

# 1 Example building on John and Jim's experiment

## 1.1 The parameters

marginal abatement cost function:  $18 - \theta e$

80 firms.

Half of them are *High MC* subjects ( $\theta = \theta_H = 1$ ) and half *Low MC* subjects ( $\theta = \theta_L = 2$ ).

To be compatible with this assumption we assume that

$$\begin{aligned} c(e) &= 200 - 18e + \frac{\theta}{2}e^2 \\ \text{de tal manera que} \\ c'(e) &= -18 + \theta e \\ -c'(e) &\stackrel{y}{=} 18 - \theta e \end{aligned}$$

We took their *low aggregate standard experiment*:  $L = 280$ , and their "uniform" allocation ( $l_H^0 = 3$  and  $l_L^0 = 4$ ).

The marginal penalty function is  $f'(e - l) = \phi + \gamma(e - l)$ .

We assume  $f(e_i - s_i) = \phi(e_i - s_i) + \frac{\gamma}{2}(e_i - s_i)^2$

The *High* enforcement strategy is  $\pi_H = 0.7$ ,  $\phi = 17.5$ , and  $\gamma = 1.43$ . This enforcement strategy should induce perfect compliance by risk neutral firms.

The *Low* enforcement strategy is  $\pi_H = 0.35$ ,  $\phi = 2$ , and  $\gamma = 2.9$ .

In our problem  $\pi$  is a choice variable.

## 1.2 Perfect information

### 1.2.1 Equal monitoring costs

$$f''(0) = \gamma = 1.43$$

$$f'(0) = \phi = 17.5$$

We assume.

$$\mu = \mu_i = 5$$

$$\beta = \beta_i = 0.5$$

**The Cost-Effectiveness of Inducing Perfect Compliance** The condition to induce compliance or not is therefore

$$\mu_i \frac{f''(0)}{f'(0)} \leq \beta_i f'(0)$$

=>

$$5 \times \frac{1.43}{17.5} \leq 0.5 \times 17.5$$

?

$$0.40857 < 8.75$$

It is therefore cost effective to induce perfect compliance.

**The allocation of emissions and probabilities in a perfectly enforced program based on emission standards** The regulator's problem is

$$\min_{(s_L, s_H) \atop (\pi_L, \pi_H)} 40 \times \left( 200 - 18 \times s_H + \frac{s_H^2}{2} \right) + 40 \times (200 - 18 \times s_L + s_L^2) + 5 \times 40 \times (\pi_L + \pi_H)$$

subject to:

$$40 \times s_H + 40 \times s_L = 280$$

$$40 \times \left( 200 - 18 \times s_H + \frac{s_H^2}{2} \right) + 40 \times (200 - 18 \times s_L + s_L^2) = 20s_H^2 - 720s_H + 40s_L^2 - 720s_L + 16000$$

$$40 \times s_H + 40 \times s_L = 280 \Rightarrow s_H + s_L = 7$$

$$\min_{(s_L, s_H) \atop (\pi_L, \pi_H)} 20s_H^2 - 720s_H + 40s_L^2 - 720s_L + 16000 + 5 \times 40 \times (\pi_L + \pi_H)$$

subject to:

$$s_H + s_L = 7$$

$$Y \text{ como el regulador va a elegir } \pi_L^* = \frac{-c'_i(s_L^*)}{f'(0)} = \frac{18-2s_L^*}{17.5} \text{ y } \pi_H^* = \frac{-c'_i(s_H^*)}{f'(0)} = \frac{18-s_H^*}{17.5}$$

Lagrange:

$$L = 20s_H^2 - 720s_H + 40s_L^2 - 720s_L + 16000 + 5 \times 40 \times (\pi_L + \pi_H) + \lambda \times (7 - s_H - s_L)$$

$$L = 20s_H^2 - 720s_H + 40s_L^2 - 720s_L + 16000 + 5 \times 40 \times \left( \frac{18-2s_L}{17.5} + \frac{18-s_H}{17.5} \right) + \lambda \times (7 - s_H - s_L)$$

$$\begin{aligned} \frac{\partial L}{\partial s_H} &= \frac{\partial (20s_H^2 - 720s_H + 40s_L^2 - 720s_L + 16000 + 5 \times 40 \times \left( \frac{18-2s_L}{17.5} + \frac{18-s_H}{17.5} \right) + \lambda \times (7 - s_H - s_L))}{\partial s_H} \\ &= 40s_H - \lambda - 731.43 = 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial s_L} &= \frac{\partial (20s_H^2 - 720s_H + 40s_L^2 - 720s_L + 16000 + 5 \times 40 \times \left( \frac{18-2s_L}{17.5} + \frac{18-s_H}{17.5} \right) + \lambda \times (7 - s_H - s_L))}{\partial s_L} \\ &= 80s_L - \lambda - 742.86 = 0 \end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial \lambda} &= \frac{\partial (20s_H^2 - 720s_H + 40s_L^2 - 720s_L + 16000 + 5 \times 40 \times (\frac{18-2s_L}{17.5} + \frac{18-s_H}{17.5}) + \lambda \times (7 - s_H - s_L))}{\partial \lambda} \\ &= 7 - s_L - s_H = 0\end{aligned}$$

