**1. Do you really understand how Snow and Warren model of ambiguity aversion? I don't. In particular I'm confused by the following: We have *p* as the objective probability of an inspection, *pi* as the firm's subjective probability of inspection, and the integral term as the perceived probability of inspection. I don't understand why the firm is uncertain about its *subjective probability of inspection*, and why this is different from its *perceived probability of inspection*. Do you understand this?**

John, I have finished reading the literature on ambiguity that I considered relevant to try to understand what Snow and Warren were doing. Snow and Warren’s approach does not resemble any of the models of ambiguity that use a second order probability distribution (surveyed by Camerer and Weber (1992)). It does not resemble either a model that incorporates decision weights to the second order probability. So my answer, after reading this literature, is the same as the one I gave you last time: their modeling is rather ad-hoc.

According to the literature, people distort objective probabilities. Typically, they overweight small probabilities and underweight large probabilities. This is captured by “*probability weighting functions*”. As you know, these are *subjective probabilities* that people construct. The probability weighting function reflects the “psychophysics of risk”, that is, the way by which theindividuals subjectively “distort” objective probabilities. The *decision weight* then determines the way in which the probability weights enter the value function. (See the summary of Starmer (2000) I attach).

Turning to ambiguity aversion, according to Camerer and Weber (1992), one of the ways that ambiguity aversion has been modeled is to assume that agents know or assign a (unique) Second-Order Probability (SOP) Distribution to the possible values of *p*, the probability of the event. These theories model ambiguity aversion as “risk aversion” in the SOP distribution. A short illustration of what this means follows. Assume, as Snow and Warren, that taxpayers do not know the value of *p*, but they think it may take the possible values . Agents assign probabilities to each of these according to the distribution function . This a subjective probability distribution over the possible values that the agent thinks *p* may take. The parameter *a* captures the *ambiguity*, ***which is the degree of uncertainty*** about the subjective probabilities (this interpretation of ambiguity is clearly stated by Ellsberg (1961)). See the table below. Say that without ambiguity (*a = 0*) the taxpayer think that *p,* the probability of an inspection, may take the values ½ or ¾, each with probability ½. So his expected probability of being inspected is 0.625. In the absence of ambiguity, there is no uncertainty about the . We are in the case of Subjective Expected Utility Theory of Savage. There is no ambiguity aversion, the subjective utility captures only the attitudes toward risk. (Ellsberg, 1961). Assume now that the taxpayer is now ambiguous about the possible values that you think *p* may take, for whatever the reason. He thinks that the probability of inspection may take the values ¼, ½, ¾ and 1, each with probability ¼. Note that this last distribution is a mean preserving spread from the previous one (in which you assigned zero probabilities to *p = 1,* and to *p = ¼*)*.* Even though the expected probability of being inspected does not change, if you are ambiguity averse you are going to prefer the first distribution (*a* = 0) to the second one (*a* >0). This is why Snow and Warren modeled an increase in the parameter *a*, the index of ambiguity, as a *mean-preserving spread* of the Second Order Probability distribution .

Table

|  |  |  |
| --- | --- | --- |
|  |  |  |
| 1 | 0 | ¼ |
| ¾ | ½ | ¼ |
| ½ | ½ | ¼ |
| ¼ | 0 | ¼ |

Now, Snow and Warren modeling of ambiguity does not resemble any of the formal models that incorporate ambiguity aversion based on assuming a known second-order probability distribution. (These models are summarized in Camerer and Weber (1992). See summary attached). Snow and Warren’s modeling is more similar to Segal (1987) in the sense that they assume the agent assigns a subjective probability to *p,* but also weights each of these in a fashion that guarantees ambiguity aversion. But their weights differ. In Snow and Warren the decision weight is what they call the *perceived probability of inspection*, a sort of distorted expected probability. According to Segal, the decision weight would be different. (See summary of Segal (1987) attached).

In conclusion, Snow and Warren’s use of a is “ ad-hoc”, as far as I can tell. It is clear why they use it though. Without such a non-linear function, an increase in ambiguity as a mean preserving spread would have no effect in the total expected costs of the firm because it would not change the perceived probability of an audit. *(If you used a perceived probability of being inspected of the form*, instead of the one they used ).

**2. Carlos and I got criticized for assuming that the regulator's probability of inspection is independent of the firm's choice of emissions and its emissions report. We then asked whether it is possible for a regulator to reduce expected enforcement costs by conditioning its inspection probability on these variables. This is worth doing here as well. Is it possible to include *p*(*e*, *r*) in your model? I suppose that then we would have  *pi*(*e*,*r*) as well. I'm attaching the last version of our emissions taxes paper, in case you don't have it, where in section 4 we answer the question of whether a regulator can reduce expected enforcement costs by conditioning its inspection probability on a firm's emissions and report and allowing noncompliance.  In that paper we assumed that the regulator was uncertain about the firm's abatement costs. I don't think we need to assume that for the ambiguity aversion paper.**

**3. What is the perceived probability of inspection (the integral term) when the firm is compliant? Is it equal to *p* or is it different? The central question depends on this.**

**4. Any chance I can get you to write in Word? I don't use Latek and the word document you sent didn't have the equations. This is not a huge deal (that is why it is number 4), but if you are comfortable writing in Word, then I prefer that we do that. Otherwise I have to rewrite all the equations.**

Of course I have no problem in writing in word.

**References**

Camerer and Weber, “Recent Developments in Modeling Preferences: Uncertainty and Ambiguity”, *Journal of Risk and Uncertainty,* 5: 325-370 (1992).

Ellsberg, D. (1951). “Risk, Uncertainty, and the Savage Axioms”, *Quarterly Journal of Economics,* **75,** 643 – 669.

Epstein, L. (1999). “A Definition of Uncertainty Aversion”, *Review of Economic Studies,* **66,** 579 – 608.

Epstein, L. and J. Zhang. (2001). “Subjective Probabilities on Subjectively Unambiguous Events”, *Econometrica,* **69** (2) 265 – 306.

Grant S. and J. Quiggin. (2005). “Increasing uncertainty: a definition”, *Mathematical Social Sciences,* **49,**117 – 141.

Segal, U. (1987). “The Ellsberg Paradox and Risk Aversion: An Anticipated Utility Approach”, *International Economic Review,* **28** (1), 175 – 202.

Starmer, C. (2000). “Developments in Non-Expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk”, *Journal of Economic Literature,* **38** (2), 332-382.