not be due to a neglect of the rural area. Since the rural area is less densely populated, we expect fewer public facilities there even if the Samuelsonian optimality condition for the supply of public goods is satisfied both for the urban and the rural areas.

5. To speak in terms of marginal utilities we are using a cardinal utility framework. It is true that the efficiency conditions (Eqs. 5-7 in Section III) can be expressed in the form of MRS, but the equity condition (Eq. 4) for a welfare maximum has to be expressed in the form of the welfare weighted marginal utilities. As the existence of an individualistic BERGSON-SAMUELSON SWF (Eq. 1 in Section III) presupposes interpersonal comparison of cardinal utilities [KEMP and NG 1976, 1977, PARKS 1976, POLLAK 1979 and in particular NG 1985], we lose no generality but gain in pedagogic simplicity in using a cardinal utility approach. On the cardinal measurability of utilities in principle and in practice, see NG [1975]. Those who do not like the marginal utility approach may use Figure 3 instead.

Given the situation depicted in *Figure 2*, a pure utilitarian SWF would divide income at the point A where the marginal utilities are equalized. But consider even an egalitarian (in terms of income) SWF dictating the division at the mid point B or a SWF so much sympathetic to the low utility rural resident (W_r/W_u much larger than one, where $W_r \equiv \partial W/\partial u^r$ is the marginal social welfare weight for the rural resident, and W^u that for the urban resident) as to dictate an approximately equal division (say at the point B).

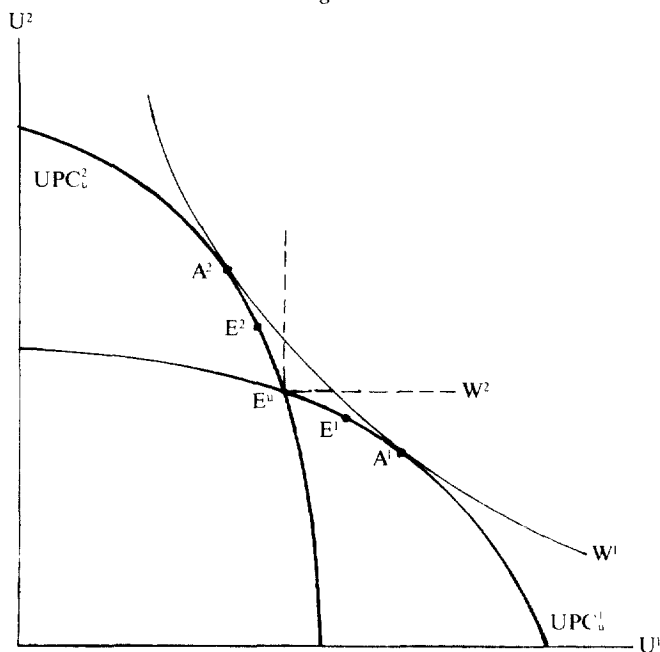
Suppose, as is likely, that urban living offers so many more opportunities that, with equality between urban and rural incomes, people much prefer to be urban residents. To make people indifferent between urban and rural livings, the per capita income of rural residents may have to be much higher than that of urban residents (a point like C). This may grossly violate the optimality condition for the equality of the marginal welfare in income. If society does not want this to happen, freedom of choice of residence may have to be restricted.

Alternatively, if the incomes of urban and rural residents are equalized or determined at levels to satisfy the equality of marginal welfare, and if there is also freedom of choice of residence, too many people may flood to the cities. While an urban resident may still enjoy the same utility level as a rural resident, the marginal contribution to social welfare of an urban resident may well be negative, taking account of the external diseconomies imposed on others in the form of congestion and reduced job opportunities. In other words, the efficient division of the population between urban and rural residents may have to be sacrificed. In general, freedom and fairness are thus inconsistent with the achievement of either equity of efficiency or both. QED.

It may be mistakenly thought that equity and efficiency can be achieved without sacrificing freedom by the imposition of a congestion tax on urban residents. At an appropriate level, a congestion tax can achieve the Pareto optimal division of the population between urban and rural residency. However, except by coincidence, the net of tax incomes need not be consistent with the equality of marginal welfare of income unless the social welfare function (SWF) is Rawlsian, giving an infinitely large weight to the individual with the lowest utility.

