#### Notas Montevideo

# 1 Estándares, *n* firmas, información incompleta, multas dadas.

2 políticas a comparar:

- i) inducir cumplimiento esperado
- ii) inducir no-cumplimiento esperado con multas convexas.

Inducir expected compliance con multas crecientes lineales o convexas tiene el mismo costo en equilibrio porque con cumplimiento perfecto no hay costos de sanción. Inducir no-cumplimiento esperado con multas lineales is ruled out by Proposition 1 below; it is never cost-effective to induce noncompliance when the marginal fine is linear. Therefore, the comparison at last is between i) and ii). ¿Cuál es la menos costosa?

#### 1.1 El problema del regulador:

$$\min E\left[c(.)\right]_{\substack{(s_1, s_2, \dots, s_n)\\(\pi_1, \pi_2, \dots, \pi_n)}} = E\left[\sum_{i=1}^n c_i(e_i, \theta_i) + \mu \sum_{i=1}^n \pi_i + \beta \sum_{i=1}^n \pi_i f(e_i - s_i)\right]$$
s.t.

1) 
$$e_i = \bar{e}_i(s_i, \pi_i, \theta_i)$$
  
2)  $\sum_{i=1}^n \bar{e}(s_i, \pi_i, \theta_i) = E$   
3)  $s_i \le \bar{e}_i \ \forall i = 1, ...n$ 

$$L = E\left[\sum_{i=1}^{n} c_{i}(\bar{e}_{i}, \theta_{i}) + \mu \sum_{i=1}^{n} \pi_{i} + \beta \sum_{i=1}^{n} \pi_{i} f(\bar{e}_{i} - s_{i})\right] + \lambda_{1}\left[\sum_{i=1}^{n} \bar{e}_{i} - E\right] + \sum_{i=1}^{n} \lambda_{2}^{i}(s_{i} - \bar{e}_{i})$$

Las  $n \times 2$  CPO respecto a las variables de elección son:

$$\frac{\partial L}{\partial s_i} = E\left[c'_i(.)\frac{\partial \bar{e}_i}{\partial s_i} + \beta \pi_i f'(.)(\frac{\partial \bar{e}_i}{\partial s_i} - 1)\right] + \lambda_1 \frac{\partial \bar{e}_i}{\partial s_i}$$
(1)  
+ $\lambda_2^i(1 - \frac{\partial \bar{e}_i}{\partial s_i}) = 0, i = 1, ..., n$ 

$$\frac{\partial L}{\partial \pi_i} = E \left\{ c'_i(.) \frac{\partial \bar{e}_i}{\partial \pi_i} + \mu + \beta \left[ f(\bar{e} - s_i) + \pi_i f'(\bar{e} - s_i) \frac{\partial \bar{e}_i}{\partial \pi_i} \right] \right\}$$

$$+ \lambda_1 \frac{\partial \bar{e}_i}{\partial \pi_i} - \lambda_2^i \frac{\partial \bar{e}_i}{\partial \pi_i} = 0, i = 1, ..., n$$
(2)

Las n + 1 CPO respecto de los multiplicadores son

$$\frac{\partial L}{\partial \lambda_1} = \sum_{i=1}^n \bar{e}_i - E = 0, \lambda_1 \ge 0 \tag{3}$$

$$\frac{\partial L}{\partial \lambda_2^i} = s_i - \bar{e}_i \le 0, \lambda_2^i \ge 0, \lambda_2^i \times (s_i - \bar{e}_i) = 0$$
 (FOC 4)

# 1.2 ¿Bajo qué condiciones es costo-efectivo inducir perfecto cumplimiento esperado, $s_i = \bar{e}_i \ \forall i = 1, ...n$ ?

Si  $\bar{e}_i = s_i \Longrightarrow \lambda_2^i \ge 0, \lambda_1 \ge 0$ . We re-write the FOC:

$$\frac{\partial L}{\partial s_i} = E\left\{c'_i(.)\frac{\partial \bar{e}_i}{\partial s_i} + \beta \pi_i f'(.)(\frac{\partial \bar{e}_i}{\partial s_i} - 1)\right\} + \underbrace{\lambda_1 \frac{\partial \bar{e}_i}{\partial s_i} + \lambda_2^i(1 - \frac{\partial \bar{e}_i}{\partial s_i})}_{\frac{\partial L}{\partial s_i}} = 0$$

$$\frac{\partial L}{\partial s_i} = E\left\{c'_i(.)\frac{\partial \bar{e}_i}{\partial s_i} + \beta \pi_i f'(.)(\frac{\partial \bar{e}_i}{\partial s_i} - 1)\right\} + \left(\lambda_1 - \lambda_2^i\right)\frac{\partial \bar{e}_i}{\partial s_i} + \lambda_2^i = 0$$

$$= \left[ \left\{ E\left[c_i'\left(s_i\right)\right] + \beta \pi_i f'(0) + \left(\lambda_1 - \lambda_2^i\right) \right\} \frac{\partial \bar{e}_i}{\partial s_i} - \beta \pi_i f'(0) + \lambda_2^i = 0 \right]$$

$$\frac{\partial L}{\partial \pi_i} = E\left[c_i'\left(s_i\right)\right] \frac{\partial \bar{e}_i}{\partial \pi_i} + \mu + \beta \times f(0) + \beta \pi_i f'(0) \frac{\partial \bar{e}_i}{\partial \pi_i} + \left(\lambda_1 - \lambda_2^i\right) \frac{\partial \bar{e}_i}{\partial \pi_i} = 0$$
$$= \left[\left\{E\left[c_i'\left(s_i\right)\right] + \beta \pi_i f'(0) + \left(\lambda_1 - \lambda_2^i\right)\right\} \frac{\partial \bar{e}_i}{\partial \pi_i} + \mu = 0\right]$$

Dividiendo:

$$\frac{\frac{\partial \bar{e}_i}{\partial s_i}}{\frac{\partial \bar{e}_i}{\partial \pi_i}} = \frac{\beta \pi_i f'(0) - \lambda_2^i}{-\mu}$$
$$\mu \frac{\pi_i f''(0)}{f'(0)} = \pi_i \beta f'(0) - \lambda_2^i$$

Si  $\lambda_2^i \ge 0$ ,

$$\mu \frac{\pi_i f''(0)}{f'(0)} \le \pi_i \beta f'(0)$$

$$\mu \frac{f''(0)}{f'(0)} \le \beta f'(0)$$

**Proposition 1** The cost-effective policy induces expected compliance for all *i* if and only if  $\mu \frac{f''(0)}{f'(0)} \leq \beta f'(0)$ .<sup>1</sup> If this condition is not met, the cost effective policy induces positive expected violations for all *i*.

