To Comply or Not To Comply? Pollution Standard Setting Under Costly Monitoring and Sanctioning

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Abstract   In this paper, we characterize optimal regulatory policies composed of a pollution standard, a probability of inspection and a fine for non-compliance, in a context where both monitoring and sanctioning are socially costly, and the penalty may include gravity and non-gravity components at the regulator's discretion. Under given penalties, the optimal policy entails compliance with the standard as long as a quite intuitive condition is met. Non-compliant policies may include standards even below the pollution levels that minimize the sum of abatement costs and external damages. Interestingly, the appropriate structure of the penalty under non-compliance is highly progressive, while the best possible shape of the fine under compliance is linear. If the regulator is entitled to choose the structure of the fine, linear penalties are socially preferred and the optimal policy induces compliance.

Keywords   Standards · Monitoring · Convex fines · Non-compliance · Non-gravity sanctions

JEL Classification   K32 · K42 · L51 · Q28

1 Introduction

Environmental regulations often require polluting agents to comply with *pollution limits* or *standards*. For example, the EPA's National Pollutant Discharge Elimination System Program of the Clean Water Act requires facilities which discharge pollutants into waters of the US to "obtain a permit to release specific amounts of pollution." Such facilities include direct and indirect dischargers, as well as Publicly Owned Treatment Works (POTWs), that is, wastewater treatment plants owned by municipalities and local sewer districts.

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According to Harrington (2003), the rates of non-compliance from the mid-1980s to the mid-1990s ranked between 6% and 14% for direct dischargers, between 9% and 11% for POTWs, and about 54% in the case of indirect dischargers. This included violations of standards and other requirements also, such as self-monitoring or reporting, although 35% of the non-compliant indirect dischargers were in violation of the standards. The EPA Office of Solid Waste and Emergency Response Directive 9610.12 provides some examples of imposed penalties, following the Civil Penalty Policy of the Clean Water Act. These penalties contain a *gravity* component, directly related to the degree of non-compliance; and also a *non-gravity* part, which considers extra conditions such as the economic impact of the penalty on the violator or economic benefits of non-compliance, such as illegal profits or competitive advantage.[1] According to the above mentioned Directive, the final structure of the sanction is case-based: the non-gravity component varies from nearly 3% to almost 84% of the total penalty.

These numbers suggest that non-compliant behavior is significant, and also that there does not exist a clear pattern of the appropriate shape of the penalties. Surprisingly, the existing theoretical literature on optimum enforcement has not considered the mentioned binary structure of the fines, see Polinsky and Shavell (2000) for an excellent survey on public enforcement of the law.[2] However, empirical studies confirm the use of both gravity and non-gravity components. For example, in a study about the structure of the penalties for water quality violations in Georgia, Oljaca et al. (1998) find that the seriousness of the violation, historical compliance records and the size of the company strongly influence penalty levels. While the first two factors are gravity-based, however the third one is not. Moreover, non-compliance has not been rationalized; that is, the problem of finding the optimal policy composed of pollution standards, inspection probabilities and fines, considering that the best possible policy may induce non-compliance, has not been solved. In the present paper, we address these issues.[3]

We consider a simple model composed of a regulator and a polluting firm. The regulator sets a pollution standard and a probability of inspection which minimizes social costs (i.e., the sum of abatement costs, external damages and enforcement costs, i.e., monitoring and sanctioning costs), considering the optimal behavior of the firm with respect to the policy. Sanctions for non-compliance contain both a non-gravity and a gravity component, the latter being strictly increasing and convex in the degree of the violation. In the first part of the paper, we consider given fines, i.e., chosen by an institution different from the regulator. Later on, we allow the regulator to choose the penalty as well.

Under given fines, we find that the optimal policy induces compliance under low monitoring costs relative to sanctioning costs, and when sanctions are not very progressive in the

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[1] The civil penalties imposed within the framework of the Clean Air Act also exhibit this structure, consult http://www.epa.gov for more information on this issue.

[2] Some examples include Polinsky and Shavell (1979, 1991), Bebchuk and Kaplow (1991) or Bose (1993). Within the environmental context, see the literature reviews by Heyes (2000) and Cohen (1999), and also Heyes (1996), Segerson and Tietenberg (1992) or Arguedas (2005).

[3] A recent paper by Stranlund (2007) on the design of emissions trading programs is the closest paper to ours, although the (overall) induced pollution level is fixed there, while it is a decision variable of regulators here. Therefore, the present paper also allows to obtain desired pollution levels. Arguedas and Hamoudi (2004) consider a similar framework in the context of pollution standards without sanctioning costs. The purpose of that paper is not to find the best possible fine structure, but to analyze polluters' incentives to invest in environmentally friendly technologies, given possible fine reductions contingent on adoption and alternative timings for policy announcements.

degree of non-compliance. Interestingly, this condition reduces to the requirement found in Stranlund (2007) under his particular assumptions on the penalty function.[4]