If we go along with the maximization of ex-ante expected utility, the choice in *Figure 2* would be the utilitarian one of dividing income at the point A [cf. HARSANYI 1953, 1955]. A person who does not know whether

Figure 3



he will be a rural or an urban resident would maximize his expected utility by agreeing in advance that, if his lot is to be a rural resident, he will receive $O'A$ and O^uA if urban. Thus, the choice even at B involves losses in terms of *ex-ante* expected utility, not to mention the choice at C .

An alternative to the marginal utility approach of *Figure 2* is the utility possibility curves of *Figure 3*, where UPC_u^1 is the utility possibility curve if individual 1 is in the urban area, and individual 2 is in the rural area, and UPC_u^2 is that for the reverse situation. The points for equal income levels might be at E^1 and E^2 respectively. But a moderately concave *SWF* as represented by the welfare contour W^1 is indifferent between A^1 and A^2 , both involving a higher income level to the urban residents. A Rawlsian *SWF* as represented by the angular welfare contour W^2 chooses however a point of equal utility E^u .

An explanation why the urban-rural or the military-civilian choice tends to create *E-F conflict* is that the either-or choice creates in general non-convexity, a condition conducive to unfairness. That an indivisible

either-or choice creates non-convexity was first discussed in NG [1965]. But the discussion of HILLMAN and SWAN [1979, 1983] is closer in its presentation to *Figure 3* above.

While the example of rural-urban residency above refers to the externality of congestion, etc. to increase the magnitude of the *E-F conflict*, the existence of external effects is not essential. Unless the distribution of personal income in accordance to the marginal productivities of factors owned happens to correspond to the interpersonal equalization of the marginal welfare of income, the *E-F conflict* is still present in the absence of external effects. This can be best seen in the soldiers vs. civilians example. In time of war, the productivity (in the wide sense of safeguarding properties and lives) of defence may be so high and the life of a soldier so risky that, if a soldier is paid his marginal product, his income may have to be many times that of a civilian before voluntary recruitment will result in the efficient number of soldiers. But this may grossly deviate from the interpersonal equality of the marginal welfare of income. On the other hand, if a soldier is paid less than his marginal product, voluntary recruitment may not yield sufficient soldiers to satisfy efficiency.

In our simple analysis, while people prefer urban to rural residency, their preference (including their adaptation to the living conditions, capacity to enjoy, etc.) itself is not affected by their place of residence. With such a simple assumption, the *E-F conflict* can easily be resolved if it is not too costly to rotate urban and rural residents. Thus, if it is optimal to have m (a fraction) of the population living in the urban and $(1 - m)$ in the rural area, each and every individual can spend m of their life in the urban and $(1 - m)$ in the rural area. While in the urban area, he receives an income equal to O^uA in *Figure 2* and while in the rural area, he receives O^rA . Then the total lifetime utility is maximized and equalized across individuals.

However, apart from the physical costs (transportation, change of accommodation, etc.) of the rotation, there are the more important considerations of preference formation such as adaptation, accustomization, learning, etc. A person born and brought up in the city (countryside) is more accustomed to urban (rural) living and working conditions. Due to the better facilities and opportunities in the urban area, a farmer's son or daughter may yet prefer to move to a city. But the gain in utility of such a transfer may be much smaller than the loss in utility and produc-

tivity suffered by the reverse transfer. A person accustomed to the facilities of urban living may suffer a tremendous loss if posted to a rural area. This was overwhelmingly witnessed by the 'sent-down' youths in China. Despite strong indoctrination and years of hardship in the countryside, most of these 'sent-down' youths could not accustom themselves to rural living. Now, virtually all of them have been allowed to return to the cities.

A further consideration is that the relatives and friends one has are usually related to one's place of residence. It would be quite impracticable to transfer all friends and relatives to the same village since the relationships of friends and relatives are interconnected rather than in the form of distinct unconnected groups. This consideration increases the cost of rotation.

For the division between military and civilian services, there is of course the significant cost of training, both to the government and to the trainees. Unless it is deemed desirable to have the whole population trained (as in a very threatened country), it may be too costly to have a system of rotation in military services. It may thus be better (in terms of maximizing expected utility) to have a system of selective conscription. Typically, young men between the ages of 18 and 30 are the principal target. But economists may be in favour of making the probabilities of being drafted subject to purchase and sale. A person hating military service very much can then have his probability reduced at the cost of paying someone else whose probability is correspondingly increased⁶.