We conclude that Prop. 1 in Arguedas (2007) is robust to n firms and to imperfect information. Stranlund (2007) reached the same result for the case of transferable permits.

<sup>&</sup>lt;sup>1</sup>Justificar el if and only if

## 1.3 Characterization of the cost-effective program to control emissions with standards when penalties are given and it is cost-effective to induce expected perfect compliance

**Proposition 2** : if the optimal policy  $(\pi_1^*, \pi_2^*, ..., \pi_n^*, s_1^*, s_2^*, ..., s_n^*)$  induces expected compliance, it is characterized by

$$E\left[c_i'(s_i^*,\theta)\right] + \mu \frac{d\pi_i^*}{ds_i} + \lambda_1 = 0$$
$$\pi_i^* = \frac{E\left[-c_i'(s_i^*,\theta)\right]}{f'(0)}$$

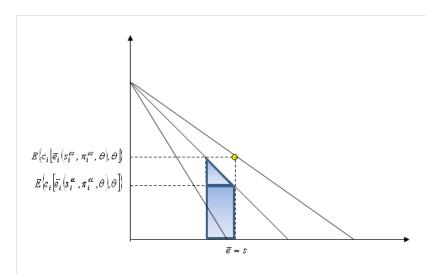
**Proof.** When  $\bar{e}_i = s_i$ , expected violations are zero and therefore there are only two types of expected costs; monitoring and abatement. Moreover,  $\bar{e}_i = s_i$  implies:  $E\left[-c'_i(s^*_i, \theta)\right] \leq \pi^*_i f'(0)$ , or  $\pi^*_i \geq \frac{E\left[-c'_i(s^*_i, \theta)\right]}{f'(0)}$ . But if the regulator can induce  $\bar{e}_i = s_i$  with  $\pi^*_i = \frac{E\left[-c'_i(s^*_i, \theta)\right]}{f'(0)}$  it would not be costeffective to select  $\pi^*_i > \frac{E\left[-c'_i(s^*_i, \theta)\right]}{f'(0)}$  because it would increase monitoring costs. Therefore,  $\pi^*_i = \frac{E\left[-c'_i(s^*_i, \theta)\right]}{f'(0)}$ . Moreover, the Lagrangean of the problem when  $\bar{e}_i = s_i$  and  $\pi^*_i = \frac{E\left[-c'_i(s^*_i, \theta)\right]}{f'(0)}$  is  $\blacksquare$  $L = E\left\{\sum_{i=1}^n c_i(s_i, \theta_i) + \mu \sum_{i=1}^n \pi^*_i\right\} + \lambda_1\left[\sum_{i=1}^n s_i - E\right]$ 

and therefore

$$\frac{dL}{ds_i} = E\left[c'_i(s_i, \theta_i)\right] + \mu \frac{d\pi_i^*}{ds_i} + \lambda_1 = 0 \qquad i = 1, 2, ...n$$

Note that  $E\{c'_i(s^*_i, \theta_i)\} + \mu \frac{d\pi^*_i}{ds_i} = E\{c'_j(s^*_j, \theta_j)\} + \mu \frac{d\pi^*_j}{ds_j}$  for all  $i \neq j$ , (i, j) = 1, ..., n. When it is cost-effective to induce expected compliance, the regulator has to set  $s_i, s_j$  such that the *sum* of marginal expected abatement and monitoring costs are equal between firms. This result was already obtained by Chávez, et. al (2009). Malik (1992) also reached the same result but in a different context; with perfect information on abtement costs and an exogenously given objective of perfect compliance.

Note also that allocating emissions responsibilities in this way does not imply perfect compliance with certainty. In the presence of imperfect information about abatement costs, perfect compliance would require the regulator to set  $\pi_i^* = \frac{-c'_i(s_i^*, \theta_L^i)}{f'(0)}$ , with  $c'_i(s_i^*, \theta_L^i)$  being the largest possible value of the marginal abatement cost of complying with the standard for firm *i*. It follows then that the monitoring probability that the regulator has to choose to induce perfect compliance with certainty is larger than the one that it has to choose to induce expected compliance. An immediate corollary that follows from this conclusion is that a program designed to induce perfect compliance with certainty does not minimize the expected costs of the program. The point is illustrated in the Figure 1 below, where  $E\left\{c_i\left[\bar{e}_i(s_i^{ec}, \pi_i^{ec}, \theta), \theta\right]\right\} < E\left\{c_i\left[\bar{e}_i(s_i^{cc}, \pi_i^{cc}, \theta), \theta\right]\right\}, \forall i = 1, ... n$ 



## 1.4 Characterization of the cost-effective program to control emissions with standards when penalties are given and it is cost-effective not to induce expected prefect compliance

**Proposition 3** if the optimal policy  $(\pi_1^*, \pi_2^*, ..., \pi_n^*, s_1^*, s_2^*, ..., s_n^*)$  induces expected non-compliance, it is characterized by

$$E\left[c_{i}'(\bar{e}_{i},\theta_{i})\right] + \mu \frac{\partial \pi_{i}^{*}/\partial s_{i}}{\partial \bar{e}_{i}/\partial s_{i}}$$
$$+\beta \left(\frac{\partial \pi_{i}^{*}/\partial s_{i}}{\partial \bar{e}_{i}/\partial s_{i}}f(\bar{e}_{i}-s_{i}) + \pi_{i}^{*}f'(\bar{e}_{i}-s_{i})\left(\frac{\partial \bar{v}_{i}/\partial s_{i}}{\partial \bar{e}_{i}/\partial s_{i}}\right)\right)$$
$$= E\left[c_{j}'(\bar{e}_{j},\theta_{j})\right] + \mu \frac{\partial \pi_{j}^{*}/\partial s_{j}}{\partial \bar{e}_{j}/\partial s_{j}}$$
$$+\beta \left(\frac{\partial \pi_{j}^{*}/\partial s_{j}}{\partial \bar{e}_{j}/\partial s_{j}}f(\bar{e}_{j}-s_{j}) + \pi_{j}^{*}f'(\bar{e}_{j}-s_{j})\left(\frac{\partial \bar{v}_{j}/\partial s_{j}}{\partial \bar{e}_{j}/\partial s_{j}}\right)\right)$$
for all  $i \neq j$ 

and

$$\pi_i^* = \frac{E\left[-c_i'(\bar{e}_i, \theta)\right]}{f'(\bar{e}_i - s_i)} \text{ for all } i = 1, \dots n.$$

