"Ambiguity about Audit probability and tax evasion"

(Snow and Warren, 2005)

The expected utility theory provides an inadequate framework for analyzing the importance of biased estimates of inspections probabilities. Expected utility maximizers would not change their decisions with increasing uncertainty regarding the outcome probabilities if the expected probability does not change. To overcome this, the alternative literature has introduced nonlinearity dependence on the outcome probabilities. This is done by introducing a nonlinear *probability weighting function* that systematically biases the subjective probability. The attitutes toward ambiguity are associated with different shapes of the probability weighting function.

1 Tax Evasion with Ambiguity

Suppose an individual taxpayer with a fixed taxable income W, a tax t and undeclared income x. When the taxpayer is not audited the income is $W_N = W(1-t)+tx$. When the tax payer is audited the income is $W_A = W_N - \theta tx$, $\theta > 1$ is the gross penalty rate. The taxpayer is risk averse; its utility function $U(W)$ is assumed to be strictly concave in W.

The *objective probability of being audited* is $p \in (0,1)$. *But the tax payer is uncertain about* p. *It faces ambiguity.* Let π denote the taxpayer *subjective probability of being audited*. The taxpayer uncertainty about π is described by the cumulative distribution function $F(\pi; a, p)$. (Because the uncertainty of the taxpayer is over probabilities instead of outcomes, the distribution function $F(\pi; a, p)$ is known in the decision theory literature as Second Order Probability (SOP) distribution). The parameter a is an *index of ambiguity.*

The authors assume that the expectations about π is unbiased in the sense that

$$\int_0^1 \pi dF(\pi; a, p) = p$$

for all values of a.

The taxpayer perception about π, however, is distorted according to the *probability weighting function* $\varphi(\pi, p)$, which may have a value greater or less than π. **The probability weighting function introduces a systematic bias in the perceived probability of an audit in a manner that dependes on the concavity of φ with respect to π. ¿Cómo es que la subjective probability π influye la perceived probability $\varphi(\pi, p)$? π es un prior/belief.** However, they assume that when $\pi = p$, $\varphi(\pi, p) = p$.

The taxpayer chooses an amount of undeclared income x^* to maximize the objective function

$$\begin{aligned} E(U) &= \left[1 - \int_0^1 \varphi(\pi, p) dF(\pi; a, p)\right] U(W_N) + \int_0^1 \varphi(\pi, p) dF(\pi; a, p) \times U(W_A) \\ &= U(W_N) - \left[\int_0^1 \varphi(\pi, p) dF(\pi; a, p)\right] [U(W_N) - U(W_A)] \end{aligned}$$

Where

$$\int_0^1 \varphi(\pi, p) dF(\pi; a, p) \in (0, 1)$$

is defined as the *perceived probability of an audit.* This is an *expected probability* that is determined by the distorted values that the taxpayer believes the probability may take, $\varphi(\pi, p)$,and the uncertainty about these values $(F(\pi; a, p))$.

The authors assume that this expected probability is always sufficiently low so that $x^* > 0$.

In the *absence of ambiguity* $(a = 0)$, F is the improper distribution function equal to 0 for all $\pi < p$, and equal to 1 otherwise. That is

$$F(\pi; 0, p) = \begin{matrix} 0 \text{ if } \pi < p \\ 1 \text{ if } \pi \geq p \end{matrix}$$

In other words, $\pi = p$. Consequently

$$\int_0^1 \varphi(\pi, p) dF(\pi; 0, p) = \varphi(p, p) = p$$

In this case, the taxpayer´s objective function reduces to the expected utility of wealth with an audit probability of p. The authors **assume** that an increase in the index of ambiguity results in a mean preserving spread of the SOP distribution. In the presence of ambiguity $(a > 0)$, $F(\pi; a, p)$ is a mean preserving spread of the improper distribution with mass at $\pi = p$.

The authors **assume** that an increase in p causes a *first-order stochastic dominance* (FSD) shift in F. This is $F_p < 0$. The effect of such an increase in the expected utility of the taxpayer is given by

$$\frac{\partial E[U]}{\partial p} = -\left[\int_0^1 \varphi_p dF + \int_0^1 \varphi dF_p\right] \times [U(W_N) - U(W_A)]$$

Integrating by parts (VER):

$$\left[-\int_0^1 \varphi_p dF + \int_0^1 \varphi_\pi F_p d\pi\right] \times [U(W_N) - U(W_A)]$$

As an FSD shift, $F_p < 0$. So to assure that the taxpayer's utility declines when p increases the authors has to assume that φ is monotonically increasing in π ($\varphi_\pi > 0$) and the expected value (WHY EXPECTED VALUE?) of φ_p is positive.

