Hola Carlos. Estuve leyendo detenidamente el paper de Arguedas (2007). El mismo es perfectamente aplicable al caso que estamos tratando: regulador con información perfecta que quiere llegar a un límite agregado de emisiones E aplicando estándares, con dos tipos de multas (lineal y convexa en la violación) y cuyo objetivo es minimizar la suma de los costos de abatimiento y los costos esperados de control (monitoreo y sanción).

Lo que sí es que algunas de sus conclusiones respecto a la forma de la función de multa que minimiza los costos no se cumplirían de acuerdo a mis resultados más abajo.

Te copio abajo el modelo y los resultados de Arguedas con nuestra notación y para nuestro caso (objetivo E en lugar de nivel eficiente de emisiones). También le agrego un grupo de firmas heterogéneas (Arguedas lo hace para una sola firma).

Por último, Arguedas hace que la función de multa tenga un parámetro fijo, F_0, el cual no depende de la violación de la siguiente forma,

$$F(e-s) = F_0 + f(e-s)$$

donde, como siempre, $f(e-s) > 0$, $f\prime(e-s) > 0$ y $f''(e-s) > 0$ para todo $e > s$ y $f(e-s) = 0$ para todo $e \leq s$. Arguedas llama a F_0 the non-gravity component y a $f(e-s)$ the gravity component.

Pero al final concluye que el mejor diseño de la multa en cualquier circunstancia es con $F_0 = 0$, así que me parece que lo podemos ignorar tranquilamente.

1 The behavior of the Firm

Es idéntico a lo que tenemos planteado y ya sabemos.

Su lema 1.

Lemma 1:

$$e_i(\pi_i, s_i) = \left\{ \begin{array}{ccc} s_i & \text{si} & c_i'(s_i) + \pi_i f'(0) \geq 0 \\ e_i^* > s_i & \text{si} & c_i'(s_i) + \pi_i f'(0) < 0 \end{array} \right\}$$

2 The Optimal Policy under given penalties

The regulator selects (π_i, s_i) for a given fine structure $f(e_i - s_i)$:

The regulator minimizes in (π_i, s_i)

$$\sum_i c_i(e_i) + \mu \sum_i \pi_i + \beta \sum_i \pi_i f(e_i - s_i) \tag{1}$$

subject to

$$e_i = e_i(\pi_i, s_i)$$
$$e_i \geq s_i$$
$$s_i \geq 0$$
$$\sum_i e_i = E$$

where $e_i(\pi_i, s_i)$ is the firm´s best response characterized by Lemma 1.

A sufficient condition for the Total-Cost-Effective policy to induce compliance is the following:

Proposition 1 ***1**: Let (s_i^*, π_i^*) be the solution to (1), then (s_i^*, π_i^*) induces compliance if*

$$\mu \frac{f''(0)}{f'(0)} \leq \beta f'(0)$$

Proof of proposition 1

The lagreangean of the problem is

$$\begin{aligned} L(\pi_i, s_i, \lambda_1^i, \lambda_2^i, \lambda_3^i, \lambda_4) &= \sum_i c_i(e_i) + \mu \sum_i \pi_i + \beta \sum_i \pi_i f(e_i - s_i) \\ &+ \sum_i \lambda_1^i \left[c_i'(e_i) + \pi_i f'(e_i - s_i)\right] \\ &+ \sum_i \lambda_2^i \left[-e_i + s_i\right] - \sum_i \lambda_3^i s_i + \lambda_4 \left[E - \sum_i e_i\right] \end{aligned}$$

For "analytical conveniencie2 Arguedas considers the pollution level as a choice varibale of the regulator. "The problem is mathematically equivalent to the one where the regulator chooses (π_i, s_i) knowing that the firm chooses $e_i(\pi_i, s_i)$ as a response to the policy". (CARLOS YO COPIO EL PROCEDIMIENTO DE ARGUEDAS. SI DISCREPAS CON ALGO AVISA PORQUE A MI HAY COSAS QUE NO ME GUSTAN).