**Proof.** Define  $\bar{v} = \bar{e}_i - s_i$ . If  $\bar{e}_i > s_i$ ,  $\lambda_2^i = 0 \quad \forall i = 1, ...n$ . Using this and  $\pi_i^* = \frac{E\left[-c_i'(\bar{e}_i, \theta)\right]}{f'(\bar{e}_i - s_i)}$  we can write the Lagrangean of the regultor's problem as  $L = E\left[\sum_{i=1}^n c_i(\bar{e}_i, \theta_i) + \mu \sum_{i=1}^n \pi_i^* + \beta \sum_{i=1}^n \pi_i^* f(\bar{e}_i - s_i)\right] + \lambda_1 \left[\sum_{i=1}^n \bar{e}_i - E\right]$ 

and the relevant FOC for the problem of choosing  $s_i$  is

$$\begin{aligned} \frac{\partial L}{\partial s_i} &= E\left[c_i'(\bar{e}_i, \theta_i)\frac{\partial \bar{e}_i}{\partial s_i}\right] + \mu \frac{\partial \pi_i^*}{\partial s_i} \\ + \beta \left(\frac{\partial \pi_i^*}{\partial s_i} f(\bar{e}_i - s_i) + \pi_i^* f'(\bar{e}_i - s_i) \left(\frac{\partial \bar{e}_i}{\partial s_i} - 1\right)\right) + \lambda_1 \frac{\partial \bar{e}_i}{\partial s_i} = 0 \\ \frac{\partial L}{\partial s_i} &= E\left[c_i'(\bar{e}_i, \theta_i)\right] + \mu \frac{\partial \pi_i^*/\partial s_i}{\partial \bar{e}_i/\partial s_i} + \\ \beta \left(\frac{\partial \pi_i^*/\partial s_i}{\partial \bar{e}_i/\partial s_i} f(\bar{e}_i - s_i) + \pi_i^* f'(\bar{e}_i - s_i) \left(\frac{\partial \bar{e}_i/\partial s_i - 1}{\partial \bar{e}_i/\partial s_i}\right)\right) = -\lambda_1 \\ \frac{\partial L}{\partial s_i} &= E\left[c_i'(\bar{e}_i, \theta_i)\right] + \mu \frac{\partial \pi_i^*/\partial s_i}{\partial \bar{e}_i/\partial s_i} + \\ \beta \left(\frac{\partial \pi_i^*/\partial s_i}{\partial \bar{e}_i/\partial s_i} f(\bar{e}_i - s_i) + \pi_i^* f'(\bar{e}_i - s_i) \left(\frac{\partial \bar{v}_i/\partial s_i}{\partial \bar{e}_i/\partial s_i}\right)\right) = -\lambda_1 \end{aligned}$$

It can be seen that the second term in the LHS,  $\mu \frac{\partial \pi_i^* / \partial s_i}{\partial \bar{e}_i / \partial s_i}$ , is the marginal monitoring cost. When  $\bar{e}_i = s_i$ , the marginal monitoring costs of moving the standard is  $\partial \pi_i^* / \partial s_i$ , because  $\partial \bar{e}_i / \partial s_i = 1$ . But with expected violations, moving the standard has an additional effect on expected emissions  $\bar{e}$ ,  $0 < \partial \bar{e}_i / \partial s_i < 1$ . If the regulator laxes the standard, expected emissions increase proportionally less than the standard (the expected violation decreases). Therefore the decrease in  $\pi_i^*$  is larger than in the case of perfect compliance. The effect of moving the standard on the cost-effective level of monitoring  $\pi_i^*$ , which is negative, is now more negative than in the case of perfect expected compliance,  $\left| \frac{\partial \pi_i^* / \partial s_i}{\partial \bar{e}_i / \partial s_i} \right| > |\partial \pi_i^* / \partial s_i|$ .

Lastly, the last term of the LHS is the marginal expected sanctioning costs. It can be seen that moving the standard has two effects on the sanctioning costs. First, the decrease in the costs-effective level of monitoring  $(\pi_i^*)$ caused by an increase in the standard affects the sanctioning costs, because less sanctions are discovered, in the amount  $\beta \frac{\partial \pi_i^* / \partial s_i}{\partial \bar{e}_i / \partial s_i} f(\bar{e}_i - s_i)$ . Second, the change in the standard affects the level of violations in  $\frac{\partial \bar{v}_i / \partial s_i}{\partial \bar{e}_i / \partial s_i}$ . The numerator of this expression is the direct change in violations due to the change in the standard. The denominator introduces the fact that a change in the standard put pressure on the constraint by an amount  $\partial \bar{e}_i / \partial s_i$ , and therefore requires the level of violations to decrease decrease even more  $(0 < \partial \bar{e}_i / \partial s_i < 1)$ . **Remark 4** When it is cost-effective to induce a positive expected level of noncompliance, the cost-effective design of a program based on firm-specific emission standards requires the regulator to set these standards such that the sum of the expected marginal abatement, monitoring sanctioning costs are equal for all firms.

Assuming that  $\mu$  and  $\beta$  to be the same for all firms, we can conclude from Proposition 3 that the cost-effective level of emission standards are firm-specific, and that the only reason behind this result is the heterogeneity in marginal abatement costs  $c'_i(\bar{e}_i, \theta_i)$ , given that these costs generate the variation  $\frac{\partial \pi_i^*/\partial s_i}{\partial \bar{e}_i/\partial s_i}$  in the required monitoring and in the optimal size of the violation, and ultimately in the marginal cost of imposing sanctions (last term of the LHS). The expression also suggests that even if marginal abatement costs were the same for all firms, differences in monitoring costs and sanctioning costs among firms ( $\mu_i \neq \mu_j, \beta_i \neq \beta_j$ ) could also call for differences in the cost-minimizing standards.