However, the pollution level is endogenously determined in our model, while it is given in Stranlund (2007). This allows us to show that a *compliant policy* (that is, the combination of a standard, an inspection probabilities and a fine which induce the firm to comply with the standard) may be characterized by a lenient standard, in contrast with a *non-compliant policy* (i.e., one which induces the firm to pollute more than the allowed limit). In fact, under compliance, we find that the optimal standard is above the pollution level which minimizes the sum of abatement costs and external damages, a well-known result in the literature, see Polinsky and Shavell (2000). Conversely, under non-compliance, the optimal standard is below, as long as sanctioning costs are small enough. In fact, the optimal standard is zero if there are no sanctioning costs, as shown in Arguedas and Hamoudi (2004). The intuition is simple. Consider a policy composed of a (positive) standard, an inspection probability and a fine. This policy induces a particular pollution level. That pollution level can be kept constant decreasing both the standard and the probability of inspection, accordingly. A decrease in the standard is equivalent to an increase in the fine, since, for a given pollution level, the degree of the violation increases. Increasing the fine is socially costless. But the corresponding decrease in the probability decreases monitoring costs. Therefore, it is socially convenient to decrease the standard as much as possible: this guarantees the largest possible fine and, consequently, the lowest possible inspection probability. This trade-off between the fine and the probability is in the same spirit as Becker's (1968).

Regarding the appropriate shape of the fine, the regulator can cheaply maintain compliance if the marginal sanction of an infinitesimal violation is large enough. This is possible when the *linear* component of the sanction is large. When the optimal policy induces non-compliance, the optimal non-gravity sanction is zero, since it does not marginally affect the behavior of the firm and it only causes sanctioning costs. The preferred shape of the penalty in this case is sufficiently progressive. This result seems contrary to the literature on crime, where an increase in the sanction increases compliance (although Kambhu 1989; Malik 1990a; Harrington 1988; Livernois and McKenna 1999 are exceptions). However, there the standard is given. Here, the standard is endogenously determined, and therefore, it changes when the sanction changes. A particular pollution level is then induced with a sufficiently large progressiveness of the sanction and a sufficiently stringent standard.

Given a non-compliant policy, it is always possible to find an equivalent compliant policy which achieves the same emission level at lower social costs, in accordance with Stranlund (2007). As a result, the optimal policy induces compliance, and it is amazingly simple: a standard above the pollution level which minimizes the sum of abatement costs and external damages, the minimum probability necessary to induce compliance and a linear gravity sanction. As a consequence, the strictly convex gravity factor of the penalty as well as the non-gravity part are not useful components of the optimal policy.

This paper contributes to the literature on standard setting, and more specifically, on non-compliance and the design of optimal fines. Downing and Watson (1974) were the first to present a theoretical model of environmental policy enforcement. Harford (1978) focusses on firms' behavior with respect to imperfectly enforceable emission standards and taxes. In the context of emissions trading policies, several papers assume imperfect enforcement, such

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[4] Stranlund (2007) assumes a perfectly competitive market for tradable permits, where firms are price-takers. Firms cannot strategically react to modify penalties (for polluting more than the quantity of permits they hold) in their favor either. In that sense, our paper and Stranlund (2007) have a common feature, that is, to assume that firms cannot contest the terms of the policy. This then leads to a similar condition for the social preference of compliance.

as Malik (1990b), Stranlund and Dhanda (1999) or Montero (2002). Ellis (1992), Stranlund and Chavez (2000) and Amacher and Malik (1996) study optimal policies (that is, policies which minimize social costs, including abatement costs, external damages and enforcement costs) constrained to induce compliance. More recently, Arguedas (2005) finds that optimal policies induce non-compliance, when firms and regulators negotiate on the level of the fines in exchange for firms' adoption of clean technologies.

The remainder of the paper is organized as follows. In the next section, we present the model. In Sect. 3, we study the optimal behavior of the firm. In Sect. 4, we analyze the optimal policy under given penalties. In Sect. 5, we discuss the appropriate shape of the fines and present the characteristics of the optimal policy. We conclude in Sect. 6. All the proofs are in the Appendix.

2 The Model

A single firm generates pollution as a result of its production activity. The pollution level is denoted by $e \in [0, e^0]$, where e^0 is the level emitted in the absence of any regulation. Pollution can be abated at a cost $c(e)$, with the usual assumptions $c'(e) < 0$, $c''(e) > 0$ and $c(e^0) = 0$.[5] Pollution generates external damages measured by the function $d(e)$, such that $d'(e) > 0$, $d''(e) \geq 0$ and $d(0) = 0$.

Let e^w be the pollution level that minimizes the sum of abatement costs and external damages, that is, $e^w = \arg\min_{e \geq 0} \{c(e) + d(e)\}$. Our assumptions ensure $0 \leq e^w < e^0$.