The $n \times 6 + 1$ optimality conditions are:

$$\frac{\partial L}{\partial e_i} = c_i'(e_i) + \beta \pi_i f'(e_i - s_i) + \lambda_1^i \left[c_i''(e_i) + \pi_i f''(e_i - s_i) \right] - \lambda_2^i - \lambda_4 = 0, \;\; i = 1, ...n \tag{2}$$

$$\frac{\partial L}{\partial \pi_i} = \mu + \beta f(e_i - s_i) + \lambda_1^i f'(e_i - s_i) = 0, \;\; i = 1, ...n \tag{3}$$

$$\frac{\partial L}{\partial s_i} = -\beta \pi_i f'(e_i - s_i) - \lambda_1^i \pi_i f''(e_i - s_i) + \lambda_2^i - \lambda_3^i = 0, \;\; i = 1, ...n \tag{4}$$

$$\frac{\partial L}{\partial \lambda_1^i} = c_i'(e_i) + \pi_i f'(e_i - s_i) = 0, \;\; i = 1, ...n \tag{5}$$

$$\frac{\partial L}{\partial \lambda_2^i} = -e_i + s_i \leq 0; \lambda_2^i \geq 0; \left(-e_i + s_i\right) \lambda_2^i = 0, \;\; i = 1, ...n \tag{6}$$

$$\frac{\partial L}{\partial \lambda_3^i} = -s_i \leq 0; \lambda_3^i \geq 0; \lambda_3^i s_i = 0, \;\; i = 1, ...n \tag{7}$$

$$\frac{\partial L}{\partial \lambda_4} = E - \sum_i e_i = 0 \tag{8}$$

If (s_i^*, π_i^*) induces compliance, then $e_i^* = s_i^*, \lambda_2^i \geq 0$ and $\lambda_3^i \geq 0$ then by 4

$$-\beta \pi_i f'(0) - \lambda_1^i \pi_i f''(0) = -\lambda_2^i + \lambda_3^i$$

By 3

$$\begin{aligned} \lambda_1^i f'(0) &= -\left[\mu + \beta f(0)\right] = -\mu \\ \lambda_1^i &= -\frac{\mu}{f'(0)} \end{aligned}$$

And combining these two

$$-\beta \pi_i f'(0) + \frac{\mu}{f'(0)} \pi_i f''(0) = -\lambda_2^i + \lambda_3^i$$

Since $e_i^* > 0, e_i^* = s_i^*$ implies $s_i^* > 0$ and by 7 $\lambda_3^i = 0$.
Then,

$$-\beta \pi_i f'(0) + \frac{\mu}{f'(0)} \pi_i f''(0) = -\lambda_2^i \leq 0$$

$$\mu \frac{\pi_i f''(0)}{f'(0)} \leq \beta \pi_i f'(0)$$

$$\mu \frac{f''(0)}{f'(0)} \leq \beta f'(0) \tag{9}$$

QED

Intuición:

El regulador se encuentra en $e_i^* = s_i$ y se pregunta si existe alguna otra combinación de s_i y π_i que induzca non-compliance y que alcance E a un costo menor. El regulador debe mover ambos s_i y π_i, porque de otra manera no logra E. Por ejemplo, si baja s_i solamente, e_i baja y por lo tanto $E > \sum_i e_i$. Lo que tendría que hacer también es bajar π_i para que$\sum_i e_i$ vuelva a subir. ¿Cuál es la correspondiente baja en π_i ante una baja marginal en s_i? Nosotros sabemos que $\partial e_i / \partial \pi_i = -f' / \left(c_i'' + \pi_i f'' \right) < 0$ y $\partial e_i / \partial s_i = \pi_i f'' / \left(c_i'' + \pi_i f'' \right) > 0$, por lo tanto la respuesta es

$$\frac{\partial e_i / \partial s_i}{\partial e_i / \partial \pi_i} = \left. \frac{\partial \pi_i}{\partial s_i} \right|_{e_i^* = s_i} = \frac{-\pi_i f''(0)}{f'(0)}$$