#### 2 The Choice of the Appropriate Penalties

We follow Arguedas: "First we derive the most appropriate shape of the penalties under the two possible scenarios: compliance and non-compliance. Next, we select the socially preferred scenario."

The result for the first Objective is Proposition 4:

**Proposition 4:** If the optimal policy induces expected compliance, the best shape of the fine is such that the linear component is set as high as possible and the progressive component is irrelevant in equilibrium. If the optimal policy induces expected non-compliance, the best shape of the fine is one in which the linear component  $\phi = 0$  and the progressive component is set "as high as possible" for all firms.

**Proof.** The fine f(e-s)

$$f(e-s) = \phi(e-s) + \frac{\gamma}{2}(e-s)^2$$

 $\phi$  is the linear component.

 $\gamma$  is the progressive

If the optimal policy induces compliance, sanctioning costs are zero.

By Proposition 2

$$\pi_i^* = \frac{E\left[-c_i'(s_i^*, \theta)\right]}{f'(0)} = \frac{E\left[-c_i'(s_i^*, \theta)\right]}{\phi}$$

From here we can conclude:

(1) The regulator must choose the linear component  $\phi$  of the fine structure as high as possible because this will decrease the optimum level of the inspection probability,  $\pi_i^*$ , and by this way the monitoring costs. Conceptually, this calls for  $\phi = \infty$  because this will make the monitoring costs equal to zero. But in the real world there may be limits to the upper value of  $\phi$ . These limits may be given by...CITATIONS FROME THE LITERATURE.

(2) The size or value of  $\gamma$  does not matter. The program has the same minimum expected costs  $= \sum_{i=1}^{n} c_i(\bar{e}_i, \theta_i) + \mu \sum_{i=1}^{n} \pi_i^*$  for all  $\gamma \ge 0$ , with  $\pi_i^* = \frac{E[-c'_i(s_i^*, \theta)]}{\phi}$ .

(3) The structure of the fine does not matter as long as  $\mu_{\phi}^{\gamma} \leq \beta \phi$ ,

Therefore, Arguedas (2008) is wrong when she concludes: "the larger the linear gravity component the lower the minimum probability to achieve compliance and therefore the social costs. Therefore, the optimal fine is one on which f'(0) is as high as possible and f''(0) is as low as possible, since only the first component affects the probability."

If the optimal policy induces non-compliance, how to choose  $\phi$  and  $\gamma$  in order to minimize the costs of a program that produces E?

To answer this question, first note that in the n-firm scenario, it is impossible to keep fines constant for all firms for different fine structures if  $\phi$  and  $\gamma$  are common for all firms. If  $f(e-s) = \phi(e-s) + \frac{\gamma}{2}(e-s)^2$ , changing  $\phi$  and  $\gamma$  so as to keep f constant requires  $\frac{e-s}{2} = -\frac{d\gamma}{d\phi}$ . But with n firms, it is impossible to move  $\phi$  and  $\gamma$  such that  $\frac{e_i-s_i}{2} = -\frac{d\gamma}{d\phi}$  for all i. Keeping f contant for all i requires a firm-specific fine parameters. We assume that this is the case and we show that the optimal design of the program calls for a uniform fine structure.

If the fine structure is firm-specific, we have  $f_i(\bar{e}_i - s_i) = \phi_i(\bar{e}_i - s_i) + \frac{\gamma_i}{2}(\bar{e}_i - s_i)^2$ , and  $f'_i(\bar{e}_i - s_i) = \phi_i + \frac{\gamma_i}{2}(\bar{e}_i - s_i)$  for each *i*. Then we ask how to choose  $\phi_i$  and  $\gamma_i$  in order to minimize the costs of a program that produces *E* when it is optimal to induce expected violations. Following Arguedas (2007), we ask ourselves whether we can decrease the costs of a program that induces a certain expected level of violation for each firm changing the fine structure

(changing the values of  $\phi_i$  and  $\gamma_i$ ) while choosing  $\pi_i$  optimally. In order to answer this question, we evaluate the Lagrangean of the regulator's problem at  $\pi_i = \pi_i^* = \frac{E\left[-c'_i(\bar{e}_i, \theta_i)\right]}{f'(\bar{e}_i - s_i)}$  when  $\bar{e}_i > s_i$  and  $\sum_i \bar{e}_i = E$  and change  $\phi_i$  and  $\gamma_i$  such that  $df_i = 0$ , that is and  $-\frac{d\phi_i}{d\gamma_i} = \frac{\bar{e}_i - s_i}{2}$ .