Most people regard discrimination by sex and age with respect to military service not necessarily unfair. Since women would probably suffer more hardship and contribute less to the defence of the nation if drafted, it makes sense to draft young men who have fewer family responsibilities. However, most people regard the possibility of reducing one's probability of being drafted by paying someone else unfair, since the rich could then afford to be free from military services. This belief ignores the fact that, by widening the sphere open to monetary transaction, the utility of having more money is increased and hence the rich can be taxed more without increasing the disincentive effect. Thus,

6. See BERGSTROM [1982] for the argument that an army recruited by a draft lottery with probabilities of being drafted subject to purchase and sale is ex-ante Pareto-superior to an all-volunteer army.

provided that the widening of the sphere open to monetary transaction is accompanied (at least in the long run) by increasing the progressivity of the income taxation system, both the rich and the poor can be made better off [NG 1984]. Nevertheless, before people are persuaded by this argument, making the draft probabilities subject to sale may weaken the morale of the soldiers. Typically, one is prone to think, 'since the rich do not have to sacrifice to defend the country, why should I fight?' This reasoning is wrong because those who pay to reduce their draft probabilities contribute to defence by reducing resources at their command (both through paying and through a higher tax). Moreover, it is Pareto-optimal for them to make their contribution this way. However, before people accept this argument, the morale weakening effect will persist. Taking account of this morale effect, it may then not be desirable to make the draft probabilities subject to sale.

While the conflict between equity/efficiency and fairness (but not freedom) with respect to military services may be partially solved by a selective draft (at a considerable cost to efficiency if the system of sale in draft probabilities is not acceptable to the population), that with respect to the rural/urban residency is more difficult to solve by a similar method. This is so for the following reason. The children of soldiers do not lead the life of soldiers before they themselves become soldiers. But the children of rural (urban) residents are themselves rural (urban) residents by the fact of living with their parents. They are thus accustomed to their respective places of residence and interwoven into their respective groups of friends and relatives. As discussed above, it may then be very costly to transfer them.

From the above considerations, if it is desirable to restrict the number of urban residents, the most efficient and equitable (but not fair) method may be to disallow the movements of rural residents into the urban area except for those who distinguish themselves through the education process as capable of contributing significantly in the urban area. This is the policy practiced by the Chinese government. Such a policy is of course not without significant costs. The inefficiency created by the almost complete immobility must be very substantial. More importantly, there are the issues of freedom and fairness. Is the gain in restricting an excess influx into cities worth the costs involved? The Chinese policy is of course also partly if not mainly a matter of administrative and political control of the population. We shall however ignore such factors.

Consider the following two situations: (A) If a person, by accident of birth, happens to be a boy, he is destined to serve in the armed forces on the reasoning that a boy makes a better soldier than a girl; (B) If a person, by accident of birth, happens to be a farmer's child, he or she is destined to live in the rural area on the reasoning that a person brought up in a rural family makes a better farmer than one from the city. For reasons difficult to explain, most people (myself included) find situation A not necessarily unfair but find situation B very unfair. The *E-F conflict* is thus made more acute for the issue of rural-urban residency of an underdeveloped economy like China or India.

Roughly speaking, China opted for control while India opted to have freedom and fairness in the choice of residency. The cost of this freedom and fairness is the overcrowded situation in most Indian cities. The marginal migrants into cities probably contribute negatively to production and also have per capita income below those remaining in the countryside. They nevertheless choose to go to the city as there is some chance they may be successful. Each migrant into the city imposes significant external diseconomies on others in terms of the congestion of public facilities and of the reduction in the probability of success (in getting jobs, etc.). The average living standard is much higher in the city than in the countryside due to the choice of most of the well-to-do to live in cities. But the per capita incomes of many of the low income groups may well be not much higher or even lower than of those in the countryside. Given the need for and the usefulness or higher income for urban living, a congestion tax on urban residency sufficiently high to internalize the externality completely may be too high for the low income groups to be deemed 'equitable' (i.e. equalization of the marginal welfare of income).