Cuando s_i baja marginalmente, los sanctioning costs aumentan en $\beta \pi_i f'(0)$. $\left(\frac{\partial e_i^*}{\partial s_i} > 0 \right)$

Cuando π_i baja en la cantidad necesaria mantener constante las emisiones , los monitoring costos bajan $\mu \frac{\pi_i f''(0)}{f'(0)}$. $\left(\frac{\partial e_i^*}{\partial \pi_i} < 0 \right)$

Si este ahorro marginal de costos por bajar π es menor o igual al incremento de costos marginales por bajar s_i, no conviene moverse. No conviene otra cosa que full-compliance.

Más importante, (9) nos dice que la forma de f afecta la optimalidad de inducir compliance o no. Aún más, si f es lineal ($f''(0) = 0$), la condición se reduce a $0 \leq \beta f'(0)$, la cual se cumple siempre.

Es decir, si f es lineal, siempre va a ser el caso que el regulador minimiza los costos totales del programa haciendo full compliance.

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Lo que queda sin responder es si los costos totales del programa (mínimos) son menores con:

(1) $f''(.) = 0$ y full compliance

(2) $f''(.) > 0$ y full compliance (Prop. 1 se cumple)

(3) $f''(0)$ >0 y no full-compliance (Prop.1 no se cumple).
(1) y (2) son iguales porque si hay full compliance f'' es irrelevante
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Proposition 2: *If the optimal policy* (s_i^*, π_i^*) *induces compliance, it is characterized by*

$$c_i'(s_i^*) - \lambda_4 + \mu \frac{d\pi_i(s_i^*)}{ds_i} = 0$$

where $\pi_i^* = \pi_i(s_i^*) = \frac{-c_i'(s_i^*)}{f'(0)}$.

Proof of Proposition 2:

Sustituyendo $e_i^* = s_i^*$ and $\pi_i^* = \pi_i(s_i^*)$ en las CPO del problema del regulador, éstas quedan:

$$c_i'(s_i^*) + \beta\pi_i f'(0) + \lambda_1^i \left[c_i''(s_i^*) + \pi_i f''(0)\right] - \lambda_2^i - \lambda_4 = 0, \ i = 1, ...n \qquad (10)$$

$$\mu + \lambda_1^i f'(0) = 0, \ i = 1, ...n \qquad (11)$$

$$-\beta\pi_i f'(0) - \lambda_1^i \pi_i f''(0) + \lambda_2^i - \lambda_3^i = 0, \ i = 1, ...n \qquad (12)$$

$$c_i'(s_i^*) + \pi_i f'(0) = 0, \ i = 1, ...n \qquad (13)$$

$$-e_i + s_i = 0; \lambda_2^i \geq 0, \ i = 1, ...n \qquad (14)$$

$$-s_i < 0; \lambda_3^i = 0, \ i = 1, ...n \qquad (15)$$

$$E - \sum_i e_i = 0 \qquad (16)$$

Summing (10) y (12) and using $\lambda_3^i = 0$,

$$c_i'(s_i^*) + \beta\pi_i f'(0) + \lambda_1^i \left[c_i''(s_i^*) + \pi_i f''(0)\right] - \lambda_2^i - \lambda_4 - \beta\pi_i f'(0) - \lambda_1^i \pi_i f''(0) + \lambda_2^i = 0$$

$$c_i'(s_i^*) + \lambda_1^i \left[c_i''(s_i^*)\right] - \lambda_4 = 0$$

From (11)