$$\begin{split} L &= E\left[\sum_{i=1}^{n} c_{i}(\bar{e}_{i},\theta_{i})\right] + \mu \sum_{i=1}^{n} \pi_{i}^{*} + \beta \sum_{i=1}^{n} \pi_{i}^{*} f_{i}(\bar{e}_{i} - s_{i}) \\ dL &= \left[\frac{\partial L}{\partial \phi_{i}} d\phi_{i} + \frac{\partial L}{\partial \gamma_{i}} d\gamma_{i} \\ dL &= \left[\mu \frac{\partial \pi_{i}^{*}}{\partial \phi_{i}} + \beta \left[\frac{\partial \pi_{i}^{*}}{\partial \phi_{i}} f_{i}(\bar{e}_{i} - s_{i}) + \pi_{i}^{*}(\bar{e}_{i} - s_{i})\right]\right] d\phi_{i} \\ &+ \left[\mu \frac{\partial \pi_{i}^{*}}{\partial \gamma_{i}} + \beta \left[\frac{\partial \pi_{i}^{*}}{\partial \gamma_{i}} f_{i}(\bar{e}_{i} - s_{i}) + \pi_{i}^{*} \frac{(\bar{e}_{i} - s_{i})^{2}}{2}\right]\right] d\gamma_{i} \\ \frac{dL}{d\phi_{i}} &= \left[\mu \frac{\partial \pi_{i}^{*}}{\partial \phi_{i}} + \beta \left[\frac{\partial \pi_{i}^{*}}{\partial \phi_{i}} f_{i}(\bar{e}_{i} - s_{i}) + \pi_{i}^{*} \frac{(\bar{e}_{i} - s_{i})^{2}}{2}\right]\right] \frac{d\gamma_{i}}{d\phi_{i}} \\ \frac{dL}{d\phi_{i}} &= \left[\mu \frac{\partial \pi_{i}^{*}}{\partial \phi_{i}} + \beta \left[\frac{\partial \pi_{i}^{*}}{\partial \phi_{i}} f_{i}(\bar{e}_{i} - s_{i}) + \pi_{i}^{*} (\bar{e}_{i} - s_{i})\right]\right] \\ &- \left[\mu \frac{\partial \pi_{i}^{*}}{\partial \phi_{i}} + \beta \left[\frac{\partial \pi_{i}^{*}}{\partial \phi_{i}} f_{i}(\bar{e}_{i} - s_{i}) + \pi_{i}^{*} \frac{(\bar{e}_{i} - s_{i})^{2}}{2}\right]\right] \frac{2}{\bar{e}_{i} - s_{i}} \\ \frac{dL}{d\phi_{i}} &= \mu \frac{\partial \pi_{i}^{*}}{\partial \phi_{i}} + \beta \left[\frac{\partial \pi_{i}^{*}}{\partial \phi_{i}} f_{i}(\bar{e}_{i} - s_{i}) + \pi_{i}^{*} (\bar{e}_{i} - s_{i})^{2}\right] \\ - \frac{2\mu}{\bar{e}_{i} - s_{i}} \frac{\partial \pi_{i}^{*}}{\partial \gamma_{i}} - \beta \left[\frac{\partial \pi_{i}^{*}}{\partial \gamma_{i}} (2\phi_{i} + \gamma_{i}(\bar{e}_{i} - s_{i}))\right] \\ \end{split}$$

We know that  $\frac{\partial \pi_i^*}{\partial \phi_i} = \frac{-E\left[-c_i'(\bar{e}_i, \theta_i)\right]}{\left[\phi_i + \gamma_i(\bar{e}_i - s_i)\right]^2}$  and  $\frac{\partial \pi_i^*}{\partial \gamma_i} = \frac{-E\left[-c_i'(\bar{e}_i, \theta_i)\right]}{\left[\phi_i + \gamma_i(\bar{e}_i - s_i)\right]^2} \times (\bar{e}_i - s_i).$ 

Therefore,

$$\frac{dL}{d\phi_{i}} = -\frac{E\left[-c_{i}'(\bar{e}_{i},\theta_{i})\right]}{\left[\phi_{i}+\gamma_{i}(\bar{e}_{i}-s_{i})\right]^{2}} \left[\mu+\beta\left(\phi_{i}(\bar{e}_{i}-s_{i})+\frac{\gamma_{i}}{2}(\bar{e}_{i}-s_{i})^{2}\right)\right] + \frac{E\left[-c_{i}'(\bar{e}_{i},\theta_{i})\right]}{\left[\phi_{i}+\gamma_{i}(\bar{e}_{i}-s_{i})\right]^{2}} \times (\bar{e}_{i}-s_{i}) \left[\frac{2\mu}{\bar{e}_{i}-s_{i}}+\beta\left(2\phi_{i}+\gamma_{i}(\bar{e}_{i}-s_{i})\right)\right] + \frac{E\left[-c_{i}'(\bar{e}_{i},\theta_{i})\right]}{\left[\phi_{i}+\gamma_{i}(\bar{e}_{i}-s_{i})\right]^{2}} \left[\mu+\beta\left(\phi_{i}(\bar{e}_{i}-s_{i})+\frac{\gamma_{i}}{2}(\bar{e}_{i}-s_{i})^{2}\right)\right] + \frac{E\left[-c_{i}'(\bar{e}_{i},\theta_{i})\right]}{\left[\phi_{i}+\gamma_{i}(\bar{e}_{i}-s_{i})\right]^{2}} \left[2\mu+\beta\left(2\phi_{i}(\bar{e}_{i}-s_{i})+\gamma_{i}(\bar{e}_{i}-s_{i})^{2}\right)\right] + \frac{E\left[-c_{i}'(\bar{e}_{i},\theta_{i})\right]}{\left[\phi_{i}+\gamma_{i}(\bar{e}_{i}-s_{i})\right]^{2}} \left[2\mu+\beta\left(2\phi_{i}(\bar{e}_{i}-s_{i})+\gamma_{i}(\bar{e}_{i}-s_{i})^{2}\right)\right] \times \left[2\mu-\mu-\beta\phi_{i}(\bar{e}_{i}-s_{i})-\beta\frac{\gamma_{i}}{2}(\bar{e}_{i}-s_{i})^{2}+2\beta\phi_{i}(\bar{e}_{i}-s_{i})+\beta\gamma_{i}(\bar{e}_{i}-s_{i})^{2}\right] + \frac{dL}{d\phi_{i}} = \frac{E\left[-c_{i}'(\bar{e}_{i},\theta_{i})\right]}{\left[\phi_{i}+\gamma_{i}(\bar{e}_{i}-s_{i})\right]^{2}} \left[\mu+\beta\left(\phi_{i}(\bar{e}_{i}-s_{i})+\frac{\gamma_{i}}{2}(\bar{e}_{i}-s_{i})^{2}\right)\right] > 0$$

This means that the regulator can decrease the costs of a program that induces a violation  $(\bar{e}_i - s_i)$  for each firm by decreasing  $\phi_i$  and increasing  $\gamma_i$  (so as to keep the equilibrium fine constant). The intuition behind this result follows form two observations. First, is that by increasing the marginal equilibrium penalty the regulator decreases the equilibrium inspection probability  $\pi_i^*$  needed to induce a given expected level of violation  $(\bar{e}_i - s_i)$ , and this decreases monitoring costs while keeps the rest of the costs constant. Second, is that the marginal equilibrium penalty increases more if the regulator increases  $\gamma_i$  than if it increases  $\phi_i$ . The first term in the RHS of 4 is the marginal effect of a change in  $\phi_i$  on the expected costs of the program. The second term is the marginal effect of a change in  $\gamma_i$ . These two effects act in opposed directions because keeping the fine constant requires increasing one parameter and decreasing the other. Decreasing  $\phi_i$  increases the expected monitoring costs by  $\frac{-E[-c'_i(\bar{e}_i,\theta_i)]}{[f(\bar{e}_i-s_i)]^2} \times \mu$  and increases the expected sanctioning costs by  $\frac{E\left[-c'_i(\bar{e}_i,\theta_i)\right]}{\left[f(\bar{e}_i-s_i)\right]^2} \left[\beta f(\bar{e}_i-s_i)\right]$ . Increasing  $\gamma_i$  by the quantity that keeps  $f(\bar{e}_i - s_i)$  constant decreases both costs by more than this. Therefore the final effect is to decrease the total expected costs of the program (expected abatement costs do not change).