We assume there exists a regulator who sets a standard $s \in [0, e^0]$, that is, a maximum amount of permitted pollution. The regulator cannot observe the pollution level selected by the firm unless it engages in a monitoring activity, which is costly and perfectly accurate. The cost *per* inspection is $m > 0$. We assume that the firm is inspected with probability $p \in [0, 1]$. Once inspected, if the firm is discovered violating the standard ($e > s$), then it is forced to pay a penalty that depends on the degree of non-compliance, $e - s$, and it is represented by the function $F(e - s) = F_0 + f(e - s)$, where $f(e - s) > 0$, $f'(e - s) > 0$ and $f''(e - s) > 0$ for all $e > s$ and $f(e - s) = 0$ for all $e \leq s$. Thus, $F_0 \geq 0$ is the non-gravity based sanction and $f(e - s)$ is the gravity-based component. When appropriate, we discuss how our results change under linear penalties ($f'' = 0$).

Sanctioning is socially costly, too. Let $t \geq 0$ represent the per-unit social cost of collecting fines.[6] Initially, we assume that the sanction is fixed in the legislation, and study the features of the regulatory problem under given penalty structures. We relax this assumption in Sect. 5.

Given $F(e - s)$, we consider a principal-agent framework where the regulator chooses the pollution standard and the inspection probability which minimizes social costs, considering the optimal response of the firm to the policy. We consider the sub-game perfect equilibrium concept. Therefore, we solve the problem backwards, that is, we first find the firm's optimal pollution level, and we then obtain the optimal policy that minimizes social costs considering the firm's optimal response.

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[5] Throughout the paper, we assume that third order derivatives are negligible.

[6] We model sanctioning costs in the same way as Stranlund (2007) to directly compare our results with his. Polinsky and Shavell (1992) argue that sanctioning costs may increase with the level of the fines, since individuals can more strongly resist to the imposition of larger fines (concealing assets, for example). However, the bulk of the sanctioning costs that are associated with imposing fines are generally independent of the size of the penalty, as modelled in Rousseau and Proost (2005) or Polinsky and Shavell (1992). Interestingly, this alternative assumption does not change our main results, although we will point out the specific differences when needed.

Given the policy s, p and $F(e-s)$, the firm chooses the pollution level that minimizes the sum of abatement costs and expected penalties for non-compliance:

$$C(s, p) = \min_{e \geq 0} \{c(e) + p[F_0 + f(e-s)]\},$$

$$\text{s. t.} \quad e - s \geq 0. \tag{1}$$

The regulator selects the policy that minimizes social costs, which contain firm's abatement costs, generated damages, expected monitoring costs and expected sanctioning costs:

$$SC(s, p) = c(e) + d(e) + p[m + tF(e-s)], \tag{2}$$

where $e = e(s, p) \leq e^0$ is the firm's optimal response to the policy.

3 The Behavior of the Firm

Given s, p and $F(e-s)$, the firm solves problem (1). The firm's decision is made in two steps. First, the firm decides whether it is worth to comply with the standard or not. Then, it chooses the pollution level. The problem is solved by backward induction.

If the firm complies with the standard, it trivially chooses $e = s$. However, if the firm does not comply, it chooses $e = n > s$, given by:

$$c'(n) + pf'(n-s) = 0. \tag{3}$$

Implicitly differentiating (3), we obtain the relationship between the chosen pollution level and, respectively, the probability of inspection and the standard:

$$n_p(s, p) = -\frac{f'(n-s)}{c''(n) + pf''(n-s)} < 0, \tag{4}$$

$$n_s(s, p) = \frac{pf''(n-s)}{c''(n) + pf''(n-s)} > 0. \tag{5}$$

These results are in accordance with Harford (1978). The pollution level selected by the firm decreases with the inspection probability and increases with the standard. Also, the degree of the violation decreases with the standard, since $n_s(s, p) < 1$.[7]

Whether the firm decides to comply with the standard depends on the fixed component of the sanction, and also on the relationship between marginal abatement costs and marginal expected fines. The following lemma provides the result.

Lemma 1 *Given s, p and $F(e-s)$, the firm's optimal choice of pollution $e(s, p)$ is the following:*

(i) If $F_0 = 0$, then $e(s, p) = \begin{cases} s, & \text{if } c'(s) + pf'(0) \geq 0; \\ n, & \text{if } c'(s) + pf'(0) < 0. \end{cases}$

(ii) If $F_0 > 0$, then $e(s, p) = \begin{cases} s, & \text{if } p \geq \dfrac{c(s) - c(n)}{F_0 + f(n-s)}; \\ n, & \text{if } p < \dfrac{c(s) - c(n)}{F_0 + f(n-s)}. \end{cases}$

Consider first the case where $F_0 = 0$. The firm complies (does not comply) with the standard if the savings in abatement costs of infinitesimally exceeding the standard are smaller

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[7] Note that $n_s(s, p) = 0$ when either $f'' = 0$ (i.e., when the sanction is linear) or $p = 0$.

(larger) than the marginal expected penalty. If the sanction includes a non-gravity component $F_0 > 0$, the expected cost function of the firm is discontinuous at $e = s$. In this case, the firm complies (does not comply) with the standard if the expected costs of complying are smaller (larger) than those of non-complying. Everything else equal, a sufficiently large F_0 ensures firm's compliance, since F_0 does not affect marginal behavior and it increases firm's expected costs only in the event of non-compliance.

