1 CONCEPCIÓN AGOSTO 2010

1.1 The condition under which it is cost effective to induce expected compliance

The regulator's problem is:

$$\min_{\substack{(s_1,s_2,..,s_n)\\(\pi_1,\pi_2,..,\pi_n)}} E\left[\sum_{i=1}^{n} c_i(e_i,\theta_i) + \sum_{i=1}^{n}\mu_i\pi_i + \sum_{i=1}^{n}\beta_i\pi_i f(e_i - s_i)\right] \tag{1a}$$

$$\text{subject to:}$$

$$e_i = e_i(s_i,\pi_i,\theta_i) \tag{1b}$$

$$E\left[\sum_{i=1}^{n} e_i\right] = E \tag{1c}$$

$$s_i \leq E\left(e_i\right)\ \ \forall i = 1,...n \tag{1d}$$

The Lagrange of the regulator´s problem can be written as

$$\begin{aligned} L \quad = \quad & E\left[\sum_{i=1}^{n} c_i(e_i(s_i,\pi_i,\theta_i),\theta_i) + \sum_{i=1}^{n}\mu_i\pi_i + \sum_{i=1}^{n}\beta_i\pi_i f(e_i(s_i,\pi_i,\theta_i) - s_i)\right] \\ & +\lambda_1\left[E\left[\sum_{i=1}^{n} e_i(s_i,\pi_i,\theta_i)\right] - E\right] + \sum_{i=1}^{n}\lambda_2^i(s_i - E\left[e_i(s_i,\pi_i,\theta_i)\right]) \end{aligned}$$

with λ_1 and λ_2^i being the $n+1$ multipliers. The $n\times 2+n+1$ necessary Kuhn-Tucker conditions for positive levels of the standard and the auditing probability are:

$$\begin{aligned} \frac{\partial L}{\partial s_i} = & E\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i)\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i} + \beta_i\pi_i f'(e_i(s_i,\pi_i,\theta_i) - s_i)(\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i} - 1)\right] \\ & +\lambda_1 E\left(\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right) + \lambda_2^i\left[1 - E\left(\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right)\right] = 0, i = 1,...,n \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial \pi_i} = & E\left[\begin{array}{c} c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i)\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i} + \mu_i \\ +\beta_i\left(f(e_i(s_i,\pi_i,\theta_i) - s_i) + \pi_i f'(e_i(s_i,\pi_i,\theta_i) - s_i)\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right) \end{array}\right] \\ & +\lambda_1 E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right] - \lambda_2^i E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right] = 0, i = 1,...,n \end{aligned} \tag{2}$$

$$\frac{\partial L}{\partial \lambda_1} = E\left[\sum_{i=1}^{n} e_i(s_i,\pi_i,\theta_i)\right] - E = 0 \tag{3}$$

$$\frac{\partial L}{\partial \lambda_2^i} = s_i - E\left[e_i(s_i, \pi_i, \theta_i)\right] \leq 0, \lambda_2^i \geq 0, \lambda_2^i \times (s_i - E\left[e_i(s_i, \pi_i, \theta_i)\right]) = 0 \quad (4)$$

We assume that these conditions are necessary and sufficient to characterize the optimal solution of the problem.

Proof of Proposition 1' If $E\left[e_i(s_i, \pi_i, \theta_i)\right] = s_i$, from () we know that $\lambda_2^i \geq 0$. We have also that $\lambda_1 \geq 0$. We can re-write the first order conditions () and () of the regulator's problem as:

$$\frac{\partial L}{\partial s_i} = E\left(c_i'(e_i(s_i, \pi_i, \theta_i), \theta_i)\frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial s_i}\right) + \beta_i \pi_i E\left(f'(e_i(s_i, \pi_i, \theta_i) - s_i)(\frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial s_i} - 1)\right)$$
$$+\lambda_1 E\left(\frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial s_i}\right) + \lambda_2^i - \lambda_2^i E\left(\frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial s_i}\right) = 0, i = 1, ..., n$$

$$\frac{\partial L}{\partial s_i} = E\left[c_i'(e_i(s_i, \pi_i, \theta_i), \theta_i)\right] \times E\left[\frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial s_i}\right] + Cov\left[c_i'(e_i(s_i, \pi_i, \theta_i), \theta_i), \frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial s_i}\right]$$
$$+\beta_i \pi_i \left[E\left[f'(e_i(s_i, \pi_i, \theta_i) - s_i) \times \frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial s_i}\right] - E\left[f'(e_i(s_i, \pi_i, \theta_i) - s_i)\right]\right]$$
$$+\lambda_1 E\left[\frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial s_i}\right] + \lambda_2^i - \lambda_2^i E\left[\frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial s_i}\right] = 0, i = 1, ..., n$$