El sistema queda

$$40s_H - \lambda - 731.43 = 0, \lambda = 40s_H - 731.43$$

$$80s_L - \lambda - 742.86 = 0, \lambda = 80s_L - 742.86$$

Por lo que

$$40s_H - \lambda - 731.43 = 80s_L - \lambda - 742.86$$

Solution is:

$$s_L = 0.5s_H + 0.14288$$

Sustituyendo en la tercera:

$$7 - \frac{1}{2}s_H - 0.14288 - s_H = 0$$

Solution is:

$$s_H = \mathbf{4.5714}$$

$$s_L = 0.5 \times \mathbf{4.5714} + 0.14288$$

$$s_L = \mathbf{2.4286}$$

$$\pi_L^* = \frac{18 - 2 \times \mathbf{2.4286}}{17.5} = \mathbf{0.75102}$$

y

$$\pi_H^* = \frac{18 - \mathbf{4.5714}}{17.5} = \mathbf{0.76735}$$

**Costo total del programa:**

$$40 \times \left( 200 - 18 \times s_H + \frac{s_H^2}{2} \right) + 40 \times (200 - 18 \times s_L + s_L^2) + 5 \times 40 \times (\pi_L + \pi_H)$$

$$\begin{aligned}40 \times \left( 200 - 18 \times \mathbf{4.5714} + \frac{\mathbf{4.5714}^2}{2} \right) + 40 \times (200 - 18 \times \mathbf{2.4286} + \mathbf{2.4286}^2) \\ + 5 \times 40 \times (\mathbf{0.75102} + \mathbf{0.76735}) \\ = \mathbf{1191.8}\end{aligned}$$

de los cuales : 11614 son costos de abatimiento y 303.67 son costos de monitoreo.

**Total emissions:**

$$40 * \mathbf{4.5714} + 40 * \mathbf{2.4286} = \mathbf{280}$$

**The allocation of emissions and probabilities in a perfectly enforced program based on tradable permits** In a perfectly competitive emission permits market, emissions are determined such that  $-c'(e) = p^*$ , the equilibrium price of permits. In a perfectly enforced market it is also true that  $e = l$ . From the firms' behaviors we know that in equilibrium

$$-c'_H(l_H) = 18 - l_H = p^*$$

$$-c'_L(l_L) = 18 - 2l_L = p^*$$

$$40 \times l_H + 40 \times l_L = 280$$

From the first two equations

$$18 - l_H = p^* = 18 - 2l_L$$

$$l_H = 2l_L$$

Substituting in the third equation

$$40 \times 2 \times l_L + 40 \times l_L = 280$$

,

$$l_L^* = \frac{7}{3}$$

$$l_H^* = 2 \times l_L^* = \frac{14}{3}$$

$$18 - l_H^* = 18 - \frac{14}{3} = \mathbf{13.333} = p^* = 18 - 2l_L^*$$

Finally,

$$\pi^* = \frac{p^*}{f'(0)} = \frac{13.333}{17.5} = \mathbf{0.76189}$$

**Costo total del programa:**

$$40 \times \left( 200 - 18 \times l_H^* + \frac{l_H^{*2}}{2} \right) + 40 \times (200 - 18 \times l_L^* + l_L^{*2}) + 5 \times 80 \times (\pi^*) =$$

$$40 \times \left( 200 - 18 \times \frac{14}{3} + \frac{\left(\frac{14}{3}\right)^2}{2} \right) + 40 \times \left( 200 - 18 \times \frac{7}{3} + \left(\frac{7}{3}\right)^2 \right) + 5 \times 80 \times (\mathbf{0.76189}) \\ \mathbf{1191.8}$$

de los cuales 1161.3 son costos de abatimiento y : 304.76 son costos de monitoreo.

**Conclusión:** Como dice nuestro modelo, asumiendo costos de una inspección igual para todas las firmas, el costo de un programa de permisos es igual que el costo de un programa basado en estándares.

### 1.2.2 Different monitoring costs

There is no a priori relation between the type of the firm ( $H$  or  $L$ ) and the cost of auditing that firm. These costs can differ because of the localization of the plant, the number of discharge points, etc. We assume.

$$\begin{aligned}\mu_H &= 20 \\ \mu_L &= 2\end{aligned}$$

**The Cost-Effectiveness of Inducing Perfect Compliance** The condition to induce compliance or not is therefore

$$\begin{aligned}\mu_i \frac{f''(0)}{f'(0)} &\leq \beta_i f'(0) \\ \Rightarrow \quad \text{Firms } H: 20 \times \frac{1.43}{17.5} &\leq 0.5 \times 17.5\end{aligned}$$

$$1.6343 < 8.75$$

$$\text{Firms } L: 2 \times \frac{1.43}{17.5} \leq 0.5 \times 17.5$$

$$0.16343 < 8.75$$

It is therefore cost effective to induce perfect compliance on both type of firms.

