# 1 The parameters

Marginal abatement cost function: $\theta - e$

80 firms.

Half of them are *High MC* subjects ($\theta = \theta_H = 18$) and half *Low MC* subjects ($\theta = \theta_L = 9$)

$$-c'_H(e) = 18 - e_H$$

$$-c'_L(e) = 9 - e_L$$

To be compatible with this assumption we assume that

$$c_H(e) = 200 - 18e_H + \frac{e_H^2}{2}$$
de tal manera que
$$c'_H(e_H) = -18 + e_H$$
y
$$-c'_H(e_H) = 18 - e_H$$

and

$$c_L(e) = 200 - 9e_L + \frac{e_L^2}{2}$$
de tal manera que
$$c'_L(e_L) = -9 + e_L$$
y
$$-c'_L(e_L) = 18 - e_L$$

The regulator's target of total emissions is $L = 560$

We assume

$$f(e_i - s_i) = \phi(e_i - s_i)$$

with $\phi = 17.5$, (the value for these parameters are taken from Murphy and Stranlund's (2006) *High* enforcement strategy

The marginal penalty function is therefore

$$f'(.) = 17.5$$

In our problem $\pi$ is a choice variable.

# 2 Equal monitoring costs

First we assume.

$$\mu = \mu_i = 5$$

$$\beta = \beta_i = 0.5$$

## 2.1 Perfect information

### 2.1.1 The Cost-Effectiveness of Inducing Perfect Compliance

The condition to induce compliance or not is therefore

$$\mu_i \frac{f''(0)}{f'(0)} \leq \beta_i f'(0)$$

=>

$$0 \leq 0.5 \times 17.5$$

It is therefore cost effective to induce perfect compliance.

### 2.1.2 The allocation of emissions and probabilities in a perfectly enforced program based on emission standards

The regulator's problem is

$$\min_{\substack{(s_L,s_H)\\(\pi_L,\pi_H)}} 40 \times \left(200 - 18 \times s_H + \frac{s_H^2}{2}\right) + 40 \times \left(200 - 9 \times s_L + \frac{s_L^2}{2}\right) + 5 \times 40 \times (\pi_L + \pi_H)$$

$$\text{subject to:}$$

$$40 \times s_H + 40 \times s_L = 560$$

$$\min_{\substack{(s_L,s_H)\\(\pi_L,\pi_H)}} \left(200 - 18 \times s_H + \frac{s_H^2}{2}\right) + \left(200 - 9 \times s_L + \frac{s_L^2}{2}\right) + 5 \times (\pi_L + \pi_H)$$

$$\text{subject to:}$$

$$s_H + s_L = 14$$

Y como el regulador va a elegir y

$$\pi_H^* = \frac{-c_i'(s_H^*)}{f'(0)} = \frac{18 - s_H^*}{17.5}$$

$$\pi_L^* = \frac{-c_i'(s_L^*)}{f'(0)} = \frac{9 - s_L^*}{17.5}$$

Lagrange:

$$L = \left(200 - 18 \times s_H + \frac{s_H^2}{2}\right) + \left(200 - 9 \times s_L + \frac{s_L^2}{2}\right) + 5 \times (\pi_L + \pi_H) + \lambda \times (14 - s_H - s_L)$$

$$L = \left(200 - 18 \times s_H + \frac{s_H^2}{2}\right) + \left(200 - 9 \times s_L + \frac{s_L^2}{2}\right) + 5 \times \left(\frac{18 - s_H}{17.5} + \frac{9 - s_L}{17.5}\right) + \lambda \times (14 - s_H - s_L)$$

$$\frac{\partial L}{\partial s_H} = \frac{\partial\left(\left(200 - 18 \times s_H + \frac{s_H^2}{2}\right) + \left(200 - 9 \times s_L + \frac{s_L^2}{2}\right) + 5 \times \left(\frac{18 - s_H}{17.5} + \frac{9 - s_L}{17.5}\right) + \lambda \times (14 - s_H - s_L)\right)}{\partial s_H}$$
$$= s_H - \lambda - 18.286 = 0$$

$$\frac{\partial L}{\partial s_L} = \frac{\partial\left(\left(200 - 18 \times s_H + \frac{s_H^2}{2}\right) + \left(200 - 9 \times s_L + \frac{s_L^2}{2}\right) + 5 \times \left(\frac{18 - s_H}{17.5} + \frac{9 - s_L}{17.5}\right) + \lambda \times (14 - s_H - s_L)\right)}{\partial s_L}$$
$$= s_L - \lambda - 9.2857 = 0$$

$$\frac{\partial L}{\partial \lambda} = \frac{\partial\left(\left(200 - 18 \times s_H + \frac{s_H^2}{2}\right) + \left(200 - 9 \times s_L + \frac{s_L^2}{2}\right) + 5 \times \left(\frac{18 - s_H}{17.5} + \frac{9 - s_L}{17.5}\right) + \lambda \times (14 - s_H - s_L)\right)}{\partial \lambda}$$
$$= 14 - s_L - s_H = 0$$