Given the choice to have freedom and fairness as in India, an optimal policy may be a compromise between equity and efficiency (internalization of externalities). On the other hand, given the alternative choice largely to ignore freedom and fairness as in China, an optimal policy may be to have more-or-less complete efficiency (in urban/rural division) and partial equity to partly compensate the rural residents for the unfairness of rural residency imposed on them. That is, rural residents should have incomes high enough to have lower marginal utilities than urban residents. But since their respective marginal utility curves are likely to be roughly as depicted in *Figure 2*, this does not necessarily mean that rural residents should have higher incomes.

The choice whether to opt for freedom or not may be very difficult to make. It may be thought that, if one agrees to the maximization of ex-ante expected utility, then the necessity of sacrificing freedom is obvious where the two conflict. It may thus appear that freedom and fairness have no role to play in a utilitarian or even a more general welfarist (except the extreme Rawlsian) *SWF*. However, when we consider a broader concept of utility and/or the long-run effects on utilities, the picture changes. This is so because the very lack of freedom may impose a utility loss not captured in a narrow utility analysis of *Figure 2* or Section III below. Moreover, the very choice to ignore freedom and fairness may affect the future prospect by changing institutions and people's attitudes. Thus, the concern for the issues of freedom and fairness need not necessarily be a non-utilitarian one.

III. A SPECIFIC MODEL

To analyse the *E-F conflict* more rigorously in a limited space, we shall deal with a simple model (retaining the simplifying assumptions of identical taste, no disincentive effects, etc.) and ignore many complications discussed informally above.

1. *The Model*

To facilitate the use of calculus, assume an uncountable number of individuals each being identified by a point on the closed unit interval $(0,1)$. This assumption (or its relaxation) is innocuous and is used purely for mathematical convenience. The (real-valued) utility of individual i ($0 \leq i \leq 1$) is denoted u^i . The function that maps each i into u^i is denoted U , i.e. U maps the unit interval $(0,1)$ into the set of real numbers, R . Denote as $L(0,1)$ the set of all such integrable functions. Take social welfare W as a function of all individual utilities⁷. This function $W(U)$ maps $L(0,1)$ into R . It is thus a functional. However, it differs from a social welfare functional used in the social choice literature⁸. Rather, it is

7. This common practice is called welfarism and attacked by SEN [1979]. For a defence of welfarism against Sen's attack, see NG [1981, 1985].

8. This maps the s individual preference functions into a real-valued social welfare, i.e. the social welfare functional maps the set of all admissible individual preference functions into R .

the counterpart (for the case of a continuum of individuals) to the Bergson-Samuelson social welfare function (*SWF*)

$$W = W(u^1, u^2, \dots, u^s) \tag{1}$$

where s is the number of individuals. In order not to be confused with a social welfare functional in its social choice sense and to emphasize its similar nature with a common *SWF*, I shall just call the functional $W(U)$ a *SWF*.

Denote x^i as the private good consumption of individual i , and $X \equiv \int_0^1 x^i di$ as the aggregate private good consumption (= production). Also denote Y, Z as the public good provision in the city and in the countryside respectively. Since the population is homogeneous, let the interval $(0, m)$ be living in the city and the interval $(m, 1)$ be living in the countryside. While individuals have identical welfare functions, their welfare depends on their place of residence. Thus,

$$\begin{aligned} u^i &= u^u(x, Y, m) && \text{for } 0 \leq i \leq m \\ &= u^r(x^r, Z, n) && \text{for } m < i \leq 1 \end{aligned} \tag{2}$$

where the superscripts u and r stand for urban and rural respectively, and $n = 1 - m$.

Ignoring variable intermediate inputs we write the production constraint as follows, since the marginal productivities of urban and rural residents may differ,

$$F(X, Y, Z) = f(m, n) \tag{3}$$

2. Optimality Conditions

To derive the conditions for a social welfare maximum, we maximize $W(U)$ with respect to x^i, Y, Z , and m subject to (3). Assuming that the second-order conditions are satisfied, we concentrate on the following first-order conditions. (See the appendix for their derivation.)