From Lemma 1, there exists a threshold probability of inspection above which the firm complies with the standard, given by the expression:

$$p(s) = \begin{cases} -\dfrac{c'(s)}{f'(0)}, & \text{if } F_0 = 0; \\ p^c(s), & \text{if } F_0 > 0; \end{cases} \tag{6}$$

where $p^c(s)$ is the implicit relationship between p and s when $c(s) = c(n) + pF(n - s)$. There exists a negative relationship between p and s, since $c'(s) < 0$ and $c''(s) > 0$. The larger the standard, the lower the required probability to induce compliance. Also, $p^c(e^0) = 0$, that is, there is no need to monitor the firm if it is required to comply with the pollution level e^0, the one it would emit in the absence of any regulation.

4 The Optimal Policy Under Given Penalties

In this section, we assume that the regulator selects $\{s, p\}$, for a given fine structure $F(e - s)$. The problem is the following:

$$\min_{s,p} \{c(e) + d(e) + pm + ptF(e - s)\},$$
$$\text{s. t. } e = e(s, p), \ p \in [0, 1], \ s \geq 0, \tag{7}$$

where $e = e(s, p)$ is the firm's optimal response, characterized in Lemma 1.

The regulator must decide between a policy which induces compliance, with possibly larger monitoring costs but without sanctioning costs; or a policy which induces non-compliance, with sanctioning costs but possibly lower monitoring costs.

We first provide a sufficient condition for the optimal policy to induce compliance.

Proposition 1 *Let (s^*, p^*) be the solution of* (7). *Then, (s^*, p^*) induces compliance if*

$$(m + tF_0)\frac{f''(0)}{f'(0)} \leq tf'(0). \tag{8}$$

This result is quite intuitive. Assume that (s^*, p^*) induces compliance, i.e., $e(s^*, p^*) = s^*$. From (4) and (5), pollution decreases with the inspection probability and increases with the standard. Therefore, the regulator can maintain the pollution level $e(s^*, p^*)$ constant by infinitesimally decreasing the standard (this is equivalent to infinitesimally increase the sanction, since the degree of non-compliance increases) and decreasing the probability of inspection accordingly. Given an infinitesimal decrease in the standard, the probability can be reduced on the amount $-\frac{n_s}{n_p}|_{n=s^*} = \frac{pf''(0)}{f'(0)}$, to keep pollution constant.

But, changing the inspection probability and the standard affect enforcement costs. If the standard infinitesimally decreases, sanctioning costs increase on the amount $tpf'(0)$. The corresponding decrease in the probability decreases both monitoring and sanctioning costs on the amount $(m + tF_0)\frac{pf''(0)}{f'(0)}$. If the enforcement cost savings of decreasing the probability

(left hand side of (8)) are lower than the additional enforcement costs of decreasing the standard (right hand side of (8)), then it is not socially convenient to depart from a policy which induces compliance.

Condition (8) is more likely to hold under low monitoring costs relative to sanctioning costs. Then, enough effort can be devoted to induce compliance, since this allows to save on sanctioning costs. Condition (8) also depends on the specific structure of the sanction. Clearly, a larger marginal sanction (the term $f'(0)$) increases the sanctioning costs of decreasing the standard and decreases the enforcement cost savings of decreasing the probability of inspection (the latter because a larger marginal sanction increases the response of the firm to a change in the probability of inspection, see (4)). Therefore, a larger marginal sanction increases the likelihood of condition (8). By contrast, the progressiveness of the sanction, (the term $f''(0)$) crucially affects the response of the firm to a change in the standard. A larger progressiveness implies that the corresponding reduction of the probability is larger, and therefore, the enforcement cost savings of decreasing the probability are larger. Thus, (8) is more likely to hold when $f''(0)$ is small. Finally, a lower F_0 decreases the social cost savings of decreasing the probability of inspection, and (8) is more likely.[8]

Next, we present the features of the optimal policy constrained to induce compliance.

Proposition 2 *If the optimal policy* (s^*, p^*) *induces compliance, it is characterized by* $c'(s^*) + d'(s^*) + m\frac{dp(s^*)}{ds} = 0$, *where* $p^* = p(s^*)$ *is given by* (6).

The optimal compliant policy balances abatement costs and expected damages against monitoring costs. Since $\frac{dp(s)}{ds} < 0$, $c'(s^*) + d'(s^*) > 0$. The optimal standard must be set above e^w, the pollution level which minimizes the sum of abatement costs and external damages. This result is in accordance with the literature, see Polinsky and Shavell (2000), and it is only due to costly monitoring and not to the particular fine structure.

We now characterize the optimal policy that entails non-compliance.