El sistema queda

$$s_H - \lambda - 18.286 = 0$$
$$\lambda = s_H - 18.286$$

$$s_L - \lambda - 9.2857 = 0$$
$$\lambda = s_L - 9.2857$$

Por lo que

$$s_H - 18.286 = s_L - 9.2857$$

de donde

$$s_L = s_H - 9.0003$$

Sustituyendo en la tercera:

$$14 - (s_H - 9.0003) - s_H = 0$$

Solution is:

$$\mathbf{s}_H = \mathbf{11.5}$$

$$\mathbf{s}_L = 11.5 - 9.0003$$
$$\mathbf{s}_L = \mathbf{2.4997}$$

$$\pi_H^* = \frac{18 - \mathbf{11.5}}{17.5} = \mathbf{0.37143}$$

y

$$\pi_L^* = \frac{9 - \mathbf{2.4997}}{17.5} = \mathbf{0.37145}$$

### 2.1.3 Costo total del programa:

$$40 \times \left(200 - 18 \times s_H + \frac{s_H^2}{2}\right) + 40 \times \left(200 - 9 \times s_L + \frac{s_L^2}{2}\right) + 5 \times 40 \times (\pi_L + \pi_H)$$

$$40 \times \left(200 - 18 \times \mathbf{11.5} + \frac{\mathbf{11.5}^2}{2}\right) + 40 \times \left(200 - 9 \times \mathbf{2.4997} + \frac{\mathbf{2.4997}^2}{2}\right)$$
$$+5 \times 40 \times (\mathbf{0.37145} + \mathbf{0.37145})$$
$$= \mathbf{9738.7}$$

de los cuales 9590.1 son costos de abatimiento y 148.58 son costos de monitoreo.

### 2.1.4 Total emissions:

$40 * 11.5 + \mathbf{40} * \mathbf{2.5} = 560$

### 2.1.5 The allocation of emissions and probabilities in a perfectly enforced program based on tradable permits

In a perfectly competitive emission permits market, emissions are determined such that $-c'(e) = p^*$, the equilibrium price of permits. In a perfectly enforced market it is also true that $e = l$. From the firms' behaviors we know that in equilibrium

$$-c'_H(l_H) = 18 - l_H = p^*$$

$$-c'_L(l_L) = 9 - l_L = p^*$$

$$40 \times l_H + 40 \times l_L = 560$$

From the first two equations

$$18 - l_H = 9 - l_L$$

$$l_H = l_L + 9$$

Substituting in the third equation

$$40 \times (l_L + 9) + 40 \times l_L = 560$$

, Solution is:

$$\mathbf{l}_L^* = \frac{5}{2} = \mathbf{2.5}$$

$$\mathbf{l}_H^* = 2.5 + 9 = \mathbf{11.5}$$
$$18 - l_H^* = 18 - 11.5 = \mathbf{6.5} = \mathbf{p}^*$$

Finally,

$$\boldsymbol{\pi}^* = \frac{p^*}{f'(0)} = \frac{\mathbf{6.5}}{17.5} = \mathbf{0.37143}$$

### 2.1.6 Costo total del programa:

$$40 \times \left(200 - 18 \times l_H^* + \frac{l_H^{*2}}{2}\right) + 40 \times \left(200 - 9 \times l_L^* + \frac{l_L^{*2}}{2}\right) + 5 \times 80 \times (\boldsymbol{\pi}^*) =$$

$$40 \times \left(200 - 18 \times 11.5 + \frac{(11.5)^2}{2}\right) + 40 \times \left(200 - 9 \times 2.5 + \frac{2.5^2}{2}\right) + 5 \times 80 \times (\mathbf{0.37143})$$

$$\mathbf{9738.6}$$

de los cuales 9590.1 son costos de abatimiento y 148.58 son costos de monitoreo.

### 2.1.7 Conclusión:

Como dice nuestro modelo, asumiendo costos de una inspección igual para todas las firmas, el costo de un programa de permisos es igual al costo de un programa basado en estándares. *Moreover, with additive differentiation of the marginal abatement costs of the firms and equal monitoring costthere is absolutely no difference between the solutions of the programs. Standards are equal to permit holdings in equilibrium, and monitoring probabilities are the same for every firm in both programs and between programs. This is not true when the differentiation in the marginal abatement costs of the firms is multiplicative (i.e: firms differ in the slope of their marginal abatement costs).*

# 3 Different monitoring costs

There is no a priori relation between the type of the firm ($H$ or $L$) and the cost of auditing that firm. These costs can differ because of the localization of the plant, the number of discharge points, etc. We assume.

$$\begin{aligned} \mu_H &= 20 \\ \mu_L &= 2 \end{aligned}$$

## 3.1 Perfect Information

### 3.1.1 The cost-effectiveness of inducing compliance

The condition to induce compliance or not is the same as before. Therefore it is therefore cost effective to induce perfect compliance on both type of firms.

### 3.1.2 The allocation of emissions and probabilities in a perfectly enforced program based on emission standards

The regulator's problem is

$$\min_{\substack{(s_L,s_H)\\(\pi_L,\pi_H)}} 40\times\left(200-18\times s_H+\frac{(s_H)^2}{2}\right)+40\times\left(200-9\times s_L+\frac{s_L^2}{2}\right)+20\times 40\times\pi_H+2\times 40\times\pi_L$$

$$\text{subject to:}$$

$$40\times s_H+40\times s_L=560$$

$$\min_{\substack{(s_L,s_H)\\(\pi_L,\pi_H)}} \left(200-18\times s_H+\frac{s_H^2}{2}\right)+\left(200-9\times s_L+\frac{s_L^2}{2}\right)+20\times\pi_H+2\times\pi_L$$

$$\text{subject to:}$$

$$s_H+s_L=14$$

Y como el regulador va a elegir y

$$\pi_H^*=\frac{-c_i'(s_H^*)}{f'(0)}=\frac{18-s_H^*}{17.5}$$

$$\pi_L^*=\frac{-c_i'(s_L^*)}{f'(0)}=\frac{9-s_L^*}{17.5}$$

Lagrange:

$$L=\left(200-18\times s_H+\frac{s_H^2}{2}\right)+\left(200-9\times s_L+\frac{s_L^2}{2}\right)+20\times\pi_H+2\times\pi_L+\lambda\times(14-s_H-s_L)$$