$$W_i u_x^i = W_j u_y^j, \quad \forall i, j \tag{4}$$

$$\int_{0 \leq i \leq m} (u_y^i / u_x^i) di = F_Y / F_X \tag{5}$$

$$\int_{m < i \leq 1} (u_z^i / u_x^i) di = F_Z / F_X \tag{6}$$

$$\begin{aligned} [u^u(x_u^m, Y, m) - u^r(x_r^m, Z, n)] / u_x^m &= \int_{m < i \leq 1} (u_n^i / u_x^i) di - \int_{0 \leq i \leq m} (u_m^i / u_x^i) di \\ &+ (x_u^m - x_r^m) - (f_m - f_n) / F_X \end{aligned} \tag{7}$$

where a subscript denotes partial differentiation, e.g. $W_i = \partial W / \partial u^i$, $u_x^i \equiv \partial u^i / \partial x^i$, $F_X \equiv \partial F / \partial X$, etc. except for x_u^m and x_r^m which are the private good allocations to the marginal resident m , if transferred to the urban area (x_u^m) and if left in the rural area (x_r^m). These two values may differ since the utility of this person m depends on his residence and hence different allocation of the private good may be required to satisfy (4). The term $(f_m - f_n) / F_X$ is the difference in marginal productivity between an urban and a rural resident, using as numeraire the marginal cost of the private good production, F_X which we may normalize to equal unity.

Eq. (5) is the Samuelsonian condition for the optimal supply of public goods to the urban area, requiring the equality of $\Sigma MRS = MRT$ or just the marginal cost of providing Y , with F_X normalized to equal one. Eq. (6) is the same condition for the rural area.

Eq. (4) is the equity (or 'welfare-efficiency') condition in the allocation of the private good, requiring the equality of the marginal (social) welfare significance of the private good across all individuals. Since our SWF is Paretian, this welfare significance is through the marginal utility of consumption u_x^i . This condition is usually referred to as the interpersonal *equity* condition. However, since the question of fairness discussed here is quite different to this condition, we may also call Eq. (4) the welfare-efficiency condition for the (interpersonal) allocation of the private good. It is an efficiency condition in the maximization of the $SWF W(U)$ though the consideration of interpersonal equity could have been reflected in the values of W_i . If the SWF happens to be utilitarian such that $W(U) = \int_{0 \geq i \geq 1} u^i di$, the predominantly welfare-efficiency nature of (4) becomes more apparent. Alternatively, following HARSANYI [1953, 1955], we may say that the ex-ante (i.e. before knowledge of which individual one will become) expected utility of an individual has not been maximized unless (4) is satisfied, with each W_i a constant.

Eq. (7) is the 'club membership' condition concerning the optimal division of the population into the urban and the rural⁹. The LHS is the gain in total utility of the marginal resident transferred from the rural to

9. Cf. BUCHANAN [1965], NG [1973], HILLMAN and SWAN [1979, 1983] and SANDLER and TSCHIRHART [1980]. NG [1973] deals with general Pareto efficiency as such, not with a specific maximum maximorum, and hence his condition need not include the difference in the private good allocation $x_u^m - x_r^m$. The difference in the general Pareto efficiency approach and the specific welfare maximum approach is explained in NG [1978].

the urban area *with* his private good allocation adjusted in accordance to the requirement of (5), i.e. from x_r^m into x_u^m . This gain in utility is measured relative to his marginal utility of the private-good consumption, and is thus in the form of MRS. The first two terms (with integrals) on the RHS measure the cost of the transfer in the form of the cost of increased congestion (assuming $u_m' < 0$ at the margin) imposed on all urban residents minus the *reduction* in the congestion on the rural area. The remaining terms in (7) measure the cost of the transfer in terms of the increased consumption of the marginal transferee minus his increase in marginal productivity (both of the increases may be negative).

3. The E-F Conflict

Whether we assume a centrally directed economy where the government can directly determine each x^i irrespective of the marginal productivity of i , or whether we assume a market economy where the government can impose a different lump-sum tax on each individual, the government has, given the value of X , the same number of instruments (= the number of individuals less one) here as the number of equations to satisfy in (4).

While our formal model deals with an uncountable number of individuals to facilitate the use of calculus, the number of individuals is certainly finite in any real economy. If we impose the condition of anonymity such that $W_i = W_j$ if $u^i = u^j$ for all i, j , the number of equations in (4) is reduced to one, namely, $W_r u_x^r = W_u u_x^u$, where r and u stand for rural and urban residents respectively, recalling our assumption of identical utility functions (2). Given the value of X , the government still has the same number of instrument (one) as the number of equations to satisfy in (4). Once the income for either sector is decided, the other follows.