From the linearity of f',

$$\frac{\partial L}{\partial s_i} = E\left[c_i'(e_i(s_i, \pi_i, \theta_i), \theta_i)\right] \times E\left[\frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial s_i}\right] + Cov\left[c_i'(e_i(s_i, \pi_i, \theta_i), \theta_i), \frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial s_i}\right]$$
$$+\beta_i \pi_i \left[\begin{array}{c} f'(E\left[e_i(s_i, \pi_i, \theta_i)\right] - s_i) \times E\left[\frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial s_i}\right] + Cov\left[f'(e_i(s_i, \pi_i, \theta_i) - s_i), \frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial s_i}\right] \\ -f'(E\left[e_i(s_i, \pi_i, \theta_i)\right] - s_i) \end{array}\right]$$
$$+\lambda_1 E\left[\frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial s_i}\right] + \lambda_2^i - \lambda_2^i E\left[\frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial s_i}\right] = 0, i = 1, ..., n$$

Evaluated at $E\left[e_i(s_i, \pi_i, \theta_i)\right] = s_i$,

$$\frac{\partial L}{\partial s_i} = E\left[c_i'(e_i(s_i, \pi_i, \theta_i), \theta_i)\right] \times E\left[\frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial s_i}\right] + Cov\left[c_i'(e_i(s_i, \pi_i, \theta_i), \theta_i), \frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial s_i}\right]$$
$$+\beta_i \pi_i \left[f'(0) \times E\left[\frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial s_i}\right] + Cov\left[f'(e_i(s_i, \pi_i, \theta_i) - s_i), \frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial s_i}\right] - f'(0)\right]$$
$$+\lambda_1 E\left[\frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial s_i}\right] + \lambda_2^i - \lambda_2^i E\left[\frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial s_i}\right] = 0, i = 1, ..., n$$

$$\frac{\partial L}{\partial s_i} = \left(E\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i)\right] + \beta_i\pi_i f'(0) + \lambda_1 - \lambda_2^i\right) \times E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right]$$
$$-\beta_i\pi_i f'(0) + \lambda_2^i + Cov\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right]$$
$$+\beta_i\pi_i\left(Cov\left[f'(e_i(s_i,\pi_i,\theta_i) - s_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right]\right) = 0, i = 1, ..., n$$

And

$$\frac{\partial L}{\partial \pi_i} = E\left(c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right) + \mu_i + \beta_i E\left(f(e_i(s_i,\pi_i,\theta_i) - s_i)\right)$$
$$+\pi_i E\left(f'(e_i(s_i,\pi_i,\theta_i) - s_i)\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right)$$
$$+\lambda_1 E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right] - \lambda_2^i E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right] = 0, i = 1, ..., n$$

$$\frac{\partial L}{\partial \pi_i} = E\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i)\right] \times E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right] + Cov\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right]$$
$$+\mu_i + \beta_i\left(E\left[f(e_i(s_i,\pi_i,\theta_i)\right] - s_i\right) + \beta_i\pi_i\left[\begin{array}{c} E\left[f'(e_i(s_i,\pi_i,\theta_i) - s_i)\right] \times E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right] \\ +Cov\left[f'(e_i(s_i,\pi_i,\theta_i) - s_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right] \end{array}\right]$$
$$+\lambda_1 E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right] - \lambda_2^i E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right] = 0, i = 1, ..., n$$