Se puede expresar como

$$L=\left(200-18\times s_H+\frac{s_H^2}{2}\right)+\left(200-9\times s_L+\frac{s_L^2}{2}\right)+20\times\left(\frac{18-s_H}{17.5}\right)+2\times\left(\frac{9-s_L}{17.5}\right)+\lambda\times(14-s_H-s_L)$$

$$\begin{aligned}\frac{\partial L}{\partial s_H} &= \frac{\partial\left(\begin{array}{c}\left(200-18\times s_H+\frac{(s_H)^2}{2}\right)+\left(200-9\times s_L+\frac{(s_L)^2}{2}\right)+20\times\left(\frac{18-s_H}{17.5}\right)+2\times\left(\frac{9-s_L}{17.5}\right)\\+\lambda\times(14-s_H-s_L)\end{array}\right)}{\partial s_H}\\ &= s_H-\lambda-19.143=0\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial s_L} &= \frac{\partial\left(\begin{array}{c}\left(200-18\times s_H+\frac{(s_H)^2}{2}\right)+\left(200-9\times s_L+\frac{(s_L)^2}{2}\right)+20\times\left(\frac{18-s_H}{17.5}\right)+2\times\left(\frac{9-s_L}{17.5}\right)\\+\lambda\times(14-s_H-s_L)\end{array}\right)}{\partial s_L}\\ &= s_L-\lambda-9.1143=0\end{aligned}$$

$$\frac{\partial L}{\partial \lambda} = 14 - s_L - s_H = 0$$

El sistema queda

$$s_H - \lambda - 19.143 = 0$$
$$\lambda = s_H - 19.143$$

$$s_L - \lambda - 9.1143 = 0$$
$$\lambda = s_L - 9.1143 = 0$$

Por lo que

$$s_H - 19.143 = s_L - 9.1143$$

de donde

$$s_L = s_H - 10.029$$

Sustituyendo en la tercera:

$$14 - (s_H - 10.029) - s_H = 0$$

Solution is:

$$\mathbf{s}_H = \mathbf{12.015}$$

$$s_L = \mathbf{12.015} - 10.029$$
$$\mathbf{s}_L = \mathbf{1.986}$$

$$\pi_H^* = \frac{18 - \mathbf{12.015}}{17.5} = \mathbf{0.342}$$

y

$$\pi_L^* = \frac{9 - \mathbf{1.986}}{17.5} = \mathbf{0.4008}$$

### 3.1.3 Total emissions:

$40 * \mathbf{12.015} + \mathbf{40} * \mathbf{1.986} = 560.04$

### 3.1.4 Costo total del programa de estándares:

$$40\times\left(200-18\times s_H+\frac{(s_H)^2}{2}\right)+40\times\left(200-9\times s_L+\frac{s_L^2}{2}\right)+20\times40\times\pi_H+2\times40\times\pi_L$$

$$40\times\left(200-18\times\mathbf{12.015}+\frac{(\mathbf{12.015})^2}{2}\right)+40\times\left(200-9\times\mathbf{1.986}+\frac{(\mathbf{1.986})^2}{2}\right)$$
$$+20\times40\times\mathbf{0.342}+2\times40\times\mathbf{0.4008}$$
$$=\mathbf{9906}$$

de los cuales 9600.3 son costos de abatimiento y 305.66 son costos de monitoreo.

### 3.1.5 The allocation of emissions and probabilities in a perfectly enforced program based on tradable permits

In a perfectly competitive emission permits market, emissions are determined such that $-c'(e)=p^*$, the equilibrium price of permits. In a perfectly enforced market it is also true that $e=l$. From the firms' behaviors we know that in equilibrium

$$-c_H'(l_H)=18-l_H=p^*$$

$$-c_L'(l_L)=9-l_L=p^*$$

$$40\times l_H+40\times l_L=560$$

From the first two equations

$$18-l_H=9-l_L$$
$$l_H=l_L+9$$

Substituting in the third equation

$$40\times(l_L+9)+40\times l_L=560$$

, Solution is:

$$\mathbf{l}_L^*=\frac{5}{2}=\mathbf{2.5}$$
$$\mathbf{l}_H^*=2.5+9=\mathbf{11.5}$$
$$18-l_H^*=18-11.5=\mathbf{6.5}=\mathbf{p}^*$$

Finally,

$$\boldsymbol{\pi}^*=\frac{p^*}{f'(0)}=\frac{\mathbf{6.5}}{17.5}=\mathbf{0.37143}$$

### 3.1.6 Costo total del programa:

$$40 \times \left(200 - 18 \times l_H^* + \frac{l_H^{*2}}{2}\right) + 40 \times \left(200 - 9 \times l_L^* + \frac{l_L^{*2}}{2}\right) + (20 + 2) \times 40 \times \boldsymbol{\pi}^* =$$

$$40 \times \left(200 - 18 \times 11.5 + \frac{(11.5)^2}{2}\right) + 40 \times \left(200 - 9 \times 2.5 + \frac{2.5^2}{2}\right) + 22 \times 40 \times (\mathbf{0.371\,43})$$

$$\mathbf{9916.9}$$

de los cuales 9590 son costos de abatimiento y 326. 86 son costos de monitoreo.

### 3.1.7 Conclusion:

Como dice nuestro modelo, asumiendo costos de una inspección distintos para los distintos tipos de firmas, **el costo de un programa de permisos es mayor** que el costo de un programa basado en estándares. Notar también que el programa de permisos tiene unos costos de abatimiento totales que son menores a los costos totales de abatimiento del programa de estándares (9590 del primero versus 9600.3 del segundo). Por lo tanto, la diferencia se explica porque los costos de monitoreo del programa de permisos son mayores que los costos de monitoreo del programa de estandares (326. 86 del primero versus 305.66 del segundo).