Next, Y and Z can be chosen to satisfy (5) and (6). (While all these have to be done simultaneously, it is of pedagogic value to proceed one by one.) If, in addition, m is chosen to satisfy (7), the production constraint (3) determines the value of X . (Recall that $n \equiv 1 - m$.) Thus, there are as many optimality conditions to satisfy as the number of free policy instruments, *only if* the government can also choose m .

If individuals are free to choose their places of residency, it will only be by chance that the welfare optimality conditions can be satisfied. It

may be thought that taxes/subsidies can be used to correct for the non-optimality created by individual free choice. For example, if too many individuals choose to reside in the urban area thereby creating excessive congestion, a congestion tax can be imposed on urban residency. The use of such a method can certainly help us to achieve the optimal balance (between urban and rural areas) condition (7), but its use precludes the attainment of condition (4) except for coincidence.

The decision to have freedom of choice of residence thus involves in general one instrument less than the number of optimality conditions. Either some of the efficiency conditions (5)–(7) or the equity condition (4) or both have to be left unsatisfied, except when they are satisfied by coincidence.

IV. CONCLUDING REMARKS

As noted at the end of Section II, freedom and fairness may be valued by a purely utilitarian *SWF*, especially in the long run. For those who are willing to value freedom and fairness over and above their contribution to individual welfares (broadly defined), the infringement of freedom and fairness is then doubly 'costly'. Thus, many people may be unwilling to yield an inch in freedom and fairness unless their conflict with equity and efficiency is very strong, i.e. unless the insistence on freedom and fairness will exact a huge loss of equity and/or efficiency.

In principle, we may extend our analysis to many other issues, such as the division between blue and white collar jobs, between different geographical regions, etc. Potential conflict between freedom and fairness with equity and efficiency can still be established for such issues. For most issues, however, the conflict is likely to be minimal relative to the importance of freedom and fairness. It is thus desirable to insist on freedom in all issues except some special issues where the conflict is strong such as the issue of military services.

While it is important to avoid the unjustified infringement of freedom on insufficient grounds, the recognition of the conflict of freedom and fairness with equity and efficiency explains, at least partly, certain social choices (such as conscription) which may appear to economists as irrational because they violate equity, efficiency, freedom, and fairness. Since these objectives cannot all be attained, sacrificing a bit of each and every one of them may yet be an optimal policy.

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APPENDIX

Maximizing $W(U)$ subject to (3), we have our Lagrangean

$$L = W(U) - \Theta[F(X, Y, Z) - f(m, n)]$$

where the value of U is specified in (2). The first-order conditions with respect to x^i , Y , Z and m are respectively,

$$W_i u^i = \Theta F_{x_i}, \quad \forall i \in (0,1) \quad (A1)$$

$$\int_{0 \geq i \geq m} W_i u^i_Y di = \Theta F_Y \quad (A2)$$

$$\int_{m > i > 1} W_i u^i_Z di = \Theta F_Z \quad (A3)$$

$$\begin{aligned} W_m [u^u(x_u^m, Y, m) - u^r(x_r^m, Z, n)] + \int_{0 \geq i \geq m} W_i u^i_m di - \int_{m > i > 1} W_i u^i_m di \\ = \Theta F_X(x_u^m - x_r^m) - \Theta(f_m - f_r) \end{aligned} \quad (A4)$$

where $W_i = \partial W / \partial u^i$, $u^i = \partial u^i / \partial x^i$, $F_X \equiv \partial F / \partial X$, etc., and x_u^m is the private good allocated to the marginal resident transferred to the urban area, and x_r^m is his allocation if he remains in the rural area. These two values may differ since his utility depends on his residence and hence different allocation of the private good may be required to satisfy (A1). The term $(f_m - f_r)$ is the difference in marginal productivity between an urban and a rural resident.