**The allocation of emissions and probabilities in a perfectly enforced program based on emission standards** The regulator's problem is

$$\begin{aligned}\min_{(s_L, s_H), (\pi_L, \pi_H)} & 40 \times \left( 200 - 18 \times s_H + \frac{s_H^2}{2} \right) + 40 \times (200 - 18 \times s_L + s_L^2) + 20 \times 40 \times \pi_H + 2 \times 40 \times \pi_L \\ \text{subject to:} \\ 40 \times s_H + 40 \times s_L &= 280\end{aligned}$$

$$\begin{aligned}\min_{(s_L, s_H), (\pi_L, \pi_H)} & 20s_H^2 - 720s_H + 40s_L^2 - 720s_L + 16000 + 20 \times 40 \times \pi_H + 2 \times 40 \times \pi_L \\ \text{subject to:} \\ s_H + s_L &= 7\end{aligned}$$

Y como el regulador va a elegir

$$\pi_L^* = \frac{-c'_i(s_L^*)}{f'(0)} = \frac{18 - 2s_L^*}{17.5}$$

y

$$\pi_H^* = \frac{-c'_i(s_H^*)}{f'(0)} = \frac{18 - s_H^*}{17.5}$$

La ecuación de Lagrange:

$$L = 20s_H^2 - 720s_H + 40s_L^2 - 720s_L + 16\,000 + 20 \times 40 \times \pi_H + 2 \times 40 \times \pi_L + \lambda \times (7 - s_H - s_L)$$

Se puede expresar como

$$L = 20s_H^2 - 720s_H + 40s_L^2 - 720s_L + 16\,000 + 20 \times 40 \times \frac{18 - s_H}{17.5} + 2 \times 40 \times \frac{18 - 2s_L}{17.5} + \lambda \times (7 - s_H - s_L)$$

$$\begin{aligned} \frac{\partial L}{\partial s_H} &= \frac{\partial (20s_H^2 - 720s_H + 40s_L^2 - 720s_L + 16\,000 + 20 \times 40 \times \frac{18 - s_H}{17.5} + 2 \times 40 \times \frac{18 - 2s_L}{17.5} + \lambda \times (7 - s_H - s_L))}{\partial s_H} \\ &= 40s_H - \lambda - 765.71 = 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial s_L} &= \frac{\partial (20s_H^2 - 720s_H + 40s_L^2 - 720s_L + 16\,000 + 20 \times 40 \times \frac{18 - s_H}{17.5} + 2 \times 40 \times \frac{18 - 2s_L}{17.5} + \lambda \times (7 - s_H - s_L))}{\partial s_L} \\ &= 80s_L - \lambda - 729.14 = 0 \end{aligned}$$

$$\frac{\partial L}{\partial \lambda} = 7 - s_L - s_H = 0$$

El sistema queda

$$40s_H - \lambda - 765.71 = 0, \lambda = 40s_H - 765.71$$

$$80s_L - \lambda - 729.14 = 0, \lambda = 80s_L - 729.14$$

Por lo que

$40s_H - 765.71 = 80s_L - 729.14$ , Solution is:

$$s_L = 0.5s_H - 0.45713$$

Sustituyendo en la tercera:  $7 - \frac{1}{2}s_H + \mathbf{0.45713} - s_H = 0$ , Solution is:

$$s_H = \mathbf{4.9714}$$

$$s_L = \frac{1}{2} \times \mathbf{4.9714} - 0.45713$$

$$s_L = \mathbf{2.0286}$$

**Total emissions:**  $40 \times 4.9714 + 40 \times 2.0286 = 280.0$

$$\pi_L^* = \frac{18 - 2 \times 2.0286}{17.5} = 0.79673$$

y

$$\pi_H^* = \frac{18 - 4.9714}{17.5} = 0.74449$$

**Costo total del programa:**

$$40 \times \left( 200 - 18 \times s_H + \frac{s_H^2}{2} \right) + 40 \times \left( 200 - 18 \times s_L + s_L^2 \right) + 20 \times 40 \times \pi_H + 2 \times 40 \times \pi_L$$

$$\begin{aligned} & 40 \times \left( 200 - 18 \times 4.9714 + \frac{4.9714^2}{2} \right) + 40 \times \left( 200 - 18 \times 2.0286 + 2.0286^2 \right) \\ & + 20 \times 40 \times 0.74449 + 2 \times 40 \times 0.79673 \\ & = 12278 \end{aligned}$$

de los cuales 1161.5 son costos de abatimiento y 659.33 son costos de monitoreo.

**The allocation of emissions and probabilities in a perfectly enforced program based on tradable permits** In a perfectly competitive emission permits market, emissions are determined such that  $-c'(e) = p^*$ , the equilibrium price of permits. In a perfectly enforced market it is also true that  $e = l$ . From the firms' behaviors we know that in equilibrium