Proposition 3 *If the optimal policy* (s^*, p^*) *induces non-compliance, it is given by:*

$$c'(n) + d'(n) + tp^*f'(n - s^*) + \frac{m + t(F(n - s^*))}{n_p} = 0, \tag{9}$$

$$c'(n) + p^*f'(n - s^*) = 0, \tag{10}$$

$$\left(m + t\left(F(n - s^*)\right)\right)\frac{f''(n - s^*)}{f'(n - s^*)} - tf'(n - s^*) \geq 0, \tag{11}$$

$$s^*\left[\left(m + t\left(F(n - s^*)\right)\right)\frac{f''(n - s^*)}{f'(n - s^*)} - tf'(n - s^*)\right] = 0. \tag{12}$$

If sanctioning is socially costless ($t = 0$), condition (11) reduces to $m\frac{pf''}{f'} > 0$, which implies $s^* = 0$, by (4), (5) and (12). Therefore, the regulator always find it convenient to decrease the standard and the probability of inspection to save on monitoring costs, in the same spirit as Becker's (1968). By contrast, under linear fines, we have $m\frac{pf''}{f'} = 0$, $s^* \in [0, n)$ and $p^* = -\frac{c'(n)}{f'(n)}$. Intuitively, the level of the standard does not affect the decision of the firm,

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[8] It is worth to point out that (8) is a sufficient condition to ensure that the optimal policy induces compliance, as long as $F_0 > 0$. The reason is that social costs are discontinuous at $e = s$ when $F_0 > 0$. Then, a sufficiently large F_0 might be enough for the optimal policy to induce compliance even if (8) does not hold: it decreases the minimum probability to induce compliance (which decreases enforcement costs in the event of compliance, see (6)), and it increases sanctioning costs under non-compliance. Conversely, when $F_0 = 0$, social costs are continuous at $e = s$. In that case, (8) becomes an *if and only if* condition.

since the marginal fine is constant. Therefore, any standard which induces non-compliance is optimal in that case.

Under costly sanctioning, an interior standard is possible, as long as (11) holds with equality. The standard and the probability are decreased until the cost savings of decreasing the probability equal the additional costs of decreasing the standard. Under linear fines, condition (11) never holds, since our assumptions ensure $tf'(n-s) > 0$. Therefore, under linear penalties and costly sanctioning, the optimal policy always induces compliance. This result can also be deducted from Proposition 1, since condition (8) trivially holds under linear penalties.

Combining (4), (9) and (11), we obtain $tpf' + \frac{m+tF}{n_p} = -\frac{tf'c''}{f''} < 0$, which then implies $c'(n) + d'(n) > 0$. The optimal non-compliant policy induces a pollution level above e^w, the pollution level which minimizes the sum of abatement costs and environmental damages. However, in contrast with the policy that induces compliance, the standard can be set below e^w, as long as sanctioning costs are low enough. An illustration of this latter case is presented in the following:

Example 1 Abatement costs are $c(e) = \frac{e^2}{2} - 2e$ and external damages are $d(e) = \frac{e^2}{2}$. Then, $e^w = 1$. The penalty is $F(e-s) = (e-s)^2$. Since $f'(0) = 0$ and $F_0 = 0$, the optimal policy induces non-compliance for any $m > 0$, $t > 0$. From (10), $n - 2 + 2p(n-s) = 0 \Rightarrow n = \frac{2+2ps}{1+2p}$. An interior solution for s follows from (11) and (12), which lead to $m = t(n-s)^2$. Since $n_p = -\frac{f'}{c''+pf''} = -\frac{2(n-s)}{1+2p}$ from (4), (9) reduces to $n = \frac{2-ts}{2-t} > s$. The latter implies $s < (>) 1$ as long as $t < (>) 2$. In any case, $n > 1$. For example, when $m = 1$ and $t = 1$, the optimal policy is $\left[s = \frac{1}{2}; p = \frac{1}{4} \right]$, which induces $n = \frac{3}{2}$. The resulting penalty is $(n-s)^2 = 1$ and the marginal penalty is $2(n-s) = 2$.

Therefore, the likelihood of standards lower than e^w crucially depends on the sanctioning costs. In the limiting case where $t = 0$, the optimal standard is zero, in accordance with Arguedas and Hamoudi (2004).

5 The Choice of the Appropriate Penalties

In this section, we discuss the selection of the penalty shape as part of the regulatory policy. The fine can be approximated by a second order degree polynomial as follows:

$$F(e-s) \simeq F_0 + f'(0)(e-s) + \frac{f''(0)}{2}(e-s)^2, \tag{13}$$

where $F_0 \geq 0$ is the non-gravity part of the sanction, and $f'(0) \geq 0$ and $f''(0) \geq 0$ are, respectively, the linear and progressive gravity components. In this section, we restrict ourselves to linear-quadratic penalty functions. While this constraint may affect the selection of the best possible policy which induces non-compliance (second part of Proposition 4), however it does not affect the main result of this section (Proposition 5). Consistent with real world laws (for example, the Civil Penalty Policy of the Clean Water Act), we assume that there is an upper limit on the fine to be levied on the polluting firm.

We proceed in two steps. First, we derive the most appropriate shape of the penalties under the two possible scenarios, namely, compliance and non-compliance. Next, we select the socially preferred scenario.

