# Bogota airport / TACA 133

## Individual choices under increasing marginal penalty

$$\min_{e_i} \; c_i(e_i) + \pi_i\left[\phi(e_i - s_i) + \gamma(e_i - s_i)^2/2\right]$$

$$\text{s.t. } e_i - s_i \geq 0, \quad s_i > 0$$

$$\mathcal{L} = c_i(e_i) + \pi_i\left[\phi(e_i - s_i) + \gamma(e_i - s_i)^2/2\right] + \lambda(e_i - s_i)$$

(1) $\dfrac{\partial \mathcal{L}}{\partial e_i} = c_i'(e_i) + \pi_i \times \left[\phi + \dfrac{2}{2}\gamma(e_i - s_i)\right] + \lambda = 0$

la firma cumple $\Leftrightarrow$ $\boxed{\pi_i \times [\phi] \geq -c_i(s_i)}$

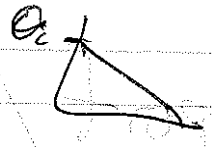
Si no cumple $(e_i \geq s_i)$

(2) $\dfrac{\partial \mathcal{L}}{\partial \lambda} = e_i - s_i \geq 0, \; \lambda \geq 0, \; \lambda \times (e_i - s_i) = 0$

$\Downarrow$

Si $e_i > s_i$, $\lambda = 0$ y $\boxed{c_i'(e_i) = \pi_i \times [\phi + \gamma(e_i - s_i)]}$

$s_i$




$$-C_i'(e_i) = \theta_i - \delta_i e_i$$

Si viola, entonces:

$$\theta_i - \delta_i e_i = \pi_i \times [\phi + \gamma \underbrace{(e_i - s_i)}_{v_i}]$$

$e_i^*(s_i)$:

$$\theta_i - \delta_i \times e_i = \pi_i \times \phi + \pi_i \gamma e_i - \pi_i \gamma s_i$$

$$\theta_i - \pi_i \cdot \phi + \pi_i \gamma s_i = \pi_i \gamma e_i + \delta_i e_i$$

$$\theta_i + \pi_i(\gamma s_i - \phi) = e_i(\pi_i \gamma + \delta_i)$$

$$e_i^*\left(\frac{s_i}{\pi_i}; \theta_i, \delta_i, \phi, \gamma\right) = \frac{\theta_i + \pi_i(\gamma s_i - \phi)}{\pi_i \gamma + \delta_i}$$

Nosotros queremos $\sum_i e_i^*(s_i, \pi_i) = E$

Determinamos la violación $v_i$ como función de los parámetros de costos, fiscalización $\frac{}{}$ y $\pi_i$

$$v_i^* = e_i^* - s_i = \frac{\theta_i + \pi_i(\gamma s_i - \phi)}{\pi_i \gamma + \delta_i} - s_i$$

$$V_i^* = \frac{\theta_i - \pi_i \phi}{\pi_i \gamma + \delta_i} + \left( \frac{\pi_i \gamma}{\pi_i \gamma + \delta_i} - 1 \right) s_i$$

$$= \frac{\theta_i - \pi_i \phi}{\pi_i \gamma + \delta_i} + \left( \frac{\pi_i \gamma - \pi_i \gamma - \delta_i}{\pi_i \gamma + \delta_i} \right) s_i$$

$$= \frac{\theta_i - \pi_i \phi}{\pi_i \gamma + \delta_i} - \left( \frac{\delta_i}{\pi_i \gamma + \delta_i} \right) s_i$$

La violación depende de los parámetros de la función de costos, parámetros de fiscalización y la elección del estándar

$$\pi_i(s_i) = \frac{-c_i(e_i^*)}{\left[\phi + \gamma(e_i^* - s_i)\right]}$$

$$\pi_i(s_i; \theta_i, \delta_i, \phi, \gamma) = \frac{\theta_i - \delta_i \left[ \frac{\theta_i + \pi_i(\gamma s_i - \phi)}{\pi_i \gamma + \delta_i} \right]}{\left[ \phi + \gamma \left[ \frac{\theta_i + \pi_i(\gamma s_i - \phi)}{\pi_i \gamma + \delta_i} - s_i \right] \right]}$$

$\pi_i$ varía entre firma porque $s_i$ (?) y los parámetros de la función de costos varían entre firmas, aunque los parámetros de la multa no varían entre firmas.

$$\pi_i(s_i; \theta_i, \delta_i, \phi, \gamma) = \frac{\theta_i - \delta_i \, e_i^*(s_i, \pi_i)}{\phi + \gamma\left[e_i^*(s_i, \pi_i) - s_i\right]}$$

## The regulatory choice of standards under increasing marginal penalty

$$\min_{s_i} \sum_i c_i\left[e_i(s_i, \pi_i)\right] + \sum_i \mu \cdot \pi_i(s_i) + \beta \sum_i \pi_i(s_i) \cdot v_i^*(s_i, \pi_i)$$

$\beta$: per dollar cost of collecting penalties

$$\text{s.a.} \quad \sum_i s_i + \left(\sum_i e_i(s_i, \pi_i) - s_i\right) \geq E$$

$$\mathcal{L} = \sum_i c_i(e_i(s_i, \pi_i)) + \sum_i \mu \pi_i(s_i) + \beta \sum_i \pi_i(s_i) v_i(s_i, \pi_i)$$

$$+ \lambda\left\{E - \sum_i s_i - \left[\sum_i e_i(s_i, \pi_i) + s_i\right]\right\}$$