Tax Evasion with Ambiguity

**1. The Model following Snow and Warren (2005)**

Suppose an individual firm with abatement costs function strictly decreasing and convex in emissions, faces an emissions tax. The regulator cannot costlessly observe therefore asks the firm to report its emissions. The firm can under-report emissions by reporting a level  Doing this, the firm reduces its tax payments but it also faces a chance of being audited and found under-reporting, which is fined with a unit penalty  The objective probability of being inspected is, but the firm does not know *p*. It faces ambiguity about *p*. Following Segal (1987), we assume that the uncertainty about *p* can be expressed as a set of possible Consequently, the firm uses a subjectively perceived probability of an inspection in order to decide how much to emit and report. This perceived probability of inspection plays the role of the subjective *decision weights* play in the Subjective Expected Utility Theory.

We assume that the firm determines how much to emit and report minimizing the Total Expected Costs of its decision. These Total Expected Costs are

subject to 

The Total Expected Costs are given by the abatement costs, the tax bill, and the subjective expected penalty. In order to calculate it we assume the firm uses a subjective expected value of *p*,, which we call the Following Snow and Warren (2005), which at the same time follow Sarin (1987), we assume that this perceived probability of being inspected (or decision weight) is formed as a weighted average of *subjective probabilities.* These aregiven by *the probability weighting function*. That is, the firms assigns a subjective probability value of ) to each of the possible values that the probability of being inspected *p* may take. However, it also weights these subjective values non-linearly. These subjective non-linear weights are captured by the function. We also assume that the firm assigns a second order probability distribution function The parameter *a* is an index of ambiguity. It captures the *ambiguity* (which is the degree of uncertainty about the subjective probabilities) as described below. Naturally, *F*(*0;a,p*)0, and *F*(*1;a,p*)1. So the perceived probability of being inspected, is

(1)

As it will become clear shortly, these non-linear weights are crucial. This is because the firm takes decisions about what to emit and report according to its Subjective Expected Costs, which are linear functions of the perceived probability of an inspection. But if this is just the subjective average of all possible values of *p,* and ambiguity is modeled as a mena preserving spread of the SOP, then an increase in ambiguity will not have an effect on this average, by definition. Ambiguity will not matter for choice.

Note the firm may distort the objective value of *p*. In particularly, according to the literature on non-expected utility theory, the firm may overweight “large” probabilities of being inspected (assign a value to > *p* form “large” values of *p*), and underweight “low” probabilities of being inspected (assign a value to < *p* for values of *p* “close” to zero). This form of *pessimism* can be connected to risk aversion and loss aversion. (See Starmer, 2000). But apart from distorting objectives probabilities, the firm faces *ambiguity*: it is not sure with respect to the values that it thinks the probability *p* may take. In this circumstance, *ambiguity aversion* may arise: the firm may prefer situations in which its subjective weights are formed with more information about *p* to situations in which these weights are formed with less information about *p*. In other words, it may prefer situations in which it is more confident with respect to its subjective probabilities, to situations in which it is less confident. In this settings, ambiguity averse may explain apparently non-rational choices (in terms of the Subjective Probability Theory, at least), such as the one first described by the Ellseberg Paradox (Ellsberg, 1961).

To study just the effect of ambiguity in the firm’s choices of *e* and *r*, we assume that the firm’s perceived probability of being inspected remains constant for all values of *a*. That is

for all *a*

where *pe* is the constant value that the perceived probability of being inspected takes, for all *a,* given *p*. In the absence of ambiguity the subjective probabilities, in the words of Ellsberg, embody all the firm`s attitudes toward risk. In the absence of ambiguity, Subjective Probability Theory applies.

Following Snow and Warren (2005), we assume that an increase in ambiguity, that is, an increase in the index of ambiguity *a* results in a mean preserving spread of *F* and that an increase in causes a first-order stochastic dominance shift in that is.