$$\lambda_1^i = -\frac{\mu}{f'(0)}$$

Then

$$c_i'(s_i^*) - \frac{\mu}{f'(0)}\left[c_i''(s_i^*)\right] - \lambda_4^i = 0$$

Since $\pi_i^* = \pi_i\left(s_i^*\right) = \frac{-c_i'(s_i^*)}{f'(0)}$,

$$\frac{d\pi_i\left(s_i^*\right)}{ds_i} = \frac{-c_i''(s_i^*)}{f'(0)}$$

Then,

$$c_i'(s_i^*) + \mu\frac{d\pi_i\left(s_i^*\right)}{ds_i} - \lambda_4 = 0$$

QED

Note: If the optimal policy induces compliance the optimal policy does not imply an allocation of emissions such that marginal abatement costs are equal, unless $c_i''(s_i^*)$ are equal for all i.

Proposition 3: *If the optimal policy* (s_i^*, π_i^*) *induces non-compliance, it is given by*

$$c_i'(e_i^*) + \beta\pi_i^* f'(e_i^* - s_i^*) + \frac{\mu + \beta f(e_i^* - s_i^*)}{\partial e_i/\partial\pi_i} - \lambda_4 = 0 \tag{17}$$

$$c_i'(e_i^*) + \pi_i f'(e_i^* - s_i^*) = 0 \tag{18}$$

$$\left[\mu + \beta f(e_i^* - s_i^*)\right]\frac{f''(e_i^* - s_i^*)}{f'(e_i^* - s_i^*)} - \beta f'(e_i^* - s_i^*) \geq 0 \tag{19}$$

$$\left[\left[\mu + \beta f(e_i^* - s_i^*)\right]\frac{f''(e_i^* - s_i^*)}{f'(e_i^* - s_i^*)} - \beta f'(e_i^* - s_i^*)\right] s_i = 0 \tag{20}$$

Proof of Proposition 3: Under non-compliance, the FOC of the regulator's problem are

$$c_i'(e_i^*) + \beta\pi_i f'(e_i^* - s_i^*) + \lambda_1^i\left[c_i''(e_i) + \pi_i f''(e_i - s_i)\right] - \lambda_2^i - \lambda_4 = 0 \tag{21}$$

$$\mu + \beta f(e_i^* - s_i^*) + \lambda_1^i f'(e_i^* - s_i^*) = 0 \tag{22}$$

$$-\beta\pi_i f'(e_i^* - s_i^*) - \lambda_1^i \pi_i f''(e_i^* - s_i^*) + \lambda_2^i - \lambda_3^i = 0 \tag{23}$$

$$c_i'(e_i^*) + \pi_i f'(e_i^* - s_i^*) = 0 \tag{24}$$

$$-e_i + s_i < 0; \lambda_2^i = 0 \tag{25}$$

$$-s_i \leq 0; \lambda_3^i \geq 0; \lambda_3^i s_i = 0 \tag{26}$$

$$E - \sum_i e_i = 0 \tag{27}$$

From (22)

$$\lambda_1^i = -\frac{\mu + \beta f(e_i^* - s_i^*)}{f'(e_i^* - s_i^*)}$$

Substituting into (23) and using $\lambda_2^i = 0$,

$$[\mu + \beta f(e_i^* - s_i^*)] \frac{\pi_i f''(e_i^* - s_i^*)}{f'(e_i^* - s_i^*)} - \beta \pi_i f'(e_i^* - s_i^*) = \lambda_3^i \geq 0$$

which is the third equation in proposition 3.
Then, using 21

$$c_i'(e_i^*) + \beta \pi_i f'(e_i^* - s_i^*) + \lambda_1^i [c_i''(e_i) + \pi_i f''(e_i - s_i)] - \lambda_2^i - \lambda_4^i = 0$$

and

$$\lambda_1^i = -\frac{\mu + \beta f(e_i^* - s_i^*)}{f'(e_i^* - s_i^*)}$$

we obtain

$$c_i'(e_i^*) + \beta \pi_i f'(e_i^* - s_i^*) - \frac{\mu + \beta f(e_i^* - s_i^*)}{f'(e_i^* - s_i^*)} [c_i''(e_i) + \pi_i f''(e_i - s_i)] - \lambda_4^i = 0$$