$$-c'_H(l_H) = 18 - l_H = p^*$$

$$-c'_L(l_L) = 18 - 2l_L = p^*$$

$$40 \times l_H + 40 \times l_L = 280$$

From the first two equations

$$18 - l_H = p^* = 18 - 2l_L$$

$$l_H = 2l_L$$

Substituting in the third equation

$$40 \times 2 \times l_L + 40 \times l_L = 280$$

,

$$l_L^* = \frac{7}{3} = 2.333\bar{3}$$

$$l_H^* = 2l_L^* = \frac{14}{3} = 4.666\bar{7}$$

$$18 - l_H^* = 18 - \frac{14}{3} = 13.333 = p^* = 18 - 2l_L^*$$

Finally,

$$\pi^* = \frac{p^*}{f'(0)} = \frac{13.333}{17.5} : 0.761\bar{89}$$

#### Costo total del programa:

$$40 \times \left( 200 - 18 \times l_H^* + \frac{l_H^{*2}}{2} \right) + 40 \times (200 - 18 \times l_L^* + l_L^{*2}) + (20 + 2) \times 40 \times \pi^* =$$

$$40 \times \left( 200 - 18 \times \frac{14}{3} + \frac{\left(\frac{14}{3}\right)^2}{2} \right) + 40 \times \left( 200 - 18 \times \frac{7}{3} + \left(\frac{7}{3}\right)^2 \right) + (20 + 2) \times 40 \times (0.761\bar{89})$$

$$12284$$

de los cuales 11613 son costos de abatimiento y 670.46 son costos de monitoreo.

**Conclusión:** Como dice nuestro modelo, asumiendo costos de una inspección distintos para los distintos tipos de firmas, el costo de un programa de permisos es mayor que el costo de un programa basado en estándares. Notar también que el programa de permisos tiene unos costos de abatimiento totales que son menores a los costos totales de abatimiento del programa de estándares (11613 del primero versus 11615 del segundo). Por lo tanto, la diferencia se explica porque los costos de monitoreo del programa de permisos son mayores que los costos de monitoreo del programa de estandares (670.46 del primero versus 659.33 del segundo).

Esto sugiere que los costos de monitoreo son menos importantes que los costos de abatimiento (?); que el programa de permisos si bien no minimiza los costos totales, anda "cerca". (Con menos cantidad de firmas (8) y con costos de monitoreo más bajos (10 y 15) la diferencia de costos entre programas era casi nula. Esto nos puede servir para decir que los permisos no son tan malos (como quiere JP). Claro que esta diferencia depende del numero de firmas a inspeccionar (cuanto más firmas, más caros los permisos en relacion a los estándares). De hecho, cuanto mayor el numero de firmas, ceteris paribus, que es justo el caso donde tendría más sentido aplicar permisos porque el mercado se atomiza, más caros son los permisos.

Pero si la diferencia de costos a favor de los estándares es baja, los permisos están más vivos que en el caso contrario. Porque en un contexto de información imperfecta, es probable que el costo de adquirir la información no justifique usar estándares.

### 1.3 Imperfect Information

Now assume that the regulator can observe whether a firm has  $\theta = \theta_H = 1$  or  $\theta = \theta_L = 2$ , but it uncertain about  $-c'(0)$ . We assume that there are 80 firms, half of them are *High MC subjects* ( $\theta = \theta_H = 1$ ) and half *Low MC subjects* ( $\theta = \theta_L = 2$ ), but for *High MC subjects*

$$-c'(e) = 18 + \alpha - e$$

and for *Low MC subjects*

$$-c'(e) = 18 + \alpha - 2e$$

with  $\alpha$  being a random variable that takes the value...

*CREO QUE DEBERÍAMOS ABER HECHO EL EJERCICIO DE INFORMACIÓN PERFECTA CON DIFERENCIAS ADITIVAS ENTRE LAS FIRMAS. SI LE AGREGO INCERTIDUMBRE ADITIVA AHORA, ME QUEDAN 4 TIPOS DE FIRMAS, H Y L, CON  $-C'(0)$  ALTA Y BAJA. LO MÁS NATURAL HUBIERA SIDO ARRANCAR CON DOS TIPOS DE FIRMAS H Y L QUE SE DIFERENCIEN POR  $-C'(0)$  Y LUEGO DECIR QUE EL REGULADOR NO PUEDE OBSERVAR QUIEN ES QUIEN.*

*DE OTRA MANERA NO LO VOY A PODER COMPARAR CON EL CASO DE INFORMACION PERFECTA QUE TENGO ARRIBA.*

## 2 New example with additive differentiation and uncertainty

### 2.1 The parameters

Marginal abatement cost function:  $\theta - e$

80 firms.

Half of them are *High MC subjects* ( $\theta = \theta_H = 18$ ) and half *Low MC subjects* ( $\theta = \theta_L = 9$ )

$$-c'_H(e) = 18 - e_H$$

$$-c'_L(e) = 9 - e_L$$

To be compatible with this assumption we assume that

$$\begin{aligned} c_H(e) &= 200 - 18e_H + \frac{e_H^2}{2} \\ &\text{de tal manera que} \\ c'_H(e_H) &= -18 + e_H \\ &\quad \text{y} \\ -c'_H(e_H) &= 18 - e_H \end{aligned}$$

and

$$\begin{aligned}
c_L(e) &= 200 - 9e_L + \frac{e_L^2}{2} \\
&\text{de tal manera que} \\
c'_L(e_L) &= -9 + e_L \\
&\text{y} \\
-c'_L(e_L) &= 18 - e_L
\end{aligned}$$

The regulator's target of total emissions is  $L = 560$

We assume

$$f(e_i - s_i) = \phi(e_i - s_i) + \frac{\gamma}{2}(e_i - s_i)^2$$

with  $\phi = 17.5$ , and  $\gamma = 1.43$  (the value for these parameters are taken from Murphy and Strandlund's (2006) *High enforcement strategy*)

The marginal penalty function is therefore

$$f'(e - l) = 17.5 + 1.43 \times (e - s)$$

In our problem  $\pi$  is a choice variable.