El ejemplo sugiere que los costos de monitoreo pueden ser poco importantes con relación a los costos de abatimiento. En dichos casos los permisos pueden ser superiores en términos de costo-efectividad aun cuando no minimicen costos de monitoreo/fiscalización. ¿Cuán importantes son los costos de enforcement dentro de los costos totales de regulación?. Probablemente depende del tamaño de población regulada, meta ambiental, nivel de cumplimiento objetivo, precisión deseada para medición de emisiones, tecnología de monitoreo, nivel y poder disuasivo de las sanciones, etc. Esta diferencia en los costos de monitoreo ha de ser comparada con la diferencia a favor lograda por los permisos en cuanto a costos de abatimiento. Esta última diferencia dependerá a su vez de la heterogeneidad de los costos de abatimiento marginales entre las firmas.

## 3.2 Imperfect Information

The regulator does not know that there are 40 firms of each type, but assigns a probability of one half the vent that any firm is $H$ or $L$.

### 3.2.1 The Cost-Effectiveness of Inducing Expected Perfect Compliance

When the regulator has imperfect information, it has to decide whether to induce expected compliance or not according to

$$\begin{aligned} & \frac{\pi_i f''(0)}{f'(0)}\mu_i + \frac{\pi_i f''(0)}{f'(0)}\left(\beta_i E\left[f(e_i(s_i,\pi_i,\theta_i)-s_i]\right) \\ & +\beta_i\pi_i Cov\left[f'(e_i(s_i,\pi_i,\theta_i)-s_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right] \\ \leq \ & \beta_i\pi_i f'(0) - Cov\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right] \\ & -\beta_i\pi_i\left(Cov\left[f'(e_i(s_i,\pi_i,\theta_i)-s_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right]\right) \end{aligned} \qquad \text{((8'))}$$

evaluated at $E(e_i(s_i,\pi_i,\theta_i) = s_i$.

But if $f''(0) = 0$, as we have assumed, this condition simplifies to

$$0 \leq \beta_i\pi_i f'(0)$$

Therefore, the regulator has to induce compliance. Given that it cannot observe the abatement costs, it designs the programs such that induces expected compliance

### 3.2.2 The allocation of emissions and probabilities when inducing expected full compliance in a program based on emission standards

The regulator's problem is:

$$\min_{\substack{(s_1,s_2,..,s_n)\\(\pi_1,\pi_2,..,\pi_n)}} E\left[\sum_{i=1}^n c_i(e_i,\theta_i) + \sum_{i=1}^n \mu_i\pi_i + \sum_{i=1}^n \beta_i\pi_i f(e_i - s_i)\right]$$

$$\text{subject to:}$$

$$e_i = e_i(s_i,\pi_i,\theta_i)$$

$$E\left[\sum_{i=1}^n e_i\right] = T$$

$$s_i = E\left(e_i\right) \ \forall i = 1,...n$$

where the last constraint indicates that the regulator chooses the standards and the inspections probabilities such that the expected level of emissions of every plant is equal to the standard faced by the plant (expected compliance).

$$\begin{gathered}\min_{(s,\pi_H,\pi_L)} 80\times E\left[c(e(s,\pi,\theta),\theta)\right]+(20+2)\times 40\times\pi\\ +0.5\times 17.5\times\pi\times 80\times E(\max(e(s,\pi,\theta)-s,0))\\ \text{subject to:}\\ 80\times E\left[e(s,\pi,\theta)\right]=560\\ s=E\left(e(s,\pi,\theta)\right)\;\;\forall i=1,...n\end{gathered}$$

$$\begin{gathered}\min_{\substack{(s)\\(\pi)}} 80\times\left(\begin{array}{c}\frac{1}{2}\times\left(200-18\times e(s,\pi,\theta)+\frac{e(s,\pi,\theta)^2}{2}\right)\\ +\frac{1}{2}\times\left(200-9\times e(s,\pi,\theta)+\frac{e(s,\pi,\theta)^2}{2}\right)\end{array}\right)\\ +(20+2)\times 40\times\pi+0.5\times 17.5\times\pi\times 80\times E(\max(e(s,\pi,\theta)-s,0))\\ \text{subject to:}\\ 80\times E\left[e(s,\pi,\theta)\right]=560\\ s=E\left(e(s,\pi,\theta)\right)\;\;\forall i=1,...n\end{gathered}$$

$$\begin{gathered}\min_{\substack{(s)\\(\pi)}} 80\times\left(\frac{1}{2}e^2-\frac{27}{2}e+200\right)+(20+2)\times 40\times\pi\\ +0.5\times 17.5\times\pi\times 80\times E(\max(e(s,\pi,\theta)-s,0))\\ \text{subject to:}\\ 80\times s=560\end{gathered}$$

$$\mathbf{s^*=560/80=7}$$

$$\begin{aligned}\frac{\partial E\left[c(e,\theta)\right]}{\partial e}&=\frac{\partial\left(\frac{1}{2}e^2-\frac{27}{2}e+200\right)}{\partial e}=e-\frac{27}{2}\\ E\left[c'(e,\theta)\right]&=\frac{1}{2}\times(-18+e)+\frac{1}{2}\times(-9+e)=e-\frac{27}{2}\end{aligned}$$

$$\pi^*=\frac{-E\left[c'(s,\theta)\right]}{f'(0)}=\frac{-\mathbf{7}+\frac{27}{2}}{17.5}=\mathbf{0.37143}$$

and of course $E\left[e(s,\pi,\theta)\right]=7$.