Remembering that $\partial e_i / \partial \pi_i = -f' / (c_i'' + \pi_i f'')$, we obtain

$$c_i'(e_i^*) + \beta \pi_i f'(e_i^* - s_i^*) + \frac{\mu + \beta f(e_i^* - s_i^*)}{\partial e_i / \partial \pi_i} - \lambda_4 = 0$$

which is the first equation of proposition 3.
Finally, combining(23) with (26), we obtain

$$\left[-\beta \pi_i f'(e_i^* - s_i^*) - \lambda_1^i \pi_i f''(e_i^* - s_i^*)\right] s_i = 0$$

QED

Note that assuming $s_i^* > 0$, *the optimal policy* (s_i^*, π_i^*) that *induces non-compliance is given by*

$$c_i'(e_i^*) + \beta\pi_i^* f'(e_i^* - s_i^*) + \frac{\mu + \beta f(e_i^* - s_i^*)}{\partial e_i^*/\partial \pi_i} - \lambda_4 = 0 \tag{28}$$

$$c_i'(e_i^*) + \pi_i^* f'(e_i^* - s_i^*) = 0 \tag{29}$$

$$[\mu + \beta f(e_i^* - s_i^*)] \frac{f''(e_i^* - s_i^*)}{f'(e_i^* - s_i^*)} - \beta f'(e_i^* - s_i^*) = 0 \tag{30}$$

Here we can see again that under lnear penalties condition (30) never holds. It is never optimal to induce non-compliance.

3 The Choice of the Appropriate Penalties

"First we derive the most appropriate shape of the penalties under the two possible scenarios: complaince and non-compliance. Next, we select the socially preferred scenario."

The result for the first Objective is Proposition 4:

Proposition 4: *If the optimal policy induces compliance, the best shape of the gravity fine is such that the linear component is set as high as possible and the progressive component is zero. Conversely, if the optimal policy induces non-compliance, then the best shape of the gravity fine is such that the linear component is zero and the progressive component is set as high as possible.* [1]

A MI NO ME DA ESTO. A MI ME DA: IF THE OPRIMAL POLICY INDUCES COMPLIANCE, THE LINEAR COMPONENT IS SET AS HIGH AS POSSIBLE AND THE PROGRESSIVE COMPONENT IS IRRELEVANT (IN EQUILIBRIUM). IF THE OPTIMAL POLICY INDUCES NON-COMPLIANCE, IT DEPENDS ON A CONDITION BELOW.

Proof of Proposition 4:

[1] "In any case, the optimal non-gravity component (F_0) is zero" is the last part of the proposition in Arguedas (2007).

The fine $f(e-s)$ (a second order degree polinomial function) can be represented by the Maclaurin series (expansion around $e-s=0$) by

$$f(e-s) = f'(0)(e-s) + \frac{f''(0)}{2}(e-s)^2$$

$f'(0)$ is the linear gravity component.
$f''(0)$ is the progressive gravity component
"we restrict ourselves to linear quadratic penalty functions"

If the optimal policy induces compliance, sanctioning costs are zero. By Proposition 2

$$\pi_i^* = \pi_i\left(s_i^*\right) = \frac{-c_i'(s_i^*)}{f'(0)}$$

"the larger the linear gravity component the lower the minimum probability to achieve compliance and therefore the social costs. Therefore, the optimal fine is one on which $f'(0)$ is as high as possible and $f''(0)$ is as low as possible, since only the first component affects the probability." (**En realidad es más correcto, o correcto decir que $f''(0)$ puede ser cualquier cosa, en equilibrio, y por ende mi comentario arriba a la Proposition 4**).