## 2.2 Perfect information

### 2.2.1 Equal monitoring costs

First we assume.

$$\mu = \mu_i = 5$$

$$\beta = \beta_i = 0.5$$

**The Cost-Effectiveness of Inducing Perfect Compliance** The condition to induce compliance or not is therefore

$$\begin{aligned}
&\mu_i \frac{f''(0)}{f'(0)} \leq \beta_i f'(0) \\
\Rightarrow &5 \times \frac{1.43}{17.5} \leq 0.5 \times 17.5 \\
?
\end{aligned}$$

$$0.40857 < 8.75$$

It is therefore cost effective to induce perfect compliance.

**The allocation of emissions and probabilities in a perfectly enforced program based on emission standards** The regulator's problem is

$$\min_{(s_L, s_H) \atop (\pi_L, \pi_H)} 40 \times \left( 200 - 18 \times s_H + \frac{s_H^2}{2} \right) + 40 \times \left( 200 - 9 \times s_L + \frac{s_L^2}{2} \right) + 5 \times 40 \times (\pi_L + \pi_H)$$

subject to:

$$40 \times s_H + 40 \times s_L = 560$$

$$\min_{(s_L, s_H) \atop (\pi_L, \pi_H)} \left( 200 - 18 \times s_H + \frac{s_H^2}{2} \right) + \left( 200 - 9 \times s_L + \frac{s_L^2}{2} \right) + 5 \times (\pi_L + \pi_H)$$

subject to:

$$s_H + s_L = 14$$

Y como el regulador va a elegir y

$$\pi_H^* = \frac{-c'_i(s_H^*)}{f'(0)} = \frac{18 - s_H^*}{17.5}$$

$$\pi_L^* = \frac{-c'_i(s_L^*)}{f'(0)} = \frac{9 - s_L^*}{17.5}$$

Lagrange:

$$L = \left( 200 - 18 \times s_H + \frac{s_H^2}{2} \right) + \left( 200 - 9 \times s_L + \frac{s_L^2}{2} \right) + 5 \times (\pi_L + \pi_H) + \lambda \times (14 - s_H - s_L)$$

$$L = \left( 200 - 18 \times s_H + \frac{s_H^2}{2} \right) + \left( 200 - 9 \times s_L + \frac{s_L^2}{2} \right) + 5 \times \left( \frac{18 - s_H}{17.5} + \frac{9 - s_L}{17.5} \right) + \lambda \times (14 - s_H - s_L)$$

$$\begin{aligned} \frac{\partial L}{\partial s_H} &= \frac{\partial \left( \left( 200 - 18 \times s_H + \frac{s_H^2}{2} \right) + \left( 200 - 9 \times s_L + \frac{s_L^2}{2} \right) + 5 \times \left( \frac{18 - s_H}{17.5} + \frac{9 - s_L}{17.5} \right) + \lambda \times (14 - s_H - s_L) \right)}{\partial s_H} \\ &= s_H - \lambda - 18.286 = 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial s_L} &= \frac{\partial \left( \left( 200 - 18 \times s_H + \frac{s_H^2}{2} \right) + \left( 200 - 9 \times s_L + \frac{s_L^2}{2} \right) + 5 \times \left( \frac{18 - s_H}{17.5} + \frac{9 - s_L}{17.5} \right) + \lambda \times (14 - s_H - s_L) \right)}{\partial s_L} \\ &= s_L - \lambda - 9.2857 = 0 \end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial \lambda} &= \frac{\partial \left( \left( 200 - 18 \times s_H + \frac{s_H^2}{2} \right) + \left( 200 - 9 \times s_L + \frac{s_L^2}{2} \right) + 5 \times \left( \frac{18-s_H}{17.5} + \frac{9-s_L}{17.5} \right) + \lambda \times (14 - s_H - s_L) \right)}{\partial \lambda} \\ &= 14 - s_L - s_H = 0\end{aligned}$$