The most appropriate shape of the penalties is presented next:

Proposition 4 *If the optimal policy induces compliance, the best shape of the gravity fine is such that the linear component is set as high as possible and the progressive component is zero. Conversely, if the optimal policy induces non-compliance, then the best shape of the gravity fine is such that the linear component is zero and the progressive component is set as high as possible. In any case, the optimal non-gravity component F_0 is zero.*

First, assume that the optimal policy induces compliance. If $F_0 = 0$, the optimal probability satisfies $p = -\frac{c'(s)}{f'(0)}$, see (6). The probability can be decreased by increasing the linear component $f'(0)$, in exchange for a lower progressive component $f''(0)$. As long as $F_0 > 0$, the firm complies at the optimal inspection probability $p^c(s)$, see (6). The larger the fine, the lower the probability, and consequently, the lower the monitoring costs. In that case, only the total amount of the fine matters, since the particular structure does not affect the behavior of the firm (other than complying versus non-complying). Thus, any structure of the fine is socially equivalent to an alternative linear fine with $F_0 = 0$ which collects the same amount. Therefore, F_0 does not play a crucial role when the optimal policy induces compliance.

When the optimal policy induces non-compliance, the regulator should not impose non-gravity sanctions either: they affect sanctioning costs but they do not affect the behavior of the firm (other than complying versus non-complying). The level of non-compliance can be better controlled under a large progressive sanction. By (5), the larger $f''(0)$, the smaller the degree of non-compliance when s decreases. Therefore, enforcement costs can be lowered if the progressive part of the sanction is increased at the expense of the linear part.[9]

Summarizing, if the regulator wants to induce compliance, the preferred fine is linear in the degree of the violation. Conversely, if she wants to induce non-compliance, the most convenient structure is very progressive, that is, the marginal fine for a small violation is much lower than the marginal fine for a large violation. But, does the optimal policy induce compliance once the shape of the penalties can be chosen accordingly? The answer is presented in the following:

Proposition 5 *The optimal policy (s^*, p^*, F^*) induces compliance, and it is characterized by the conditions $c'(s^*) + d'(s^*) - m\frac{c''(s^*)}{\bar{a}} = 0$, $p^* = -\frac{c'(s^*)}{\bar{a}}$ and $F^*(e - s) = \bar{a} \cdot (e - s)$, where $\bar{a} > 0$ is the largest possible charge per unit of the violation.*

Therefore, the answer is yes, that is, the optimal policy induces compliance and fines must be linear in the degree of the violation. The reason is that any non-compliant policy (and in particular, the best possible non-compliant policy from a social view point) is dominated by a compliant policy which induces the same pollution level with the same inspection probability and the same marginal penalty (in the case of the compliant policy, this marginal penalty is constant). Abatement costs, external damages and expected monitoring costs are the same,

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[9] The result presented in Proposition 4 is valid under fixed sanctioning costs, also. On the one hand, sanctioning costs do not affect policies which induce compliance. On the other hand, all what matters in the selection of a fine which induces non-compliance is how it affects the trade-off between the standard and the probability of inspection such that the induced pollution level is kept constant (i.e., the combination of Eqs. 4, 5). For a given standard, the larger the progressiveness of the sanction, the lower the inspection probability needed to induce a particular pollution level and, therefore, the lower the monitoring costs. The peculiarity under fixed sanctioning costs is that any non-compliant policy such that $s > 0$ is socially more costly than an alternative non-compliant policy which induces the same pollution level with a lower standard (and a lower inspection probability). The reason is that, once a policy induces non-compliance, a further reduction in the standard does not cause additional sanctioning costs (since they do not depend on the degree of the violation), but it only reduces monitoring costs, since the corresponding inspection probability can be reduced. Therefore, the optimal non-compliant policy under fixed sanctioning costs is necessarily such that $s^* = 0$.

but in the case of the compliant policy, there are no sanctioning costs. Again, this result is independent of sanctioning costs being fixed or dependent on the degree of non-compliance. Thus, the result found in Stranlund (2007) can be translated to our context (as opposed to one of tradable emission permits), under more general penalty functions and where the induced pollution level is endogenously determined (instead of exogenously given).

6 Conclusions

In this paper, we have studied optimal policies composed of pollution standards, inspection probabilities and sanctions dependent both on gravity and non-gravity-based components. From a strict social point of view, the optimal policy consists of a standard above the pollution level which minimizes the sum of abatement costs and external damages, the minimum probability needed to induce compliance and a linear gravity sanction.

We have also characterized the set of (suboptimal) policies which induce non-compliance. In those cases, optimal standards are sufficiently stringent and penalties are very progressive in the degree of non-compliance. Fixed sanctions should not be imposed, since they only cause sanctioning costs but no change in the behavior of the firm.

Our model can be extended in several ways, with no change in the flavor of the main results. For example, under risk neutrality, agents are concerned about the shape of the expected fine, and not only on the specific structure of the fine. In our model, both are equivalent since the inspection probability is fixed, i.e., independent of the size of the violation. However, it is reasonable to imagine situations in which polluting agents face an extra probability of being inspected due to third party complaints (neighboring communities or environmental groups), which may provide (noisy) signals about the size of the violation, see Rousseau and Proost (2005). In those cases, the probability of inspection is likely to depend on the size of the violation as well, i.e., $p = \overline{p} + \phi(e - s)$, such that $\phi(0) = 0$, $\phi' \geq 0$ and $\phi'' \geq 0$. Our results can be easily adapted in this case. Any combination of the penalty and the inspection probability which results in a progressive expected penalty that induces non-compliance is socially dominated by an alternative policy with a linear expected penalty which induces compliance.[10] Therefore, if the inspection probability were to depend on the size of the violation, we would recommend to use a linear shape and a fixed fine F_0, i.e., non-gravity based.