### 3.2.3 Expected costs of the program based on emissions standards with imperfect information

$$\begin{gathered}80\times\left(\frac{1}{2}\times 7^2-\frac{27}{2}\times 7+200\right)+(20+2)\times 40\times\mathbf{0.37143}\\ +0.5\times 17.5\times\mathbf{0.37143}\times 80\times E(\max(e(s,\pi,\theta)-s,0))\end{gathered}$$

If there are 80 firms, half of which are $H$, with abatement costs

$$-c'(e, \theta_H) = 18 - e$$

and half of which are $L$, with abatement costs

$$-c'(e, \theta_L) = 9 - e$$

and the standard for both is

$$\mathbf{s}^* = \mathbf{7}$$

the probability for both types is

$$\pi^* = \mathbf{0.371\,43}$$

firms $H$ do not comply with the standard because

$$-c'(s = 7, \theta_H) = 18 - 7 = 11 > \pi^* \times f'(0) = \mathbf{0.371\,43} \times \mathbf{17.5} = 6.5$$

Therefore, $e_H = e(s, \pi, \theta_H)$ :

$$\begin{aligned} -c'(e, \theta_H) &= \mathbf{0.371\,43} \times \mathbf{17.5} \\ 18 - e_H &= \mathbf{0.371\,43} \times \mathbf{17.5} \end{aligned}$$

$$\mathbf{e}_H = \mathbf{11.5}$$

$$e_H - s = 11.5 - 7 = 4.5$$

On the contrary, firms $L$ comply because

$$-c'(s = 7, \theta_L) = 9 - 7 = 2 < \pi^* \times f'(0) = \mathbf{0.371\,43} \times \mathbf{17.5} = 6.5$$

$$\mathbf{e}_L = \mathbf{7} = \mathbf{s}$$

Therefore, the total expected cost of the program is

$$80 \times \left( \frac{1}{2} \times 7^2 - \frac{27}{2} \times 7 + 200 \right) + (20 + 2) \times 40 \times \mathbf{0.371\,43}$$
$$+0.5 \times 17.5 \times \mathbf{0.371\,43} \times 80 \times E(\max(e(s, \pi, \theta) - s, 0))$$

$$10400 + 326.86 + 260.0 \times \frac{1}{2} \times 4.5$$

$$\mathbf{11312}$$

#### 3.2.4 Expected level of emissions of the program based on emissions standards with imperfect information

$$80 * (\frac{1}{2} * 11.5 + \frac{1}{2} * 7) = 740$$

### 3.2.5 Actual cost of the program based on emissions standards with imperfect information

$$40\times\left(200-18\times e_H+\frac{(e_H)^2}{2}\right)+40\times\left(200-9\times e_L+\frac{e_L^2}{2}\right)+(20+2)\times 40\times\pi$$
$$+40\times 0.5\times\pi\times 17.5\times(e_H-s)+40\times 0.5\times\pi\times 17.5\times(e_L-s)$$

$$40\times\left(200-18\times\mathbf{11.5}+\frac{(\mathbf{11.5})^2}{2}\right)+40\times\left(200-9\times 7+\frac{(7)^2}{2}\right)$$
$$+(20+2)\times 40\times\mathbf{0.371\,43}$$
$$+40\times 0.5\times\mathbf{0.371\,43}\times 17.5\times(\mathbf{11.5}-7)$$
$$=8825.0+326.86+585.0$$

$$9736.9$$

### 3.2.6 Actual level of emissions of the program based on emissions standards with imperfect information

$$40*11.5+40*7=740$$

### 3.2.7 The allocation of emissions and probabilities when inducing expected full compliance in a program based on tradable permits

The regulator designs the program such that $E(l_i)=E(e_i)$, $i=1,...,n.$

Knowing that a perfectly competitive market of emission permits minimizes the total abatement costs (whatever the level of permits issued and compliance status of firms), the regulators problem is

$$\min_{(\pi_1,\pi_2,..,\pi_n)} E\left(\sum_{i=1}^{n}\mu_i\pi_i+\sum_{i=1}^{n}\beta_i\pi_i f(e_i(p^*,\theta)-l_i(p^*,\pi_i,\theta))\right)$$
subject to
$$E(e_i(p^*,\theta))=E(l_i(p^*,\pi_i,\theta)),\ i=1,...,n$$
$$E\left(\sum_{i=1}^{n}l_i(p^*,\pi_i,\theta)\right)=L=T=560$$

The solution to this problem requires an estimation of the level of the equilibrium price of permits, $p^*$.

Now, the problem that a polluting firm that participates in a competitive market for pollution permits solves is

$$\begin{array}{rc} min_{e,l} & c(e,\theta) + p(l - l_0) + \pi \times f(e - l) \\ & s.t \quad e - l \geq 0. \end{array}$$

where, $l_0$ denote the possible initial free allocation of licenses to the firm. The Kuhn-Tucker conditions for a positive level of emissions and demand for permits are:

$$\begin{aligned} c'(e,\theta) + \pi f'(e - l) - \eta &= 0 \\ p - \pi f'(e - l) + \eta &= 0 \\ e - l &\geq 0, \eta \geq 0, \eta \times (e - l) = 0 \end{aligned}$$

In the case of a constant marginal penalty, as our example:

$$\begin{aligned} c'(e,\theta) + \pi \times \phi - \eta &= 0 \\ p - \pi \times \phi + \eta &= 0 \\ e - l &\geq 0, \eta \geq 0, \eta \times (e - l) = 0 \end{aligned}$$

From these Kuhn-Tucker it is relatively easy to show that a firm chooses its emissions up to the point where its marginal abatement cost equals the market permit price regardless of its compliance status; that is, $p = -c'(e,\theta)$. Thus, the firm's choice of emissions is a function of the permit price and the shift parameter; that is $e = e(p,\theta)$.

