1 Tax Evasion with Ambiguity

Suppose an individual firm with abatement costs function $C(e_i)$, strictly decreasing and convex in emissions e_i. The firm faces an emissions tax t_i. The regulator cannot costlessly observe e_i, therefore asks the firm to repot its emissions. The firm can under-report emissions by reporting a level $r_i < e_i$. Doing this, the firm can reduce its tax payments but it also faces a chance of being audited and found under-reporting, which is fined with a unit penalty $\phi_i > t_i$.

The objective probability of being inspected is $p \in (0,1)$. But the firm does not know p. So he decides how much to emit and report according to a belief about p. Let's call π this *subjective probability of being inspected.* Nevertheless, the firm is uncertain about π. It faces ambiguity with respect to the probability of being inspected. This uncertainty is described by the cumulative distribution function $F(\pi; a, p)$. The parameter a is an *index of ambiguity.*

In addition, the firms´s perception about π, is distorted according to the *probability weighting function* $\varphi(\pi, p)$, which may have a value greater or less than π. However, when $\pi = p$, $\varphi(\pi, p) = p$.

The firm chooses a level of emissions and emissions report to minimize its total expected costs

$$TEC = C(e_i) + t_i \times r_i + \int_0^1 \varphi(\pi, p) dF(\pi; a, p) \times [\phi_i (e_i - r_i)]$$

Subject to $e_i \geq r_i \geq 0$.

Where

$$\int_0^1 \varphi(\pi, p) dF(\pi; a, p) \in (0, 1)$$

is defined as the *perceived probability of an inspection.* This is an *expected probability* that is determined by the distorted values that the firm believes the probability may take, $\varphi(\pi, p)$,and the uncertainty about these values ($F(\pi; a, p)$).

In the *absence of ambiguity* ($a = 0$), F is the improper distribution function equal to 0 for all $\pi < p$, and equal to 1 otherwise. Consequently

$$\int_0^1 \varphi(\pi, p) dF(\pi; 0, p) = \varphi(p, p) = p$$

and the firm´s objective function reduces to the classic expected cost with known inspection probability p. Snow and Warren **assume** that an increase in the index of ambiguity results in a mean preserving spread of F. In the presence of ambiguity ($a > 0$), $F(\pi; a, p)$ is a mean preserving spread of the improper distribution with mass at $\pi = p$.

Assuming (as Snow and Warren) that an increase in p causes a *first-order stochastic dominance* shift in F, that is $F_p < 0$, the effect of such an increase in the expected costs of the firm is given by

$$\frac{\partial TEC}{\partial p} = \left[\int_0^1 \varphi_p dF + \int_0^1 \varphi dF_p \right] \times \left[\phi_i \left(e_i - r_i \right) \right]$$

Integrating by parts (the second integral):

$$\frac{\partial TEC}{\partial p} = \left[\int_0^1 \varphi_p dF - \int_0^1 \varphi_\pi F_p d\pi \right] \times \left[\phi_i \left(e_i - r_i \right) \right]$$

To assure that the firm´s costs increase when p increases, one has to assume (as did Snow and Warren) that φ_π and φ_p are positive.

Because $F(\pi; a, p)$ is a mean preserving spread of the improper distribution with mass at $\pi = p$, an increase in ambiguity results in an elementary increase in risk.[1]

$$\int_0^\tau F_a(\pi; a, p) d\pi \geq 0$$

for all $\tau \in [0, 1]$, with strict inequality at some $\tau \in [0, 1]$ and with strict equality at $\tau = 1$.

The effect of an increase in ambiguity on the firm's total expected costs is given by

$$\frac{\partial TEC}{\partial a} = \left[\int_0^1 \varphi dF_a \right] \times \left[\phi_i \left(e_i - r_i \right) \right]$$

Integrating by parts (two times),

$$\frac{\partial TEC}{\partial a} = \left[\int_0^1 \varphi_{\pi\pi} \int_0^\tau F_a d\pi d\tau \right] \times \left[\phi_i \left(e_i - r_i \right) \right]$$

It follows that the firm is ambiguity neutral ($\partial TEC / \partial a = 0$) if $\varphi_{\pi\pi} = 0$ (φ is linear in π). As in the case of the absence of ambiguity, the objective function of an ambiguity neutral firm reduces to the expected costs function with the probability of an inspection equal to p. This is because the introduction of ambiguity has no effect on the expected costs of the firm if this is ambiguity neutral. Therefore, it must be the case that the expected probability of an audit remains equal to p, implying that $\varphi(\pi, p) = p$ for all $\pi \in [0, 1]$.