$$\frac{\partial L}{\partial \pi_i} = \left[E\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i)\right] + \beta_i\pi_i f'\left[E(e_i(s_i,\pi_i,\theta_i)) - s_i\right] + \lambda_1 - \lambda_2^i\right] \times E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right]$$
$$+Cov\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right]$$
$$+\mu_i + \beta_i E\left[f(e_i(s_i,\pi_i,\theta_i) - s_i\right] + \beta_i\pi_i Cov\left[f'(e_i(s_i,\pi_i,\theta_i) - s_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right] = 0, i = 1, ..., n$$

$$\frac{\partial L}{\partial \pi_i} = \left(E\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i)\right] + \beta_i\pi_i f'(0) + \lambda_1 - \lambda_2^i\right) \times E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right] + \mu_i$$
$$+Cov\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right]$$
$$+\beta_i E\left[f(e_i(s_i,\pi_i,\theta_i) - s_i\right] + \beta_i\pi_i Cov\left[f'(e_i(s_i,\pi_i,\theta_i) - s_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right] = 0, i = 1, ..., n$$

Dividing the expressions of $\frac{\partial L}{\partial s_i}$ and $\frac{\partial L}{\partial \pi_i}$, we obtain

$$\left(E\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i)\right]+\beta_i\pi_i f'(0)+\lambda_1-\lambda_2^i\right)\times E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right]=$$

$$+\beta_i\pi_i f'(0)-\lambda_2^i-Cov\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i),\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right]$$

$$-\beta_i\pi_i\left(Cov\left[f'(e_i(s_i,\pi_i,\theta_i)-s_i),\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right]\right)$$

$$\left(E\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i)\right]+\beta_i\pi_i f'(0)+\lambda_1-\lambda_2^i\right)\times E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right]=$$

$$-\mu_i-\beta_i E\left[f(e_i(s_i,\pi_i,\theta_i)-s_i\right]-Cov\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i),\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right]$$

$$-\beta_i\pi_i Cov\left[f'(e_i(s_i,\pi_i,\theta_i)-s_i),\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right]$$

$$\left.\frac{E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right]}{E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right]}\right|_{E[e_i(s_i,\pi_i,\theta_i)]=s_i}=$$

$$\frac{+\beta_i\pi_i f'(0)-\lambda_2^i-Cov\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i),\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right]-\beta_i\pi_i\left(Cov\left[f'(e_i(s_i,\pi_i,\theta_i)-s_i),\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right]\right)}{-\mu_i-\beta_i E\left[f(e_i(s_i,\pi_i,\theta_i)-s_i\right]-Cov\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i),\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right]-\beta_i\pi_i Cov\left[f'(e_i(s_i,\pi_i,\theta_i)-s_i),\frac{\partial e_i(s_i,}{\partial \pi}\right.}$$

También sabemos que

$$\partial e_i/\partial \pi_i=\frac{-f'}{c_i''+\pi_i f''}<0$$

and

$$0<\partial e_i/\partial s_i=\frac{\pi_i f''}{c_i''+\pi_i f''}<1$$

In expected terms and evaluated at $E\left[e_i(s_i,\pi_i,\theta_i)\right]=s_i$:

$$E\left(\partial e_i/\partial \pi_i\right)=E\left(\frac{-f'(0)}{c_i''+\pi_i f''(0)}\right)<0$$

and

$$0<E\left(\partial e_i/\partial s_i\right)=E\left(\frac{\pi_i f''(0)}{c_i''+\pi_i f''(0)}\right)<1$$

$$\frac{E\left(\partial e_i/\partial s_i\right)}{E\left(\partial e_i/\partial \pi_i\right)}=\frac{E\left(\frac{\pi_i f''(0)}{c_i''+\pi_i f''(0)}\right)}{E\left(\frac{-f'(0)}{c_i''+\pi_i f''(0)}\right)}$$

$$
\begin{aligned}
E\left(\frac{\pi_i f''(0)}{c_i''+\pi_i f''(0)}\right) &= E\left(\pi_i f''(0)\right) E\left(\frac{1}{c_i''+\pi_i f''(0)}\right) + Cov\left(\pi_i f''(0), \frac{1}{c_i''+\pi_i f''(0)}\right) \\
&= \pi_i f''(0) E\left(\frac{1}{c_i''+\pi_i f''(0)}\right) + Cov\left(\pi_i f''(0), \frac{1}{c_i''+\pi_i f''(0)}\right)
\end{aligned}
$$