Now, decreasing  $\phi_i$  has a limit and this limit is  $\phi_i = 0$ . Under a negative value of  $\phi_i$  it will always exist a (sufficiently small) level of violation that makes the fine negative. But a negative fine violates our assumption that  $f \geq 0$  for all levels of violations. On the other hand, there is no theoretical maximum value for  $\gamma_i$ . In theore this value is infinite, and therefore it is not firm-specific. Therefore, the expected cost minimizing design of a program based on standards calls for a uniform penalty structure for all firms:  $f(\bar{e}_i - s_i) = \gamma(\bar{e}_i - s_i)^2$  for all *i*. The regulator always decreases monitoring costs by increasing  $\gamma$ , for the same level of violation. This is true for all firms and therefore it must set  $\gamma$  as high as possible for all firms.

# 3 The expected-cost-minimizing design of a program based on standards when the regulator can choose the fine structure

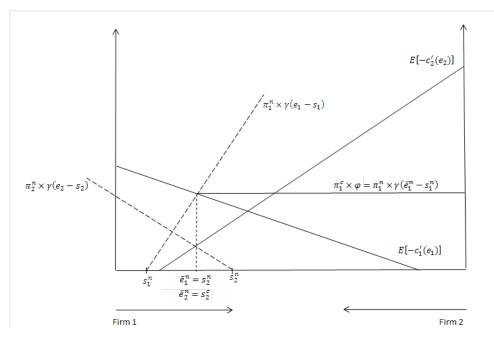
Having characterized the optimal program when it is optimum to induce compliance and when it is optimum to induce non-compliance, we now allow the regulator to choose the fine structure and therefore the optimality of inducing expected compliance or not. The result of this comparison is given in the next Proposition:

**Proposition 5** The optimal policy  $(s_1^*, s_2^*, ..., s_n^*, \pi_1^*, \pi_2^*, ..., \pi_n^*, f^*)$  induces compliance and it is characterized by (1)  $E\{c_i'(s_i^*, \theta_i)\} + \mu \frac{d\pi_i^*}{ds_i} = E\{c_j'(s_j^*, \theta_j)\} + \mu \frac{d\pi_j^*}{ds_j}$  for all  $i = 1, ...n, i \neq j$ , (2)  $\pi_i^* = \frac{E[-c_i'(s_i^*, \theta_i)]}{f'(0)}$ , and (3)  $f(e_i - s_i) = \phi(e_i - s_i) + \frac{\gamma}{2}(e_i - s_i)^2$  for all i, with  $\phi$  set as high as necessary to induce all firms to comply and  $\gamma$  is set at any value as long as  $\mu\gamma \leq \beta\phi^2$ .

**Proof.** Following Arguedas (2008), assume that it is optimum to induce expected non-compliance, and call the optimal policy  $P^n = (s_1^n, s_2^n, ..., s_n^n, \pi_1^n, \pi_2^n, ..., \pi_n^n, f^n)$ , with  $f^n = \frac{\gamma}{2}(e_i - s_i)^2$  for all i, with  $\gamma$  as high as possible (following Proposition 4),  $\pi_i^n = \frac{E[-c'_i(\bar{e}_i^n, \theta_i)]}{\gamma(\bar{e}_i^n - s_i^n)}$  and  $\sum_{i=1}^n \bar{e}_i^n = E$ . Now consider an alternative policy  $P^c = (s_1^c, s_2^c, ..., s_n^c, \pi_1^c, \pi_2^c, ..., \pi_n^c, f^c)$  such that  $s_i^c = \bar{e}_i^n$  and  $\pi_i^c = \pi_i^n$  for all i, and  $f^c = \phi(e_i - s_i)$  for all i with  $\phi = \gamma \times \max_i [\bar{e}_i^n - s_i^n]$ . By construction, this policy induces expected compliance because  $\pi_i^c \phi = \pi_i^c \gamma \times \max_i [\bar{e}_i^n - s_i^n] \ge E[-c'_i(\bar{e}_i^r, \theta)] = E[-c'_i(s_i^c, \theta)]$  for all i. Moreover,  $P^c$  is cheaper than  $P^n$  in

expected terms because expected abatement costs are the same under both programs  $(s_i^c = \bar{e}_i^n \text{ for all } i)$ , expected monitoring costs are the same under both programs  $(\pi_i^c = \pi_i^n \text{ for all } i)$ , but under policy  $P^c$  there are no expected sanctioning costs because there are no expected violations. QED.

The proof is illustrated in the following graph with n = 2:



In the above Figure emissions of firm 1 are measured from left to right and emissions of firm 2 from right to left. The initial situation is assumed to be one in which is optimum to induce violations and  $f^{n'} = \gamma(e_i - s_i)$ . The regulators sets  $s_1^n$  and  $s_2^n$  and the firms expected level of emissions are  $\bar{e}_1^n$  and  $\bar{e}_2^n$ , such that  $\bar{e}_1^n + \bar{e}_2^n = E$ , the size of the box. This is policy  $P^n$ . But if the regulator changes the fine structure and sets a constant marginal penalty  $\phi$  for both firms equal to the larger marginal penalty in  $P^n$ , which is that of firm 1, and increases both emission standards up to  $\bar{e}_1^n$  and  $\bar{e}_2^n$ , respectively, without changing the probabilities of inspection, the result is another policy  $P^c$  that induces expected compliance with constant marginal penalties  $\phi = \gamma \times (\bar{e}_1^n - s_1^n)$  and that meets the policy objective E with lower expected costs: expected abatement costs and monitoring costs are the same and expected sanctioning costs are zero.