[1] By definition, an elementary increase in risk is a mean preserving spread. We know that if $G(.)$ is a mean-preserving spread of $F(.)$, then $\int_0^x G(t)dt \geq \int_0^x F(t)dt$ for all x. (See Mas-Colell, Whinston and Green (1995), pg. 198). The result follows from this definition.

In contrast, the introduction of ambiguity increases the expected costs of the **ambiguity averse firm** ($\partial TEC/\partial a > 0$). (This is a definition, as presented by Snow and Warren). For this to happen, for a firm that under reports, we need $\varphi_{\pi\pi} > 0$. Note that if the firm reports truthfully an increase in ambiguity has no effect on the firm´s expected costs because the firm face no expected penalty. Hence, **in the presence of ambiguity the perceived probability of an audit is greater than the true probability**, $\int_0^1 \varphi dF > p$ for taxpayers that are ambiguity averse.[2]

2 The Effect of Ambiguity on Taxpayer Compliance

Calling $\mathcal{L}$ the Lagrange equation, and λ_i the multiplier corresponding to the constraint $e_i - r_i \geq 0$, the FOCs for the choice of emissions and emissions report are:

$$
\begin{aligned}
\mathcal{L}_e &= C_e(e_i) + \left[\int_0^1 \varphi(\pi, p) dF(\pi; a, p) \right] \times \phi_i - \lambda_i = 0 \\
\mathcal{L}_r &= t_i - \left[\int_0^1 \varphi(\pi, p) dF(\pi; a, p) \right] \times \phi_i + \lambda_i \geq 0, r_i \geq 0, \mathcal{L}_r \times r_i = 0 \\
\mathcal{L}_\lambda &= r_i - e_i \geq 0, \lambda_i \geq 0, \lambda_i \times (r_i - e_i) = 0)
\end{aligned}
$$

(SOC to be seen)

With a constant marginal penalty, the firm will under-report (report $r_i = 0$)if the tax is larger than the expected penalty. In this case it will report zero emissions. Assuming we are in this situation, namely

$$
\begin{aligned}
r_i^* &= 0 \\
t_i &> \left[\int_0^1 \varphi(\pi, p) dF(\pi; a, p) \right] \times \phi_i
\end{aligned}
$$

an increase in ambiguity decreases under-reporting if it makes this inequality to change. This can only happen, obviously, if an increase in ambiguity increases the expected or perceived probability of being inspected. But we have just seen

[2] $TEC = C(e_i) + t_i r_i + \int_0^1 \varphi(\pi, p) dF(\pi; a, p) \times [\phi_i (e_i - r_i)]$. Ambiguity aversity is $\partial TEC/\partial a > 0$. The only way this can happen is that the integral increase with a. The derivative of the integral with respect to a is $\int_0^1 \varphi_{\pi\pi} \int_0^\tau F_a d\pi d\tau$, which is positive in the case of ambiguity aversity, as just seen. Q.E.D.

that this is the case if the firm is ambiguity averse. (See above). **It follows that, under a constant penalty scheme, a sufficient increase in ambiguity could make an ambiguity averse firm to report truthfully.**

Because experimental tests of cumulative prospect theory suggests that individuals are ambiguity loving with respect to uncertainty about a *small* probability of loss, this would suggest that, given actual probabilities of being inspected (small), an increase in ambiguity would reduce compliance, contrary to what the IRS would want.

Nevertheless, another brand of the literature treats experimental subjects as individuals instead using the individuals responses to estimate a unique probability weighting function of a representative individual. This literature finds that a considerable proportion of individuals (70% - 80%) are ambiguity averse for low levels of the probability of an audit, a non trivial proportion is ambiguity neutral and less than 10% are ambiguity loving. This raises the question the final effect of an increase in ambiguity on compliance since it could happen that the increase in compliance of ambiguity averse is outweighted by the decrease in compliance by ambiguity lovers.