We could have considered a dynamic context where regulators base their present inspection frequencies on firms' past behavior, such as in Harrington (1988). In this context, firms could even enroll in costly avoidance activities, see Kambhu (1989). Intuitively, our analysis is robust to such extension as well. The reason is that contesting firms would bear additional

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[10] Condition (8) now becomes:

$$(m + t F_0) \left[\frac{\overline{p} f''(0) + F_0 \phi''(0) + 2\phi'(0) f'(0)}{f'(0)} \right] \leq t \left[\overline{p} f'(0) + F_0 \phi'(0) \right] + m \phi'(0),$$

where the term within brackets in the left hand side of this equation is the trade-off between $\overline{p}$ and s which keeps pollution constant, and the numerator of this term reflects the progressiveness of the expected sanction for an infinitesimally small violation. Again, the larger the progressiveness of the sanction, the larger the amount of $\overline{p}$ that can be reduced for an infinitesimal decrease in s, in order to keep pollution constant. Following an exact proof as that of Proposition 5, social costs are minimized under (constant) marginal expected fines which induce compliance. Thus, under risk neutrality, either a constant fine and a linear inspection probability or a linear fine and a constant inspection probability (such that both result in the same marginal expected fine) are equivalent alternatives from the firm's point of view (also note that both satisfy the updated condition (8)), although the first one avoids monitoring costs under compliance, since $\phi(0) = 0$.

costs (specially) under non-compliant policies, which should be considered as a component of the social costs.

Things may change if the standard is fixed (for example, established in the legislation) and it is sufficiently stringent.[11] Then, non-compliance might be the desired outcome, and our model predicts the use of very progressive gravity-based sanctions in that case.

Alternative scenarios where our prediction may change are under incomplete information or when the regulator faces a group of heterogeneous firms. Our model is one of complete information in which, in particular, the regulator knows a representative firm's abatement costs. If the regulator did not perfectly know that information or, if alternatively, she regulated a group of heterogeneous firms (instead of a representative firm), several issues are worth to mention. First, it is interesting to point out that condition (8) does not depend on the specific characteristics of the firm. Therefore, for a given structure of the fine, the condition on the social preference of compliance continues to hold. But, of course, the optimal policy (and in particular, the threshold probability of inspection defined in (6)) depends on the specific characteristics of the firm. This alternative analysis may help us to know whether non-gravity factors such as the size of the firm should be included in the fine structure, as in Oljaca et al. (1998). Also, partial compliance may appear as a distinctive feature of the optimal policy under this setting, especially if threshold probabilities for some firms are particularly large. Whether the optimal policy is uniform (i.e., the same for all the firms regardless of their specific features) or separating (i.e., contingent on their characteristics) remains unknown and constitutes an area where further research is needed.

Appendix

Proof of Lemma 1   The Lagrangian of (1) is $L(e, \kappa) = c(e) + pF(e - s) - \kappa(e - s)$, where $\kappa \geq 0$ is the Kuhn-Tucker multiplier. The optimality conditions are $c'(e) + pf'(e - s) - \kappa = 0$; $\kappa(e - s) = 0$; $\kappa \geq 0$; and $e - s \geq 0$.

First consider $F_0 = 0$ and $\kappa \geq 0$, $e = s$. Then, $\kappa = c'(s) + pf'(0) \geq 0$. If this does not hold, then $\kappa = 0$ and $e = n$, such that $c'(n) + pf'(n - s) = 0$. If $F_0 > 0$, $e(s, p) = s$ as long as $c(s) \leq c(n) + pF(n - s)$, and $e(s, p) = n$, otherwise.

Proof of Proposition 1   To prove the result, we find the condition under which the induced pollution level converges to s. The Lagrangian of the problem is $L(s, p, \lambda, \mu_i) = c(e) + d(e) + pm + ptF(e - s) + \lambda\{c'(e) + pf'(e - s)\} - \mu_1(e - s) - \mu_2 s$.

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[11]   In this case, condition (8) becomes

$$p\left[tf'(0) - \frac{mf''(0)}{f'(0)}\right] + \left[c'(s) + d'(s) - \frac{m}{f'(0)}c''(s)\right] \geq 0.$$

If the standard is set at the optimal level, the second expression within brackets is zero (see Proposition 2), and therefore, condition (8) remains the same. But, if the standard is set below the optimal level, then the second expression within brackets is negative. Therefore, the updated condition (8) may not hold even with linear penalties, as long as either t or $f'(0)$ are sufficiently small.