The effect of an increase *p* in the expected costs of the firm is given by



Integrating by parts the second integral,



Thus, to assure that the firm´s expected costs increase when  increases, one has to assume that is increasing ( is positive).

By definition, because we model an increase in ambiguity as a mean preserving spread, is an elementary increase in risk from for every, or equivalently second – order stochastically dominates. In this case, it also true that for all , or

 (2)

for all  with strict inequality at some  and with strict equality at 

**The effect of an increase in ambiguity on the firm's total expected costs** is given by



Integrating by parts,



According to equation (2) above, the integral in brackets is equal to zero. So an increase in ambiguity does not have an effect in the TEC. This is a natural result following the definition of ambiguity as a *mean preserving spread* of *F.*

Snow and Warren stated that in this case the firm is *ambiguity neutral* (In their model  happened when  ( is linear in ). In our model there is no , just . But I think this is wrong. By definition of ambiguity as an mean preserving spread it is always true that an increase in ambiguity does not changes you expected compliance costs because it does change your expected probability of being inspected. Unless you introduce the function , and you make it non-linear. Both of which are very ad-hoc.

I think an ambiguity averse agent does not maximize total expected costs. By definition of ambiguity, the objective function of an ambiguity neutral firm reduces to the expected costs function with the probability of an inspection equal to the so-called perceived probability (expected probability of *p*).

In contrast, the definition of an ambiguity averse firm for Snow and Warren is  But, again, I think this is wrong. By the way, different versions of a definition of ambiguity in the literature appears formally in very recent papers: Epstein (1999), Epstein and Zhang (2001) and Grant and Quiggin (2005). I am now trying to incorporate a “more correct” definition of ambiguity in our model based on these papers. TRATAR DE USAR LA DEFINCIÓN DE AMBIGUITY AVERSE QUE LEÍ EN ALGUN LADO. For this to happen, for a firm that under reports, we need  Note that if the firm reports truthfully an increase in ambiguity has no effect on the firm´s expected costs because the firm face no expected penalty. Hence, in the presence of ambiguity the perceived probability of an audit is greater than the true probability,  for taxpayers that are ambiguity averse.[[1]](#footnote-2)

The Effect of Ambiguity on Taxpayer Compliance

Calling  the Lagrange equation, and  the multiplier corresponding to the constraint  the FOCs for the choice of emissions and emissions report are:



(SOC to be seen)

With a constant marginal penalty, the firm will under-report (report  if the tax is larger than the expected penalty. In this case it will report zero emissions. Assuming we are in this situation, namely



an increase in ambiguity decreases under-reporting if it makes this inequality to change. This can only happen, obviously, if an increase in ambiguity increases the expected or perceived probability of being inspected. But we have just seen that this is the case if the firm is ambiguity averse. (See above). It follows that, under a constant penalty scheme, a sufficient increase in ambiguity could make an ambiguity averse firm to report truthfully.

Because experimental tests of cumulative prospect theory suggests that individuals are ambiguity loving with respect to uncertainty about a small probability of loss, this would suggest that, given actual probabilities of being inspected (small), an increase in ambiguity would reduce compliance, contrary to what the IRS would want.

Nevertheless, another brand of the literature treats experimental subjects as individuals instead using the individuals responses to estimate a unique probability weighting function of a representative individual. This literature finds that a considerable proportion of individuals (70% - 80%) are ambiguity averse for low levels of the probability of an audit, a non trivial proportion is ambiguity neutral and less than 10% are ambiguity loving. This raises the question the final effect of an increase in ambiguity on compliance since it could happen that the increase in compliance of ambiguity averse is outweighted by the decrease in compliance by ambiguity lovers.

**2. The Model with inspection probability conditioned on the firm’s emissions and reports**

1.  Ambiguity aversity is  The only way this can happen is that the integral increase with  The derivative of the integral with respect to  is  which is positive in the case of ambiguity aversity, as just seen. Q.E.D. [↑](#footnote-ref-2)