As for the firm's demand for permits with a constant marginal penalty, the firms goes to the market and compares the marginal gain from being in violation (the license price), with the marginal expected cost (the marginal expected penalty, $\pi \times \phi$), according to the second and third condition. Therefore, the demand function for permits of a firm is

$$l(p,\pi,\theta) = \left\{ \begin{array}{c} 0 \text{ if } p > \pi \times \phi \\ (0, e(p,\theta)) \text{ if } p = \pi \times \phi \\ e(p,\theta) \text{ if } p < \pi \times \phi \end{array} \right\}$$

for $\theta = (\theta_L, \theta_H)$,and $e(p,\theta_H)$ is the solution to

$$-c'(e,\theta_H) = 18 - e = p$$

and $e(p,\theta_L)$ is the solution to

$$-c'(e,\theta_L) = 9 - e = p$$

, or

$$\begin{aligned} e(p,\theta_H) &= 18 - p \\ e(p,\theta_L) &= 9 - p \end{aligned}$$

Therefore, the aggregate demand for permits in our example is

$$\sum_{i=1}^{80} l_i(p,\pi,\theta) = L^D$$

$$\left\{ \begin{array}{c} 0 \text{ if } p > \pi \times 17.5 \\ (0, 40 \times (18-p) + 40 \times (9-p)) \text{ if } p = \pi \times 17.5 \\ 40 \times (18-p) + 40 \times (9-p) \text{ if } p < \pi \times 17.5 \end{array} \right\}$$

Of course, the regulator does not know that there are 40 firms of each type. It has therefore to estimate the aggregate demand for permits, and from this estimate the equilibrium price of permits. We call this estimate $\bar{p}$.

TENGO QUE REPETIR EL ANALISIS ESTIMANDO LD Y EN BASE A ESO P BARRA Y PASAR ESTE ANALISIS DEL ACTUAL LEVEL OF P* PARA LA SIGUIENTE SUBSECCION.

¿What is the equilibrium price of permits $p^*$ for a supply of permits $L^S = 560$?

The equilibrium price of permits is $p^* = \pi \times 17.5$ if $560 \leq \hat{L}$, where $\hat{L}$ is the value of the aggregate demand for permits $L^D = 40 \times (18-p) + 40 \times (9-p) = 1080 - 80p$ for which its inverse is equal to $\pi \times 17.5$, or

$$\frac{1080 - \hat{L}}{80} = \pi \times 17.5$$

$$\hat{L} = 1080 - 1400 \times \pi$$

Therefore, the equilibrium price of permits is $p^* = \pi \times 17.5$ if $560 \leq 1080 - 1400 \times \pi$, or if

$$\pi \in \left[0, \frac{13}{35}\right]$$

If $\pi = \frac{13}{35} = 0.371\,43, p^* = \frac{13}{35} * 17.5 = 6.5$ and $L^D = 1080 - 80 \times 6.5 = 560$. It is clear that any value of $\pi < 0.37143$ would make $L^D$ possibly larger than 560. Therefore, the minimum value of the monitoring probability that assures the regulator $L^D = L^S = 560$ is $\pi = 0.371\,43$. On the other hand, if $\pi > 0.371\,43$, $p^*$ :

$$\begin{aligned} 1080 - 80 \times p^* &= 560 \\ p^* &= 6.5 \end{aligned}$$

but it will not be a cost effective solution because the regulator could decrease and still attain $L^D = L^S = 560$. Therefore, assuming that $l(p,\pi,\theta) = e(p,\theta)$ when $p = \pi \times 17.5$, we have that

$$\begin{aligned} p^* &= 6.5 \\ \pi_H &= \pi_L = \boldsymbol{\pi}^* = \frac{p^*}{17.5} = 0.371\,43 \end{aligned}$$

$$\min_{\pi} (\mu_H + \mu_L) \times 40 \times \pi$$
$$+0.5 \times \pi \times 17.5 \times 80 \times E\left(\max(e\left(p^*, \theta\right) - l\left(p^*, \pi, \theta\right), 0)\right)$$
$$\text{subject to}$$
$$E(e_i\left(p^*, \theta\right)) = E\left(l_i\left(p^*, \pi_i, \theta\right)\right),\ i = 1, ..., n$$
$$80 * E\left(l_i\left(p^*, \pi_i, \theta\right)\right) = 560$$

Because

$$\begin{aligned} E\left(\max(e\left(p^*, \theta\right) - l\left(p^*, \pi, \theta\right), 0)\right) &= \frac{1}{2} \times \max\left(e\left(p^*, \theta_H\right) - l\left(p^*, \pi, \theta_H\right), 0\right) \\ &\quad + \frac{1}{2} \times \max\left(e\left(p^*, \theta_L\right) - l\left(p^*, \pi, \theta_L\right), 0\right) \end{aligned}$$

We have

$$\min_{\pi} (\mu_H + \mu_L) \times 40 \times \pi$$
$$+0.5 \times \pi \times 17.5 \times 80 \times \begin{pmatrix} \frac{1}{2} \times \max\left(e\left(p^*, \theta_H\right) - l\left(p^*, \pi, \theta_H\right), 0\right) \\ +\frac{1}{2} \times \max\left(e\left(p^*, \theta_L\right) - l\left(p^*, \pi, \theta_L\right), 0\right) \end{pmatrix}$$
$$\text{subject to}$$
$$E(e_i\left(p^*, \theta\right)) = E\left(l_i\left(p^*, \pi, \theta\right)\right),\ i = 1, ..., n$$
$$80 * E\left(l_i\left(p^*, \pi_i, \theta\right)\right) = 560$$