El sistema queda

$$s_H - \lambda - 18.286 = 0$$

$$\lambda = s_H - 18.286$$

$$s_L - \lambda - 9.2857 = 0$$

$$\lambda = s_L - 9.2857$$

Por lo que

$$s_H - 18.286 = s_L - 9.2857$$

de donde

$$s_L = s_H - 9.0003$$

Sustituyendo en la tercera:

$$14 - (s_H - 9.0003) - s_H = 0$$

Solution is:

$$s_H = \mathbf{11.5}$$

$$s_L = 11.5 - 9.0003$$

$$s_L = \mathbf{2.4997}$$

$$\pi_H^* = \frac{18 - \mathbf{11.5}}{17.5} = \mathbf{0.37143}$$

y

$$\pi_L^* = \frac{9 - \mathbf{2.4997}}{17.5} = \mathbf{0.37145}$$

**Costo total del programa:**

$$\begin{aligned}40 \times \left( 200 - 18 \times s_H + \frac{s_H^2}{2} \right) + 40 \times \left( 200 - 9 \times s_L + \frac{s_L^2}{2} \right) + 5 \times 40 \times (\pi_L + \pi_H) \\ 40 \times \left( 200 - 18 \times \mathbf{11.5} + \frac{\mathbf{11.5}^2}{2} \right) + 40 \times \left( 200 - 9 \times \mathbf{2.4997} + \frac{\mathbf{2.4997}^2}{2} \right) \\ + 5 \times 40 \times (\mathbf{0.37145} + \mathbf{0.37145}) \\ = \mathbf{9738.7}\end{aligned}$$

de los cuales 9590.1 son costos de abatimiento y 148.58 son costos de monitoreo.

**Total emissions:**

$$40 * 11.5 + 40 * 2.5 = 560$$

**The allocation of emissions and probabilities in a perfectly enforced program based on tradable permits** In a perfectly competitive emission permits market, emissions are determined such that  $-c'(e) = p^*$ , the equilibrium price of permits. In a perfectly enforced market it is also true that  $e = l$ . From the firms' behaviors we know that in equilibrium

$$-c'_H(l_H) = 18 - l_H = p^*$$

$$-c'_L(l_L) = 9 - l_L = p^*$$

$$40 \times l_H + 40 \times l_L = 560$$

From the first two equations

$$18 - l_H = 9 - l_L$$

$$l_H = l_L + 9$$

Substituting in the third equation

$$40 \times (l_L + 9) + 40 \times l_L = 560$$

, Solution is:

$$l_L^* = \frac{5}{2} = 2.5$$

$$l_H^* = 2.5 + 9 = 11.5$$

$$18 - l_H^* = 18 - 11.5 = 6.5 = p^*$$

Finally,

$$\pi^* = \frac{p^*}{f'(0)} = \frac{6.5}{17.5} = 0.37143$$

**Costo total del programa:**

$$40 \times \left( 200 - 18 \times l_H^* + \frac{l_H^{*2}}{2} \right) + 40 \times \left( 200 - 9 \times l_L^* + \frac{l_L^{*2}}{2} \right) + 5 \times 80 \times (\pi^*) =$$

$$40 \times \left( 200 - 18 \times 11.5 + \frac{(11.5)^2}{2} \right) + 40 \times \left( 200 - 9 \times 2.5 + \frac{2.5^2}{2} \right) + 5 \times 80 \times (\mathbf{0.371\,43})$$

**9738.6**

de los cuales 9590.1 son costos de abatimiento y 148.58 son costos de monitoreo.

**Conclusión:** Como dice nuestro modelo, asumiendo costos de una inspección igual para todas las firmas, el costo de un programa de permisos es igual que el costo de un programa basado en estándares. *Moreover, with additive differentiation of the marginal abatement costs of the firms and equal monitoring cost there is absolutely no difference between the solutions of the programs. Standards are equal to permit holdings in equilibrium, and monitoring probabilities are the same for every firm in both programs and between programs. This is not true when the differentiation in the marginal abatement costs of the firms is multiplicative (i.e: firms differ in the slope of their marginal abatement costs).*

### 2.2.2 Different monitoring costs

There is no a priori relation between the type of the firm ( $H$  or  $L$ ) and the cost of auditing that firm. These costs can differ because of the localization of the plant, the number of discharge points, etc. We assume.

$$\begin{aligned}\mu_H &= 20 \\ \mu_L &= 2\end{aligned}$$

**The Cost-Effectiveness of Inducing Perfect Compliance** The condition to induce compliance or not is therefore

$$\begin{aligned}\mu_i \frac{f''(0)}{f'(0)} &\leq \beta_i f'(0) \\ \Rightarrow \quad \text{Firms } H: 20 \times \frac{1.43}{17.5} &\leq 0.5 \times 17.5\end{aligned}$$

$$1.6343 < 8.75$$

$$\begin{aligned}\text{Firms } L: 2 \times \frac{1.43}{17.5} &\leq 0.5 \times 17.5 \\ 0.16343 &< 8.75\end{aligned}$$

It is therefore cost effective to induce perfect compliance on both type of firms.

**The allocation of emissions and probabilities in a perfectly enforced program based on emission standards** The regulator's problem is

$$\min_{(s_L, s_H) \atop (\pi_L, \pi_H)} 40 \times \left( 200 - 18 \times s_H + \frac{s_H^2}{2} \right) + 40 \times \left( 200 - 9 \times s_L + \frac{s_L^2}{2} \right) + 20 \times 40 \times \pi_H + 2 \times 40 \times \pi_L$$

subject to:

$$40 \times s_H + 40 \times s_L = 560$$

$$\min_{(s_L, s_H) \atop (\pi_L, \pi_H)} \left( 200 - 18 \times s_H + \frac{s_H^2}{2} \right) + \left( 200 - 9 \times s_L + \frac{s_L^2}{2} \right) + 20 \times \pi_H + 2 \times \pi_L$$

subject to:

$$s_H + s_L = 14$$

Y como el regulador va a elegir y

$$\pi_H^* = \frac{-c'_i(s_H^*)}{f'(0)} = \frac{18 - s_H^*}{17.5}$$

$$\pi_L^* = \frac{-c'_i(s_L^*)}{f'(0)} = \frac{9 - s_L^*}{17.5}$$

Lagrange:

$$L = \left( 200 - 18 \times s_H + \frac{s_H^2}{2} \right) + \left( 200 - 9 \times s_L + \frac{s_L^2}{2} \right) + 20 \times \pi_H + 2 \times \pi_L + \lambda \times (14 - s_H - s_L)$$