The optimality conditions with respect to (n, s, p) are, respectively, the following:

$$\left[c'(n) + d'(n) + tpf'\right] + \lambda \left[c''(n) + pf''\right] - \mu_1 = 0 \tag{14}$$

$$m + tF + \lambda f' = 0 \tag{15}$$

$$\mu_1 - \mu_2 - tpf' - \lambda pf'' = 0 \tag{16}$$

$$\mu_1(n - s) = 0; \quad \mu_2 s = 0. \tag{17}$$

For analytical convenience, we consider the pollution level as a choice variable of the regulator, although this variable is decided by the firm in response to the regulatory policy, as noted in (3). The problem we consider here is mathematically equivalent to the one where the regulator chooses (s, p) knowing that the firm chooses $n = e(s, p)$ in response to the policy.

The induced pollution level converges to s when $\mu_1 \geq 0$ and $\mu_2 \geq 0$. Combining (15) and (16), and substituting n by s, we have $\mu_2 - \mu_1 = -tpf'(0) + \frac{m + tF_0}{f'(0)} pf''(0) \geq 0$. Since $n = s$ and $n > 0$, we then have $\mu_2 = 0$. Thus, $\mu_1 \geq 0$ implies $-tpf'(0) + \frac{m + tF_0}{f'(0)} pf''(0) \leq 0$.

Proof of Proposition 2 The result is easily obtained from (7), substituting $e(s, p) = s$ and $p = p(s)$ given by (6).

Proof of Proposition 3 The result follows considering the case where $\mu_1 = 0$ (i.e., $n > s$) and combining the conditions (14) to (17), such that $\mu_2 \geq 0$.

Proof of Proposition 4 From (13), we have $F'(e - s) = f'(e-s) \simeq f'(0) + f''(0)(e-s)$ and $F''(e-s) = f''(e-s) \simeq f''(0)$.

If the optimal policy induces compliance, fines are not collected and, consequently, sanctioning costs are zero. Therefore, the larger the fines, the lower the probability (see (6)) and, consequently, the lower the social costs. If $F_0 = 0$, the optimal probability is $p = -\frac{c'(s)}{f'(0)}$. Therefore, the optimal fine is one where $f'(0)$ is as high as possible and $f''(0)$ is as low as possible, since only the first component affects the probability. Conversely, if $F_0 > 0$, once an exogenous limit of the fine has been achieved, it is not possible to decrease social costs changing the penalty, since $p = \frac{c(s) - c(n)}{F_0 + f(n-s)}$. Thus, any fine structure (such that the total amount collected is the same) plays the same role. In particular, this is true for fines such that $F_0 = 0$.

If the optimal policy induces non-compliance, from the Lagrangian of Proposition 1, then $F_0 = 0$. The fine is kept constant as long as $df'(0) + \frac{n-s}{2} df''(0) = 0$. Differentiating the Lagrangian of Proposition 1 with respect to $f'(0)$ and $f''(0)$, and considering the relationship between the two gravity components, we obtain:

$$\begin{aligned}
\frac{dL}{df'(0)} &= \frac{\partial L}{\partial f'(0)} + \frac{\partial L}{\partial f''(0)} \frac{df''(0)}{df'(0)} \\
&= p\{t(n-s) + \lambda\} df'(0) - p\{t(n-s) + 2\lambda\} df'(0) \\
&= -\lambda p df'(0) > 0,
\end{aligned}$$

since $\lambda < 0$, see (15). Then, decreasing $f'(0)$ and increasing $f''(0)$ reduces social costs.

Proof of Proposition 5 (Some parts of this proof have been adapted from Stranlund (2007)). First, we prove that the optimal policy induces compliance. Assume, to the contrary, that the optimal policy induces non-compliance, and call it (s^n, p^n, F^n). By Proposition 4, the optimal fine F^n, is sufficiently progressive and such that $F_0 = 0$. Thus, the fine can be approximated by the second order degree polynomial $\frac{\bar{b}}{2} \cdot (e - s)^2$, where $\bar{b}$ is the upper limit set in the legislation. Conditions (9) to (12) characterize the optimal policy. In particular, the optimal pollution level, $n^n > s^n$, is such that $c'(n^n) + p^n \bar{b} \cdot (n^n - s^n) = 0$.

Now, consider an alternative policy (s^c, p^c, F^c), such that $s^c = n^n$, $p^c = p^n$ and $F^c = a \cdot (e - s)$, where $a = \overline{b} \cdot (n^n - s^n)$. By (8), this policy induces compliance, since $f''(0) = 0$. Therefore, the induced pollution level is $n^c = s^c$. Since $s^c = n^n$ by definition, we then have $n^c = n^n$. Therefore, abatement costs and external damages are the same under both policies. Since $p^c = p^n$, expected monitoring costs are equal, too. By construction, both policies are such that the equilibrium marginal fine is the same. However, fines collected under the policy which induces compliance are zero and, therefore, there are no sanctioning costs. As a result, the alternative policy (s^c, p^c, F^c) is socially preferred, contradicting the initial assumption that the optimal policy induces non-compliance.

Now, by Proposition 4, the optimal fine is $F^c = \overline{a} \cdot (e - s)$, where $\overline{a} > 0$ is the upper limit charge per unit set in the legislation. Substituting this in Proposition 2, we obtain the desired result.

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