Therefore, the best the regulator can do is to estimate $\bar{p}$ according to

$$-E\left(c_i'\left(e, \theta\right)\right) = \bar{p}$$

for all $i = 1, ..., 80$.Given the linearity of $c'$, this can be written as

$$-c_i'\left(e, \bar{\theta}\right) = \bar{p}$$

where $\bar{\theta} = E\left(\theta\right)$. Given that all firms are $H$ or $L$ with equal probability, this condition can be written as

$$0.5 \times 18 + 0.5 \times 9 - e_i = \bar{p}$$

for all $i = 1, ..., 80$, or

$$13.5 - e_i = \bar{p}$$

Therefore,

$$E(e_i) = e_i\left(\bar{p}, \bar{\theta}\right) = \bar{\theta} - \bar{p} = 13.5 - \bar{p}$$

for all $i = 1, ..., 80$. Using $E(e_i\left(p, \theta\right)) = E\left(l_i\left(p, \pi, \theta\right)\right)$ and the market equilibrium condition $80 * E\left(l_i\left(p, \pi, \theta\right)\right) = 560$

$$\begin{aligned} 13.5 - \bar{p} &= 560/80 = 7 \\ \bar{p} &= 6.5 \end{aligned}$$

Given this estimate of the equilibrium price of permits, the regulator now estimates $e(p^*,\theta_H), l(p^*,\pi,\theta_H), e(p^*,\theta_L)$ and $l(p^*,\pi,\theta_L)$ with $e(\bar{p},\theta_H), l(\bar{p},\pi,\theta_H), e(\bar{p},\theta_L)$ and $l(\bar{p},\pi,\theta_L)$. Knowing that a firm chooses its emissions up to the point where its marginal abatement cost equals the market permit price regardless of its compliance status, $e(\bar{p},\theta_H)$ is the solution to

$$-c'(e,\theta_H) = 18 - e = \bar{p}$$

and $e(\bar{p},\theta_L)$ is the solution to

$$-c'(e,\theta_L) = 9 - e = \bar{p}$$

Therefore,

$$e(\bar{p},\theta_H) = 18 - 6.5 = 11.5$$

and

$$e(\bar{p},\theta_L) = 9 - 6.5 = 2.5$$

Asuming $l(\bar{p},\pi,\theta_H) = e(\bar{p},\theta_H) = 11.5$ when $\bar{p} = \pi \times 17.5$, the solution to the problem is $\pi = \pi^* = \frac{\bar{p}}{17.5} = \frac{6.5}{17.5} = 0.371\,43$. Finally, we also asume that $l(\bar{p},\pi,\theta_L) = e(\bar{p},\theta_L) = 2.5$ when $\bar{p} = 6.5 = \pi^* \times 17.5$.

#### 3.2.8 Expected costs of the program based on tradable permits with imperfect information

Given that the regulator does not know the actual number of firms $H$ and firms $L$, the expected cost of this program is

$$80 \times E\left[c_i(7,\theta_i)\right] + (20+2) \times 40 \times \pi^* + 0.5 \times 17.5 \times \pi^* \times 0$$

$$80 \times \left(\frac{1}{2} \times \left(200 - 18 \times 7 + \frac{7^2}{2}\right) + \frac{1}{2} \times \left(200 - 9 \times 7 + \frac{7^2}{2}\right)\right)$$
$$+ (20+2) \times 40 \times \mathbf{0.371\,43}$$

$$10400 + 326.86$$

$$\mathbf{10727}$$

#### 3.2.9 Actual cost of the program based on tradable permits with imperfect information

The actual cost of the programs is

$$40 \times \left(200 - 18 \times e(p^*,\theta_H) + \frac{e(p^*,\theta_H)^2}{2}\right) + 40 \times \left(200 - 9 \times e(p^*,\theta_L) + \frac{e(p^*,\theta_L)^2}{2}\right)$$
$$+ (20+2) \times 40 \times \pi^*$$
$$+0.5 \times \pi \times 17.5 \times \left(\begin{array}{c} 40 \times \max\left(e(p^*,\theta_H) - l(p^*,\pi,\theta_H), 0\right) \\ +40 \times \max\left(e(p^*,\theta_L) - l(p^*,\pi,\theta_L), 0\right) \end{array}\right)$$

where $p^*$, $e(p^*, \theta_H)$, $e(p^*, \theta_L)$, $l(p^*, \pi, \theta_H)$ and $l(p^*, \pi, \theta_L)$ are the solution to

$$\begin{aligned} -c_H'(e_H) &= 18 - e_H = p^* \\ -c_L'(e_L) &= 9 - e_L = p^* \\ l(p^*, \pi, \theta_H) + l(p^*, \pi, \theta_L) &= 14 \end{aligned}$$

with

$$l(p^*, \pi, \theta_H) = \left\{ \begin{array}{c} 0 \text{ if } p^* > \pi \times 17.5 \\ (0, e(p^*, \theta_H)) \text{ if } p^* = \pi \times 17.5 \\ e(p^*, \theta_H) \text{ if } p^* < \pi \times 17.5 \end{array} \right\}$$

and similarly for firms $L$.