If the optimal policy induces non-compliance,

$$f(e_i^* - s_i^*) = f'(0)(e_i^* - s_i^*) + \frac{f''(0)}{2}(e_i^* - s_i^*)^2$$

In order to know how the shape of the penalty function affects social costs, we differentiate the Lagrangean with respect to $f'(0)$ and $f''(0)$

$$dL = \frac{\partial L}{\partial f'(0)} df'(0) + \frac{\partial L}{\partial f''(0)} df''(0)$$

$$\begin{aligned} L &= \sum_i c_i(e_i) + \mu \sum_i \pi_i + \beta \sum_i \pi_i \left[f'(0)(e_i - s_i) + \frac{f''(0)}{2}(e_i - s_i)^2 \right] \\ &\quad + \sum_i \lambda_1^i \left[c_i'(e_i) + \pi_i \left[f'(0) + f''(0)(e_i - s_i) \right] \right] + \sum_i \lambda_2^i \left[-e_i + s_i \right] \\ &\quad - \sum_i \lambda_3^i s_i + \lambda_4 \left[E - \sum_i e_i \right] \end{aligned}$$

$$dL = \left[\beta \sum_i \pi_i (e_i - s_i) + \sum_i \lambda_1^i \pi_i\right] df'(0) + \left[\beta \sum_i \pi_i \frac{(e_i - s_i)^2}{2} + \sum_i \lambda_1^i \pi_i (e_i - s_i)\right] df''(0)$$

$$\frac{dL}{df'(0)} = \left[\beta \sum_i \pi_i (e_i - s_i) + \sum_i \lambda_1^i \pi_i\right] + \left[\beta \sum_i \pi_i \frac{(e_i - s_i)^2}{2} + \sum_i \lambda_1^i \pi_i (e_i - s_i)\right] \frac{df''(0)}{df'(0)}$$

In order to keep constant the total amount of fines collected, and therefore the sanctioning costs, the regulator must choose choose $df'(0)$ and $df''(0)$ such that

$$d\sum_i f(e_i^* - s_i^*) = d\sum_i \left[f'(0)(e_i^* - s_i^*) + \frac{f''(0)}{2}(e_i^* - s_i^*)^2\right] = 0$$

$$\frac{\partial \left[\sum_i \left[f'(0)(e_i^* - s_i^*) + \frac{f''(0)}{2}(e_i^* - s_i^*)^2\right]\right]}{\partial f'(0)} df'(0) + \frac{\partial \left[\sum_i \left[f'(0)(e_i^* - s_i^*) + \frac{f''(0)}{2}(e_i^* - s_i^*)^2\right]\right]}{\partial f''(0)} df''(0) = 0$$

Or

$$\sum_i (e_i^* - s_i^*) df'(0) + \sum_i \left[\frac{(e_i^* - s_i^*)^2}{2}\right] df''(0) = 0$$

$$\frac{\sum_i \left[\frac{(e_i^* - s_i^*)^2}{2}\right]}{\sum_i (e_i^* - s_i^*)} = -\frac{df'(0)}{df''(0)}$$

Then,

$$\frac{dL}{df'(0)} = \left[\beta \sum_i \pi_i (e_i - s_i) + \sum_i \lambda_1^i \pi_i\right] - \left[\beta \sum_i \pi_i \frac{(e_i - s_i)^2}{2} + \sum_i \lambda_1^i \pi_i (e_i - s_i)\right] \frac{\sum_i (e_i^* - s_i^*)}{\sum_i \left[\frac{(e_i^* - s_i^*)^2}{2}\right]}$$

$$\frac{dL}{df'(0)} = \beta \sum_i \pi_i (e_i - s_i) + \sum_i \lambda_1^i \pi_i - \left[\frac{\beta}{2} \sum_i \pi_i (e_i - s_i)^2 + \sum_i \lambda_1^i \pi_i (e_i - s_i)\right] \frac{\sum_i (e_i^* - s_i^*)}{\sum_i \left[\frac{(e_i^* - s_i^*)^2}{2}\right]}$$