Assuming additive uncertainty and quadratic abatement cost functions, the last covariance is zero, therefore

$$
\begin{aligned}
E\left(\frac{\pi_i f''(0)}{c_i''+\pi_i f''(0)}\right) &= \pi_i f''(0) E\left(\frac{1}{c_i''+\pi_i f''(0)}\right) \\
&= \frac{\pi_i f''(0)}{c_i''+\pi_i f''(0)}
\end{aligned}
$$

also,

$$
\begin{aligned}
E\left(\frac{-f'(0)}{c_i''+\pi_i f''(0)}\right) &= -E(f'(0)) E\left(\frac{1}{c_i''+\pi_i f''(0)}\right) + Cov\left(-f'(0), \frac{1}{c_i''+\pi_i f''(0)}\right) \\
E\left(\frac{-f'(0)}{c_i''+\pi_i f''(0)}\right) &= -\frac{f'(0)}{c_i''+\pi_i f''(0)}
\end{aligned}
$$

Therefore,

$$
\left.\frac{E\left(\partial e_i/\partial s_i\right)}{E\left(\partial e_i/\partial \pi_i\right)}\right|_{E[e_i(s_i,\pi_i,\theta_i)]=s_i} = \frac{E\left(\frac{\pi_i f''(0)}{c_i''+\pi_i f''(0)}\right)}{E\left(\frac{-f'(0)}{c_i''+\pi_i f''(0)}\right)} = \frac{\frac{\pi_i f''(0)}{c_i''+\pi_i f''(0)}}{-\frac{f'(0)}{c_i''+\pi_i f''(0)}} = -\frac{\pi_i f''(0)}{f'(0)}
$$

Given the two expressions tha we have for this expression,

$$
\left.\frac{E\left(\partial e_i/\partial s_i\right)}{E\left(\partial e_i/\partial \pi_i\right)}\right|_{E[e_i(s_i,\pi_i,\theta_i)]=s_i} = \frac{\pi_i f''(0)}{f'(0)}
$$

$$
= \frac{+\beta_i \pi_i f'(0) - \lambda_2^i - Cov\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right] - \beta_i \pi_i \left(Cov\left[f'(e_i(s_i,\pi_i,\theta_i) - s_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right]\right.}{\mu_i + \beta_i E\left[f(e_i(s_i,\pi_i,\theta_i) - s_i\right] + Cov\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right] + \beta_i \pi_i Cov\left[f'(e_i(s_i,\pi_i,\theta_i) - s_i), \frac{\partial e_i(s_i}{\partial}\right.}
$$

$$
\begin{aligned}
&\frac{\pi_i f''(0)}{f'(0)}\mu_i + \frac{\pi_i f''(0)}{f'(0)}\left(\beta_i E\left[f(e_i(s_i,\pi_i,\theta_i) - s_i]\right) + \beta_i \pi_i Cov\left[f'(e_i(s_i,\pi_i,\theta_i) - s_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right]\right. && \text{((8'))} \\
\leq\; &\beta_i \pi_i f'(0) - Cov\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right] - \beta_i \pi_i \left(Cov\left[f'(e_i(s_i,\pi_i,\theta_i) - s_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right]\right)
\end{aligned}
$$

This is the new condition that indicates whether it is cost effective, in expected terms, to induce plant i to comply.

Interpretation: the left-hand side of this expression is the change in the expected costs of the program caused by the necessary change in π_i^* to keep

$E\left[e_i(s_i, \pi_i, \theta_i)\right] = s_i$ when s_i changes. This change is composed by the change in the monitoring costs (first term) plus the change in the expected sanctioning costs. The latter can decomposed in a direct effect (second term) plus an indirect effect through the change in e_i (third term). The direct effect is given by the change in the discovered violations and fines when π_i is changed. The indirect effect is given by the change in violations induced by the change in π_i. The second and third terms are zero with perfect information because the fines are certainly zero and because there is no covariance term.

It can be seen that if f' is constant ($f' = \phi$, for all $e_i - s_i > 0$) the expression simplifies to

$$0 \leq \beta_i \pi_i f'(0)$$

which means that is f' is constant the regulator has no uncertainty whether it has to induce perfect complaince or not. This is another reason for using a flat marginal penalty.