Now, $l(p^*, \pi, \theta_H) = l(p^*, \pi, \theta_L) = 0$ is not and equilibrium. At the same time, $p^* < \pi \times 17.5$ is not a cost effective choice of the monitoring probability by the regulator. Therefore, assuming that when $p^* = \pi \times 17.5$, $l(p^*, \pi, \theta) = e(p^*, \theta)$, from the above three conditions we have

$$e(p^*, \theta_H) + e(p^*, \theta_L) = 14$$

and

$$e(p^*, \theta_H) - e(p^*, \theta_L) = 9$$

from where,

$$\begin{aligned} e(p^*, \theta_H) &= 11.5 = l(p^*, \pi, \theta_H) \\ e(p^*, \theta_L) &= 2.5 = l(p^*, \pi, \theta_L) \\ p^* &= 18 - 11.5 = 6.5 \\ \pi^* &= \frac{p^*}{17.5} = \frac{6.5}{17.5} = 0.371\,43 \end{aligned}$$

Therefore, the ex-post expected cost of the program is

$$40 \times \left(200 - 18 \times 11.5 + \frac{11.5^2}{2}\right) + 40 \times \left(200 - 9 \times 2.5 + \frac{2.5^2}{2}\right)$$
$$+ (20 + 2) \times 40 \times 0.37143$$

$$= 9590.0 + 326.\,86$$

$$= \mathbf{9916.\,9}$$

## 3.3 Conclusion

We have constructed an example in which firms are of two different types (40 firms of each type) and the regulator cannot distinguish a priori if a firm is type $H$ or type $L$. In this scenario, the regulator assigns the value of 1/2 to the probability that any firm is $H$ or $L$. Based on this, it designs a program based on emission standards and a program based on tradable permits to minimize the expected cost of the program, inducing expected compliance. As shown in the table below, the a priori costs of the TDP program is lower. ex post, the program based on standards is cheaper but the toal level of emissions is larges than the target $(740 - 560)$.

| | Expected Cost of the program | | Level of emissions | |
|---|---|---|---|---|
| | A priori | Ex post | Estimated a priori | Actual |
| Emission standards | 11312 | 9736.9 | 740 | 740 |
| Tradable discharge permits | 10727 | 9916.9 | | |

## 3.4 ¿How do these costs change under a different degree of "ignorance" of the regulator?

We assume that the number of $H$ firms and $L$ firms is still 40 each, but that the regulator now assigns a value of 3/4 to the probability that a firm is $H$.

### 3.4.1 The allocation of emissions and probabilities when inducing expected full compliance in a program based on emission standards

The regulator's problem is:

$$\min_{\substack{(s)\\(\pi)}} 80 \times \left( \begin{array}{c} \frac{3}{4} \times \left(200 - 18 \times e(s,\pi,\theta) + \frac{e(s,\pi,\theta)^2}{2}\right) \\ +\frac{1}{4} \times \left(200 - 9 \times e(s,\pi,\theta) + \frac{e(s,\pi,\theta)^2}{2}\right) \end{array} \right)$$
$$+ (20+2) \times 40 \times \pi + 0.5 \times 17.5 \times \pi \times 80 \times E(\max(e(s,\pi,\theta) - s, 0))$$
$$\text{subject to:}$$
$$80 \times E\left[e(s,\pi,\theta)\right] = 560$$
$$s = E\left(e(s,\pi,\theta)\right)\ \forall i = 1, ...n$$

$$\min_{\substack{(s)\\(\pi)}} 80 \times \left(\frac{1}{2}e^2 - \frac{63}{4}e + 200\right) + (20+2) \times 40 \times \pi$$
$$+0.5 \times 17.5 \times \pi \times 80 \times E(\max(e(s,\pi,\theta) - s, 0))$$
$$\text{subject to:}$$
$$80 \times s = 560$$

$$\mathbf{s^{*} = 560/80 = 7}$$

$$E\left[c'(e,\theta)\right] = \frac{3}{4} \times (-18 + e) + \frac{1}{4} \times (-9 + e) = e - \frac{63}{4}$$

$$\pi^{*} = \frac{-E\left[c'(s,\theta)\right]}{f'(0)} = \frac{-\mathbf{7} + \frac{63}{4}}{17.5} = 0.5$$

### 3.4.2 Expected costs of the program based on emissions standards with imperfect information

$$80 \times \left(\frac{1}{2}e^2 - \frac{63}{4}e + 200\right) + (20 + 2) \times 40 \times \mathbf{0.5}$$
$$+0.5 \times 17.5 \times \mathbf{0.5} \times 80 \times E(\max(e(s, \pi, \theta) - s, 0))$$

Firms $H$ do not comply with the standard because

$$-c'(s = 7, \theta_H) = 18 - 7 = 11 > \pi^* \times f'(0) = \mathbf{0.5} \times \mathbf{17.5} = 8.75$$

Therefore, $e_H = e(s, \pi, \theta_H)$ :

$$18 - e_H = 8.75$$

$$\mathbf{e}_H = \mathbf{9.25}$$

On the contrary, firms $L$ comply because

$$-c'(s = 7, \theta_L) = 9 - 7 = 2 < \pi^* \times f'(0) = \mathbf{8.75}$$

$$\mathbf{e}_L = \mathbf{7} = \mathbf{s}$$

Therefore, the total expected cost of the program is

$$80 \times \left(\frac{1}{2} \times 7^2 - \frac{63}{4} \times 7 + 200\right) + (20 + 2) \times 40 \times \mathbf{0.5}$$
$$+0.5 \times 17.5 \times \mathbf{0.5} \times 80 \times E(\max(e(s, \pi, \theta) - s, 0))$$

$$9140.0 + 440.0 + 260.0 \times \frac{3}{4} \times 2.25$$
$$10019.$$

### 3.4.3 Actual cost of the program based on emissions standards with imperfect information

$$40 \times \left(200 - 18 \times \mathbf{9.25} + \frac{(\mathbf{9.25})^2}{2}\right) + 40 \times \left(200 - 9 \times 7 + \frac{(7)^2}{2}\right)$$
$$+ (20 + 2) \times 40 \times \mathbf{0.5}$$
$$+0.5 \times 17.5 \times \mathbf{0.5} \times 40 \times (\mathbf{9.25} - 7)$$

$$= 9511.3 + 440.0 + 393.75$$

$$: 10345.$$

Actual emissions: $40 * 9.25 + 40 * 7 :\ 650.0$