Se puede expresar como

$$L = \left( 200 - 18 \times s_H + \frac{s_H^2}{2} \right) + \left( 200 - 9 \times s_L + \frac{s_L^2}{2} \right) + 20 \times \left( \frac{18 - s_H}{17.5} \right) + 2 \times \left( \frac{9 - s_L}{17.5} \right) + \lambda \times (14 - s_H - s_L)$$

$$\begin{aligned} \frac{\partial L}{\partial s_H} &= \frac{\partial \left( \left( 200 - 18 \times s_H + \frac{s_H^2}{2} \right) + \left( 200 - 9 \times s_L + \frac{s_L^2}{2} \right) + 20 \times \left( \frac{18 - s_H}{17.5} \right) + 2 \times \left( \frac{9 - s_L}{17.5} \right) + \lambda \times (14 - s_H - s_L) \right)}{\partial s_H} \\ &= s_H - \lambda - 19.143 = 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial s_L} &= \frac{\partial \left( 20s_H^2 - 720s_H + 40s_L^2 - 720s_L + 16000 + 20 \times 40 \times \frac{18-s_H}{17.5} + 2 \times 40 \times \frac{9-s_L}{17.5} + \lambda \times (7 - s_H - s_L) \right)}{\partial s_L} \\ &= 80s_L - \lambda - 729.14 = 0 \end{aligned}$$

$$\frac{\partial L}{\partial \lambda} = 7 - s_L - s_H = 0$$

El sistema queda

$$40s_H - \lambda - 765.71 = 0, \lambda = 40s_H - 765.71$$

$$80s_L - \lambda - 729.14 = 0, \lambda = 80s_L - 729.14$$

Por lo que

$40s_H - 765.71 = 80s_L - 729.14$ , Solution is:

$$s_L = 0.5s_H - 0.45713$$

Sustituyendo en la tercera:  $7 - \frac{1}{2}s_H + \mathbf{0.45713} - s_H = 0$ , Solution is:

$$s_H = \mathbf{4.9714}$$

$$s_L = \frac{1}{2} \times \mathbf{4.9714} - 0.45713$$

$$s_L = \mathbf{2.0286}$$

Total emissions:  $40 \times \mathbf{4.9714} + 40 \times \mathbf{2.0286} = 280.0$

$$\pi_L^* = \frac{18 - 2 \times \mathbf{2.0286}}{17.5} = \mathbf{0.79673}$$

y

$$\pi_H^* = \frac{18 - \mathbf{4.9714}}{17.5} = \mathbf{0.74449}$$

**Costo total del programa:**

$$40 \times \left( 200 - 18 \times s_H + \frac{s_H^2}{2} \right) + 40 \times \left( 200 - 18 \times s_L + s_L^2 \right) + 20 \times 40 \times \pi_H + 2 \times 40 \times \pi_L$$

$$\begin{aligned} & 40 \times \left( 200 - 18 \times \mathbf{4.9714} + \frac{\mathbf{4.9714}^2}{2} \right) + 40 \times \left( 200 - 18 \times \mathbf{2.0286} + \mathbf{2.0286}^2 \right) \\ & + 20 \times 40 \times \mathbf{0.74449} + 2 \times 40 \times \mathbf{0.79673} \\ & = \mathbf{12278} \end{aligned}$$

de los cuales 11615 son costos de abatimiento y 659.33 son costos de monitoreo.

**The allocation of emissions and probabilities in a perfectly enforced program based on tradable permits** In a perfectly competitive emission permits market, emissions are determined such that  $-c'(e) = p^*$ , the equilibrium price of permits. In a perfectly enforced market it is also true that  $e = l$ . From the firms' behaviors we know that in equilibrium

$$-c'_H(l_H) = 18 - l_H = p^*$$

$$-c'_L(l_L) = 18 - 2l_L = p^*$$

$$40 \times l_H + 40 \times l_L = 280$$

From the first two equations

$$18 - l_H = p^* = 18 - 2l_L$$

$$l_H = 2l_L$$

Substituting in the third equation

$$40 \times 2 \times l_L + 40 \times l_L = 280$$

,

$$l_L^* = \frac{7}{3} = 2.333\bar{3}$$

$$l_H^* = 2l_L^* = \frac{14}{3} = 4.666\bar{7}$$

$$18 - l_H^* = 18 - \frac{14}{3} = 13.333 = p^* = 18 - 2l_L^*$$

Finally,

$$\pi^* = \frac{p^*}{f'(0)} = \frac{13.333}{17.5} : 0.761\bar{89}$$

**Costo total del programa:**

$$40 \times \left( 200 - 18 \times l_H^* + \frac{l_H^{*2}}{2} \right) + 40 \times (200 - 18 \times l_L^* + l_L^{*2}) + (20 + 2) \times 40 \times \pi^* =$$