In conclusion, the expected cost minimizing policy when a regulator wants to achieve a certain level of aggregate emissions E with emission standards will be one that induces expected compliance. The structure of the fine does not play any role in equilibrium. Expected compliance could be induced with a constant marginal penalty or an increasing marginal penalty. If marginal penalties are constant the result we obtained for the case of emissions standards is the same Stranlund (2007) obtained for tradable permits: both programs need to be designed so as to achieve expected compliance and this is done using a constant marginal penalty. Arguedas (2008) proved the same result for the case of one firm.

Proposition 5 does not give a clear rule for setting  $\phi$  "as high as possible" or "as high as neccessary". In the real world  $\phi$  will be given be bounded upward by things such as the possibility that firms may go bankrupt, ... VER LITERATURE Wasserman (1992), Segerson and Tietenberg (1991), Becker (1968)

It is not difficult to think of emission control programs in the real world that were designed or are being designed by different agencies or offices inside a regulatory agency. Think for example of ....PONER EJEMPLOS. If this is the case, one agency or office may set first the environmental objective (the aggregate level of emissions E in our case) and the abatement responsibilities among firms (the standards) while another agency or office may be in charge of designing the monitoring and enforcing strategy, for which it could be using fine structures defined by the general civil or criminal law. Proposition 5 suggests that the resulting regulatory design will be probably sub-optimum, except for the cases in which the penalty structure is the appropriate to induce expected perfect compliance and the offices are coordinated so as to set standards and monitoring probabilities according to Proposition 5.

#### 3.1 Firm - specific monitoring and sanctioning costs

We have assumed so far that the monitoring and sanctioning costs ( $\mu$  and  $\beta$ ) were the same for all firms. Nevertheless, there may be good reasons to assume that these costs may vary among firms. In the case of the monitoring costs, the distance between the firm and the enforcing agency or the number of discharge points may cause  $\mu$  to vary among firms.<sup>2</sup> Similarly, sanctioning costs may differ between firms because of their differing propensity to litigate sanctions and challenge the legislation, which may be a function of their

<sup>&</sup>lt;sup>2</sup>It is also true that on-line monitoring, as it is the the case in the US SO2 program based on tradable allowances, may work in the opposite direction: it could make monitoring costs more homogeneous among firms than classic monitoring.

budget or their willingness to conserve an image. we will have that  $\mu = \mu_i$ and  $\beta = \beta_i$ , i = 1, ..n, and  $\mu_i \neq \mu_j$  and  $\beta_i \neq \beta_j$ , for at least some  $i \neq j$ . In this case the condition in Proposition 1 may be valid for some firms and not be valid for other ones. In other words, it could be cost-effective for the regulator to induce violations for some firms and compliance for the rest.

# 4 Comparing Costs Between an optimally designed program based on standards and an optimally designed program based on tradable permits

We have seen that the optimal design of a program based on emissions standards is one in which standards are firm-specific (set according to Proposition 2) and perfectly enforced (with the fine structure playing no role in equilibrium). We know from Stranlund (2007) that the optimal design of a program based on tradable permits is one in which the marginal fine is constant and the program is perfectly enforced. The question remains whether a regulator interested in controlling emissions of a given pollutant by setting a cap on aggregate emissions in an expected cost minimizing manner should implement a perfectly enforced program based on firm-specific standards as in Proposition 2 above or a perfectly enforced program based on tradable permits as in Stranlund (2007). That is, once we know the optimal design of the programs based on the two instruments, what instrument should a regulator use if it wants to minimize the total expected costs of the program? The answer is given in the following Proposition:

**Proposition 6** If a regulator wants to control the emissions of a given pollutant by setting a cap on the aggregate level of emissions of this pollutant it will minimize the expected total costs by implementing a firm-specific emissions standards and perfectly enforcing this program.

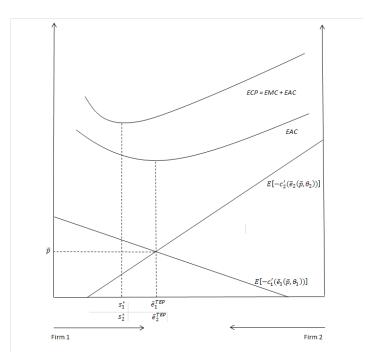
**Proof.** The total expected costs of a program that sets a cap on aggregate emissions is given by the expected abatement costs of the regulated firms and the expected monitoring and sanctioning costs of the regulator. That is,

$$ECP^k = EAC^k + EMC^k + ESC^k$$

where  $ECP^k$  is the total expected costs of the program k,  $EAC^k$  is the expected abatement costs of the program k,  $EMC^k$  is the expected monitoring costs of the program k,  $ESC^k$  is the expected sanctioning costs of the program k and k = emission standards or tradable permits. We know from Proposition 5 that the optimally designed program based on emission standards must induce expected compliance. We also know from Stranlund (2007) that an optimally designed program based on tradable permits must also induce expected compliance. As a result, our comparison of the programs does not need to take into account ESC because these are zero in both programs when optimally designed. Taking this into account, and assuming that the emission standards program is enforced with a constant marginal penalty function, we know from Proposition 2 that  $\pi_i^* = \frac{E[-c'_i(s_i^*, \theta_i)]}{\phi}$  in the optimally designed program based on standards and from Stranlund (2007) that  $\pi_i^* = \pi^* = \frac{\bar{p}}{\phi}$  for all i in the case of the optimally designed program based on tradable permits.

$$ECP^{TEP} = E\left(\sum_{i=1}^{n} c_i \left(e_i\left(\bar{p}\right), \theta_i\right)\right) + \mu n \frac{\bar{p}}{\phi}$$
$$ECP^{ES} = E\left(\sum_{i=1}^{n} c_i \left(s_i^*, \theta_i\right)\right) + \frac{\mu}{\phi} E\left(\sum_{i=1}^{n} -c_i' \left(\bar{e}_i \left(s_i^*, \theta_i\right)\right)\right)$$

where  $ECP^{TEP}$  is the expected cost of an optimally designed program based on tradable emission permits and  $ECP^{ES}$  is the expected cost of an optimally designed program based on emission standards. The proof that  $ECP^{ES} < ECP^{TEP}$  is trivial because, by definition, the emission standards and monitoring probabilities in the optimally designed ES program are allocated so as to minimize the total expected costs of the program when this is perfectly enforced. That is, when the costs of the program consist only of abatement and monitoring costs. Therefore, the total expected costs of this program are lower than the total expected costs of a program based on tradable permits, which consists of a different allocation of emissions and monitoring probabilities. QED. (¿ES SUFICIENTE ESTO PARA LA DEMOSTRACIÓN? NO DEBERIAMOS DEMOSTRAR FORMAL-MENTE QUE COMO ILUSTRA EL GRAFICO LA ASIGAN-CION DE EMISIONES DEL tep ES DISTINTA A LA DEL es PROGRAM?