Because an increase in ambiguity results in an increase in risk in the sense of Rothschild and Stiglitz (1970), (VER)

$$\int_0^\tau F_a(\pi; a, p) d\pi \geq 0$$

for all $\tau \in [0, 1]$, with strict inequality at some $\tau \in [0, 1]$ and with strict equality at $\tau = 1$.The qualitative effect of an increase in ambiguity on the taxpayer's welfare is given by

$$\frac{\partial E[U]}{\partial a} = -\left[\int_0^1 \varphi dF_a\right] \times [U(W_N) - U(W_A)]$$

integrating by parts (VER)

$$\frac{\partial E[U]}{\partial a} = -\left[\int_0^1 \varphi_{\pi\pi} \int_0^1 F_a d\pi d\tau\right] \times [U(W_N) - U(W_A)]$$

It follows that the taxpayer is ambiguity neutral if ($\partial E(U)/\partial a = 0$) if $\varphi_{\pi\pi} = 0$ (φ is linear in π). As in the case of the absence of ambiguity, the objective function of an ambiguity neutral taxpayer reduces to the expected welfare function with the probability of an audit equal to p. This is because if you introduce ambiguity this has no effect on the expeted utility of the taxpayer if this is ambiguity neutral. Therefore, it must be the case that the expected probability of an audit remains equal to p, implying that $\varphi(\pi, p) = p$ for all $\pi \in [0, 1]$.

In contrast, the introduction of ambiguity *reduces the welfare* of a taxpayer who is **ambiguity averse** ($\partial E(U)/\partial a < 0$). ESTO ES UNA DEFINICIÓN, NO UN RESULTADO. AMBIGUITY AVERSE IS $\partial E(U)/\partial a < 0$, WHICH SAYS THAT AN INCREASE IN AMBIGUITY REDUCES EXPECTED UTILITY).

For this to happen they need $\varphi_{\pi\pi} > 0$. (**¿No es que** $\int_0^1 F_a(\pi; a, p) d\pi = 0$?)

Hence, **in the presence of ambiguity the perceived probability of an audit is greater than the true probability**, $\int_0^1 \varphi dF > p$ for taxpayers that are ambiguity averse.[1]

[1] $E(U) = U(W_N) - \left[\int_0^1 \varphi(\pi, p) dF(\pi; a, p)\right] [U(W_N) - U(W_A)]$. Ambiguity aversity is

2 The Effect of Ambiguity on Taxpayer Compliance

The FOC for the choice of undeclared income is:

$$\frac{\partial E(U)}{\partial x} = \left\{ U'(W_N) - \left[\int_0^1 \varphi(\pi, p) dF(\pi; a, p) \right] [U'(W_N) - U'(W_A) \times (1 - \theta)] \right\} t = 0$$

The SOC is satisfied because the taxpayer is risk averse (VER).

An increase in ambiguity increases compliance ($\partial x^* / \partial a < 0$) if the marginal value of undeclared income declines as ambiguity increases. That is

$$- \left[\int_0^1 \varphi_{\pi\pi} \int_0^\tau \varphi F_a d\pi d\tau \right] \times [U'(W_N) - U'(W_A) \times (1 - \theta)] < 0$$

Becasue $\theta > 1$, the sign of the LHS is the same as the sign of the term in brackets. **It follows that an increase in ambiguity aversion increases compliance if the taxpayer is ambiguity averse.**

Because experimental tests of cumulative prospect theory suggests that individuals are ambiguity loving with respect to uncertainty about a *small* probability of loss, this would suggest that, given actual probabilities of being inspected (small), an increase in ambiguity would reduce compliance, contrary to what the IRS would want.

Nevertheless, another brand of the literature treats experimental subjects as individuals instead using the individuasl responses to estimate a unique probability weighting function of a representative individual. This literature finds that a considerable proportion of individuals (70% - 80%) are ambiguity averse for low levels of the probability of an audit, a non trivial proportion is ambiguity neutral and less than 10% are ambiguity loving. This raises the question the final effect of an increase in ambiguity on compliance since it could happen that the increase in compliance of ambiguity averse is outweight by the decrease in compliance by ambiguity lovers.

$\partial E(U)/\partial a < 0$. The only way this can happen is that the integral increase with a. The derivative of the integral with respect to a is $\int_0^1 \varphi_{\pi\pi} \int_0^1 F_a d\pi d\tau$, which is positive in the case of ambiguity aversity, as just seen. Q.E.D.