$$\frac{dL}{df'(0)} = \beta \sum_i \pi_i(e_i - s_i) + \sum_i \lambda_1^i \pi_i - \left[\beta \sum_i \pi_i(e_i - s_i)^2 + 2\sum_i \lambda_1^i \pi_i(e_i - s_i)\right] \frac{\sum_i (e_i^* - s_i^*)}{\sum_i (e_i^* - s_i^*)^2}$$

$$\begin{aligned} \frac{dL}{df'(0)} &= \left[\sum_i \lambda_1^i \pi_i + 2\sum_i \lambda_1^i \pi_i(e_i - s_i) \times \frac{\sum_i (e_i^* - s_i^*)}{\sum_i (e_i^* - s_i^*)^2}\right] \\ &\quad -\beta \left[\sum_i \pi_i(e_i - s_i) + \sum_i \pi_i(e_i - s_i)^2 \times \frac{\sum_i (e_i^* - s_i^*)}{\sum_i (e_i^* - s_i^*)^2}\right] \end{aligned}$$

The second term is negative and the second is negative (because $\lambda_1^i = -\frac{\mu+\beta f(e_i^*-s_i^*)}{f'(e_i^*-s_i^*)} < 0$). Therefore,

$$\frac{dL}{df'(0)} < 0$$

QUEDE ACA. VER ARGUEDAS DEMOSTRACION DE PROP 4 PARA CALCULAR MI LAMBDA Y RAZONAR PORQUE F22 TIENE QUE SER LO MAS BAJO POSIBLE.

Proposition 5: the optimal policy (s_i^*, π_i^*, f^*) induces compliance and it is characterized by:

$$c_i'(s_i^*) - \lambda_4 - \mu \frac{c_i''(s_i^*)}{\overline{a}} = 0$$

$$\pi_i^* = -\frac{c_i'(s_i^*)}{\overline{a}}$$

$$F(e_i - s_i) = \overline{a}(e_i - s_i)$$

where $\overline{a} > 0$ is the largest possible charge per unit of violation.

In other words, the optimal polciy induces compliance and have linear fines (idem Stranlund, 2007).

Proof of Proposition 5 ("adapted from Stranlund (2007)").

"First, we prove that the optimal policy induces compliance":

Assume to the contrary that the optimal policy does not induce compliance, and call it (s_i^n, π_i^n, f^n). Arguedas, following her proposition (4), assumes that the form of the $f^n = \frac{\overline{b}}{2}(e_i - s_i)^2$. where as in Arguedas, $\overline{b}$ is the (largest possible) $f''(0)$ given by the legislation.

If the policy induces non-compliance, we know from Proposition (3) that

$$c_i'(e_i^*) + \pi_i f'(e_i^* - s_i^*) = 0$$

$$c_i'(e_i^n) + \pi_i^n \left[\overline{b}(e_i^n - s_i^n)\right] = 0$$

Now consider an alternative policy (s_i^c, π_i^c, f^c), such that $s_i^c = e_i^n$ and $\pi_i^c = \pi_i^n$ *and*

$$f^c(.) = \overline{a}(e_i - s_i)$$

donde

$$\overline{a} = \overline{b}(e_i^n - s_i^n)$$

This policy induces compliance by construction since

$$-c_i'(e_i^n) = -c_i'(s_i^c) = \pi_i^n \left[\overline{b}(e_i^n - s_i^n)\right] = \pi_i^c \overline{a}$$

Therefore, $e_i^c = e_i^n$, which means that abatement costs are the same under both policies. Since $\pi_i^c = \pi_i^n$, expected monitoirng costs are equal too. However, fines collected under (s_i^c, π_i^c, f^c) are zero, therefore there are no sanctioning costs. We can conclude then that (s_i^c, π_i^c, f^c) is socially preferred, contradicting the initial assumption that the optimal policy induces non-compliance.

Substituting in Proposition (2) we obtain the desired result.