Since $W_i u^i$ is equal to ΘF_{x_i} for each and every i , we may divide (A2)–(A4) through by either $W_i u^i$ or ΘF_{x_i} to express these conditions in the form of the marginal rates of substitution (MRS), and also rewrite (A1), yielding (4)–(7) in the text

REFERENCES

- ASH, COLIN; UDIS, BERNARD and MCNOWAN, ROBERT F.: Enlistments in the all-volunteer force: a military personnel supply model and its forecasts. *American Economic Review*, Vol. 73 (1983), March, pp. 145–155.
- ATKINSON, ANTHONY B. and STIGLITZ, JOSEPH E.: The design of tax structure: direct versus indirect taxation. *Journal of Public Economics*, Vol. 6 (1976), pp. 55–75.
- ATKINSON, ANTHONY B. and STIGLITZ, JOSEPH E.: *Lectures on Public Economics* (New York: McGraw-Hill), 1980.
- BALCER, YVES and SADKA, EPHRAIM: Horizontal equity, income taxation and self-selection with an application to income tax credits. *Journal of Public Economics*, Vol. 19 (1982), pp. 291–309.
- BERGSTROM, TED: *Soldiers of fortune?* Typescript, 1982.
- BUCHANAN, JAMES M.: An economic theory of clubs. *Economica*, Vol. 32 (1965), Feb., pp. 211–212.
- COOTER, ROBERT: Optimal tax schedules and rates: Mirrlees and Ramsey. *American Economic Review*, Vol. 68 (1978), Dec., pp. 756–768.
- FISHER, ANTHONY C.: The cost of the draft and the cost of ending the draft. *American Economic Review*, Vol. 59 (1969), June, pp. 239–254.

- HARBERGER, ARNOLD C.: On the use of distributional weights in social cost-benefit analysis *Journal of Political Economy*, Vol. 86 (1978), April, pp. S87-120.
- HARSANYI, JOHN C.: Cardinal utility in welfare economics and in the theory of risk-taking. *Journal of Political Economy*, Vol. 61 (1953), Oct., pp. 434-435.
- HARSANYI, JOHN C.: Cardinal welfare, individualistic ethics, and interpersonal comparison of utility. *Journal of Political Economy*, Vol. 63 (1955), Aug., pp. 309-321.
- HILLMAN, ARYE L. and SWAN, PETER L.: Club participation under uncertainty. *Economic Letters*, Vol. 4 (1979), pp. 307-312.
- HILLMAN, ARYE L. and SWAN, PETER L.: Participation rules for Pareto-optimal clubs. *Journal of Public Economics*, Vol. 20 (1983), Feb., pp. 55-76.
- KEMP, MURRAY C. and NG, YEW-KWANG: On the existence of social welfare functions, social orderings, and social decision functions. *Economica*, Vol. 43 (1976), Feb., pp. 59-66.
- KEMP, MURRAY C. and NG, YEW-KWANG: More on social welfare functions: the incompatibility of individualism and ordinalism. *Economica*, Vol. 44 (1977), Feb., pp. 89-90.
- MEADE, JAMES E.: *Efficiency, Equality, and the Ownership of Property* (London: Allen & Unwin).
- MIRRELES, JAMES A.: An exploration in the theory of optimal income taxation. *Review of Economic Studies*, Vol. 38 (1971), April, pp. 175-208.
- NG, YEW-KWANG: Why do people buy lottery tickets? Choices involving risk and the indivisibility of expenditure. *Journal of Political Economy*, Vol. 73 (1965), Oct., pp. 530-535.
- NG, YEW-KWANG: The economic theory of clubs: Pareto optimality conditions. *Economica*, Vol. 40 (1973), Aug., pp. 291-298.
- NG, YEW-KWANG: Bentham or Bergson? Finite sensibility, utility functions and social welfare functions. *Review of Economic Studies*, Vol. 42 (1975), Oct., pp. 545-569.
- NG, YEW-KWANG: Optimal club size: a reply. *Economica*, Vol. 45 (1978), Nov., pp. 407-410.
- NG, YEW-KWANG: *Welfare Economics. Introduction and Development of Basic Concepts* (London: Macmillan), 1979/1983.
- NG, YEW-KWANG. Welfarism: a defence against Sen's attack. *Economic Journal*, Vol. 91 (1981), June, pp. 527-530.
- NG, YEW-KWANG: Quasi Pareto social improvements. *American Economic Review*, Vol. 74 (1984), Dec.
- NG, YEW-KWANG: Some fundamental issues in social welfare. In GEORGE FEIWEL (ed.): *Issues in Contemporary Microeconomics and Welfare*, Macmillan, 1985.
- OKUN, ARTHUR M.: *Equality and Efficiency. The Big Tradeoff* (Washington: Brookings Institution), 1975.
- PARISH, ROSS M. and WEISSER, MENDEL: Paying the soldier his hire: the economics of abolishing conscription. *Current Affairs Bulletin*, Vol. 46 (1970), July 27.
- PARKS, ROBERT P.: An impossibility theorem for fixed preferences: a dictatorial Bergson-Samuelson welfare function *Review of Economic Studies*, Vol. 43 (1976), Oct., pp. 447-450.