$$40 \times \left( 200 - 18 \times \frac{14}{3} + \frac{\left(\frac{14}{3}\right)^2}{2} \right) + 40 \times \left( 200 - 18 \times \frac{7}{3} + \left(\frac{7}{3}\right)^2 \right) + (20 + 2) \times 40 \times (0.761\bar{89})$$

$$12284$$

de los cuales 1161.3 son costos de abatimiento y 670.46 son costos de monitoreo.

**Conclusión:** Como dice nuestro modelo, asumiendo costos de una inspección distintos para los distintos tipos de firmas, **el costo de un programa de permisos es mayor** que el costo de un programa basado en estándares. Notar

también que el programa de permisos tiene unos costos de abatimiento totales que son menores a los costos totales de abatimiento del programa de estándares (11613 del primero versus 11615 del segundo). Por lo tanto, la diferencia se explica porque los costos de monitoreo del programa de permisos son mayores que los costos de monitoreo del programa de estandares (670.46 del primero versus 659.33 del segundo).

Esto sugiere que los costos de monitoreo son menos importantes que los costos de abatimiento (?); que el programa de permisos si bien no minimiza los costos totales, anda "cerca". (Con menos cantidad de firmas (8) y con costos de monitoreo más bajos (10 y 15) la diferencia de costos entre programas era casi nula. Esto nos puede servir para decir que los permisos no son tan malos (como quiere JP). Claro que esta diferencia depende del numero de firmas a inspeccionar (cuanto más firmas, más caros los permisos en relacion a los estándares). De hecho, cuanto mayor el numero de firmas, ceteris paribus, que es justo el caso donde tendría más sentido aplicar permisos porque el mercado se atomiza, más caros son los permisos.

Pero si la diferencia de costos a favor de los estándares es baja, los permisos están más vivos que en el caso contrario. Porque en un contexto de información imperfecta, es probable que el costo de adquirir la información no justifique usar estándares.

### **2.2.3 The Cost-Effectiveness of Inducing Expected Perfect Compliance**

### **2.2.4 The allocation of emissions and probabilities when inducing expected full compliance in a program based on emission standards**

The regulator's problem is

$$\begin{aligned} \min_{(s)} & \frac{8}{2} \times \left( \left( 100 - 18 \times s + \frac{s^2}{2} \right) + (100 - 18 \times s + s^2) \right) \\ & + 5 \times 8 \times \pi \\ & + E \left( \sum_{i=1}^8 \frac{1}{2} \times \pi \times \left( 17.5 \times (e_i - s) + \frac{1.43}{2} \times (e_i - s)^2 \right) \right) \\ & \text{subject to:} \\ & 4 \times s_H + 4 \times s_L = 28 \end{aligned}$$

$$\min_{\substack{(s) \\ (\pi)}} \frac{8}{2} \times \left( \left( 100 - 18 \times s + \frac{s^2}{2} \right) + (100 - 18 \times s + s^2) \right)$$

$$+ 5 \times 8 \times \pi$$

$$+ \frac{1}{2} \times \pi \times E \left( \sum_{i=1}^8 17.5 \times (e_i - s) + \frac{1.43}{2} \times (e_i - s)^2 \right)$$

subject to:

$$4 \times s_H + 4 \times s_L = 28$$

$$\min_{\substack{(s) \\ (\pi)}} \frac{8}{2} \times \left( \left( 100 - 18 \times s + \frac{s^2}{2} \right) + (100 - 18 \times s + s^2) \right)$$

$$+ 5 \times 8 \times \pi$$

$$+ \frac{1}{2} \times \pi \times \sum_{i=1}^8 E \left( 17.5 \times (e_i - s) + \frac{1.43}{2} \times (e_i - s)^2 \right)$$

subject to:

$$4 \times s_H + 4 \times s_L = 28$$

$$\min_{\substack{(s) \\ (\pi)}} \frac{8}{2} \times \left( \left( 100 - 18 \times s + \frac{s^2}{2} \right) + (100 - 18 \times s + s^2) \right)$$

$$+ 5 \times 8 \times \pi$$

$$+ \frac{1}{2} \times \pi \times \sum_{i=1}^8 \left( \frac{1}{2} \times \left( 17.5 \times (e_H - s) + \frac{1.43}{2} \times (e_H - s)^2 \right) + \frac{1}{2} \times \left( 17.5 \times (e_L - s) + \frac{1.43}{2} \times (e_L - s)^2 \right) \right)$$

subject to:

$$4 \times s_H + 4 \times s_L = 28$$

Where  $e_H /$

$$\begin{aligned} -c'(e; \theta &= \theta_H) = \pi \times f'(e_H - s)) \\ 18 - e_H &= \pi \times (17.5 + 1.43 \times (e_H - s)) \end{aligned}$$