Moreover, we know that abatement costs are minimized with a tradable permits program. Therefore,  $E\left(\sum_{i=1}^{n} c_i\left(e_i\left(\bar{p}\right), \theta_i\right)\right) < E\left(\sum_{i=1}^{n} c_i\left(s_i^*, \theta_i\right)\right)$ . Therefore, it must be the case that the larger expected monitoring costs of the tradable emission permits program more than compensate its lower abatement costs when compared to the (optimally designed) emission standards program.

The intuition behind this result can be illustrated with the aid of Figure XXX below:

In the Figure,  $(\bar{e}_1^{TEP}, \bar{e}_2^{TEP})$  is the expected equilibrium allocation of emissions resulting from a perfectly enforced program based on tradable permits. We call  $\bar{p}$  the equilibrium price of this market, assumed to be perfectly competitive. We know from Proposition 2 that the total expected costs of a program based on emission standards are minimized when the standards and the monitoring probabilities are chosen such that

$$E\left[c_{1}'(s_{i}^{*},\theta_{1})\right] + \mu \frac{d\pi_{1}^{*}}{ds_{1}} = E\left[c_{2}'(s_{2}^{*},\theta_{2})\right] + \mu \frac{d\pi_{2}^{*}}{ds_{2}}$$

with  $\pi_i^* = \frac{E\left[-c_i'(s_i^*,\theta_i)\right]}{f'(0)}$ , i = 1, 2. In the case of linear marginal penalties,  $\frac{d\pi_i^*}{ds_i} = \frac{E\left[-c_i''(s_i^*,\theta_i)\right]}{\phi} \le 0, i = 1, 2$ , and

$$E\left[-c_{1}'(s_{i}^{*},\theta_{1})\right] + \mu \frac{E\left[c_{1}''(.)\right]}{\phi} = E\left[-c_{2}'(s_{2}^{*},\theta_{2})\right] + \mu \frac{E\left[c_{2}''(.)\right]}{\phi}$$

If we assume that  $E[c_2''] > E[c_1''] > 0$  as in Figure XXX,  $\frac{E[c_1''(.)]}{\phi} < \frac{E[c_2''(.)]}{\phi}$ . Therefore, because  $E\left[-c_1'(\bar{e}_1^{TEP}, \theta_1)\right] = E\left[-c_2'(\bar{e}_2^{TEP}, \theta_2)\right]$  at  $(\bar{e}_1^{TEP}, \bar{e}_2^{TEP})$ , the optimal emission standards allocation must be to the left of  $(\bar{e}_1^{TEP}, \bar{e}_2^{TEP})$ , the expected allocation resulting from a tradable permits program. This allocation is drawn as  $(s_1^*, s_2^*)$ . We know that  $(\bar{e}_1^{TEP}, \bar{e}_2^{TEP})$  minimizes the expected total abatement costs (curve EAC) of all possible allocations responsibilities among firm 1 and 2 such that  $e_1 + e_2 = E$ . We also know that, by definition, the *total* expected costs of an optimally designed emission standards program such that  $e_1 + e_2 = E$  are minimized at  $(s_1^*, s_2^*)$ . These costs are represented by the curve ECP, consisting of the sum of expected abatement (EAC) and minimum expected monitoring costs (EMC).

**Remark 7** If the regulator could observe marginal abatement costs, the costs effective solution to control emissions would call for a system of perfectly enforced firm-specific emissions standards.

Of course, it is not the case that a regulator can observe firms' marginal abatement costs. In fact, it may commit relevant mistakes in the estimation of the abatement costs functions. (PONER EJEMPLOS DE ESTIMA-CIONES DE COSTOS DE ABATIMIENTO VIA PRECIO DE EQUILIB-RIO EN EL SO2 MARKET DE EEUU Y EN EL EUETS). If this is the case, the realized social costs of setting and enforcing a global cap on emissions via standards could end up being more expensive than doing it via an emissions trading scheme. In any case, it is not in the name of cost-effectiveness *per se* that we economists are to argue in favor of tradable emission permits, but in the name of information advantages: the regulator needs to know nothing about abatement costs when designing and enforcing an emissions trading scheme, and by this way it *may* be a cheaper instrument than emissions standards in terms of the realized social costs of setting a global cap on emissions. (Comparar con Weitzman (1974) y Montero (2002)?)

NOS QUEDA ESTUDIAR O COMENTAR SOBRE LOS FACTORES DE LOS QUE DEPENDE QUE EL REGULADOR TERMINE COME-TIENDO ERRORES TAL QUE LA ASIGNACIÓN DE ESTANDARES SEA TAL QUE (EL COSTO """REAL""" del  $P^{ES}$  termine siendo superior a al costo del  $P^{TEP}$ .

# 5 Comparing costs when it is cost-effective to induce non-compliance

RESTARIA COMPARAR AMBOS PROGRAMAS CUANDO ES OPTIMO IDUCIR VIOLACIONES EN TERMINOS ESPERADOS. mOTIVATION: as discussed above, it may be common that the fine structure is given to the environmental authority. Assume that  $\gamma > 0$ . In this case, whether the regulator has to perfectly enforce the program or not depends on the relative size of the monitoring and sanctioning parameters (i.e: whether  $\mu \gamma \leq \beta \phi^2$ ). Assume that  $\mu \gamma > \beta \phi^2$ , then it is cost-effective to design a program that induce a given expected level of non-compliance. How do the cost of such a program based on emission standards compare with one based on tradable permits?