EQUITY AND EFFICIENCY VS. FREEDOM AND FAIRNESS

- POLLAK, ROBERT A.: Bergson-Samuelson social welfare functions and the theory of social choice. *Quarterly Journal of Economics*, Vol. 93 (1979), Feb., pp. 73-90.
- SANDLER, TODD and TSCHIRHART, JOHN: The economic theory of clubs: an evaluative essay. *Journal of Economic Literature*, Vol. 18 (1980), Dec., pp. 1481-1521.
- SEN, AMARTYA K.: Personal utilities and public judgements; or what's wrong with welfare economics? *Economic Journal*, Vol. 89 (1979), Sept., pp. 537-558

SUMMARY

Freedom (in the choice of jobs or the place of residency) and fairness (horizontal equity) may be inconsistent with the attainment of (vertical) equity (interpersonal equalization of marginal welfare of income) and/or efficiency, even abstracting from familiar factors (incentive effects, administrative costs) accounting for the equity-efficiency trade-off. The imposition of congestion taxes on urban residency or the offer of high salaries to attract volunteer soldiers may achieve an efficient division of the population consistent with freedom and fairness but is unlikely to coincide with the equalization of the marginal welfare of income. This conflict may partly explain the urban-rural segregation in China and the prevalence of conscription

ZUSAMMENFASSUNG

Freiheit (wie zum Beispiel in der Wahl des Berufes oder Wohnsitzes) und Fairness (horizontale Gerechtigkeit) können unvereinbar sein mit dem Ziel der (vertikalen) Gerechtigkeit und/oder der Effizienz, selbst wenn man von den bekannten Faktoren (wie Ansporneffekten oder Administrationskosten) abstrahiert, die für den Trade-off zwischen Gerechtigkeit und Effizienz verantwortlich sind. Freiheit und Fairness erfordern Indifferenz zwischen den Alternativen (Gleichheit des Gesamtnutzens), während (vertikale) Gerechtigkeit zwischenpersönliche Gleichsetzung des Grenznutzens der Einkommen voraussetzt. Die Aufbürdung von Ballungssteuern auf städtische Wohnsitze oder eine Solderhöhung als Ansporn für freiwilligen Militärdienst können eine effiziente Aufteilung der Bevölkerung im Einklang mit den Zielen der Freiheit und Fairness erreichen, aber sie werden kaum die Gleichsetzung der Grenznutzen der Einkommen erreichen. Dieser Konflikt mag einen Beitrag leisten zu der Erklärung der Segregation in China zwischen Stadt und Land und dem weitverbreiteten Phänomen der Wehrpflicht. Der vorliegende Beitrag bietet einen Beweis und eine schematische Illustration des Problems und eine allgemeine Diskussion des Konflikts.

RÉSUMÉ

La liberté (comme dans le choix d'une situation professionnelle ou d'un lieu de résidence) et l'impartialité (équité horizontale) peuvent être incompatibles avec la réalisation de l'équité (verticale) et/ou de l'efficacité, même en faisant abstraction des facteurs familiaux (effets d'incitation, coûts administratifs) qui interviennent dans le compromis équité-efficacité. Liberté et impartialité requièrent une indifférence individuelle (égalité des utilités totales) entre les deux possibilités, tandis que l'équité (verticale) requiert une égalisation interindividuelle du bien-être marginal du revenu. Asséoir un impôt d'encombrement sur les résidences urbaines ou offrir des salaires élevés pour attirer des soldats volontaires peuvent permettre d'obtenir une répartition efficace de la population compatible avec la liberté et l'impartialité, mais ont peu de chances de coïncider avec l'égalisation du bien-être marginal du revenu. Ce conflit peut expliquer en partie la ségrégation ville-campagne en Chine et le maintien de la conscription. Le présent article vérifie la théorie de ce conflit, illustre ce problème graphiquement, et pose le débat en termes généraux.