Exercise 5

Using the file bond.wf1 that contains USA 3, 6 and 12 months interest rates:

a) Estimate an error correction model for these variables.

b) Check whether the variables are cointegrated.

Solution

a) Before testing for cointegration we will check whether the relevant variables are I(1), that is, we are going to test whether all the univariate time series have a unit root.

We proceed, as we did it before in the exercise 3 by checking the order of integration of all the series. We use the ADF and Phillips-Perron (not reported) tests for unit root.

We first check the order of integration of the series $r3$. We use the command "Quick" "Series Statistics", "Unit Root Test" and then type the series: $r3$. You will be asked to choose the lags to be included in the estimation, initially, select 3 lags (or more).

We estimate an equation such as

$$\Delta y_t = \mu + \lambda y_{t-1} + \alpha_1 \Delta y_{t-1} + ... + \alpha_3 \Delta y_{t-3} + \varepsilon_t$$

In this case we chose to augment the regression with 3 lags. To choose the order of augmentation of the DF regression several procedures have been proposed in the literature. Some of these consist in:

(i) *choosing k as a function of the number of observations as in Schwert (1989)*

$k = INT(12(T/100)^{1/12})$

(ii) *information based rules such as AIC and BIC.*

(iii) *Sequential rules*

Our results are based on a sequential rule procedure.

ADF Test Statistic	-2.441812	1% Critical Value*		-3.4770
		5% Critical Value		-2.8817
		10% Critical Value		-2.5774
MacKinnon critical values for				
rejection of hypothesis of a unit root.				
Augmented Dickey-Fuller Test Equation				
Dependent Variable: D(R3)				
Method: Least Squares				
Date: 07/07/03 Time: 03:06				
Sample(adjusted): 5 147				
Included observations: 143 after adjusting endpoints				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
R3(-1)	-0.096052	0.039336	-2.441812	0.0159
D(R3(-1))	-0.125051	0.085594	-1.460974	0.1463
D(R3(-2))	-0.205069	0.082687	-2.480058	0.0143
D(R3(-3))	0.234407	0.082536	2.840051	0.0052
C	0.631056	0.268351	2.351607	0.0201
R-squared	0.196513	Mean dependent var		0.015664
Adjusted R-squared	0.173223	S.D. dependent var		1.249541
S.E. of regression	1.136173	Akaike info criterion		3.127547
Sum squared resid	178.1427	Schwarz criterion		3.231143
Log likelihood	-218.6196	F-statistic		8.437831
Durbin-Watson stat	1.998974	Prob(F-statistic)		0.000004

The results show that we cannot reject the null hypothesis of unit root. Nevertheless, given that the statistic is close to the critical value is recommended that a KPSS test is conducted as well.

We perform a similar analysis on the 6 and 12 month bond and find that we cannot reject the unit root hypothesis for any of these maturities.

ADF Test Statistic	-2.345686	1% Critical Value*		-3.4770
		5% Critical Value		-2.8817
		10% Critical Value		-2.5774
MacKinnon critical values for rejection of hypothesis of a unit root.				
Augmented Dickey-Fuller Test Equation				
Dependent Variable: D(R6)				
Method: Least Squares				
Date: 07/07/03 Time: 03:08				
Sample(adjusted): 5 147				
Included observations: 143 after adjusting endpoints				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
R6(-1)	-0.089630	0.038211	-2.345686	0.0204
D(R6(-1))	-0.138039	0.085920	-1.606605	0.1104
D(R6(-2))	-0.187253	0.083790	-2.234796	0.0270
D(R6(-3))	0.196331	0.083212	2.359399	0.0197
C	0.603450	0.265094	2.276360	0.0244
R-squared	0.165350	Mean dependent var		0.015944
Adjusted R-squared	0.141157	S.D. dependent var		1.163283
S.E. of regression	1.078058	Akaike info criterion		3.022539
Sum squared resid	160.3849	Schwarz criterion		3.126135
Log likelihood	-211.1115	F-statistic		6.834698
Durbin-Watson stat	2.025045	Prob(F-statistic)		0.000048

ADF Test Statistic	-2.282988	1% Critical Value*	■	-3.4770
■	■	5% Critical Value	■	-2.8817
■	■	10% Critical Value	■	-2.5774
MacKinnon critical values for rejection of hypothesis of a unit root.	■	■	■	■
■	■	■	■	■
■	■	■	■	■
Augmented Dickey-Fuller Test Equation	■	■	■	■
Dependent Variable: D(R12)	■	■	■	■
Method: Least Squares	■	■	■	■
Date: 07/07/03 Time: 03:10	■	■	■	■
Sample(adjusted): 5 147	■	■	■	■
Included observations: 143 after adjusting endpoints	■	■	■	■
Variable	Coefficient	Std. Error	t-Statistic	Prob.
R12(-1)	-0.087306	0.038242	-2.282988	0.0240
D(R12(-1))	-0.187770	0.086451	-2.171993	0.0316
D(R12(-2))	-0.146831	0.085905	-1.709223	0.0897
D(R12(-3))	0.153265	0.083822	1.828466	0.0696
C	0.592195	0.265230	2.232763	0.0272
R-squared	0.142806	Mean dependent var	■	0.014825
Adjusted R-squared	0.117959	S.D. dependent var	■	1.075138
S.E. of regression	1.009737	Akaike info criterion	■	2.891596
Sum squared resid	140.7005	Schwarz criterion	■	2.995192
Log likelihood	-201.7491	F-statistic	■	5.747582
Durbin-Watson stat	2.034437	Prob(F-statistic)	■	0.000262

Before testing for cointegration we should find the order of the VECM. Tests for cointegration are only valid if we chose the order of the VECM correctly. We estimate a VECM(8) in order to perform lag deletion tests using the Lag length option of Eviews. We estimate the following model:

$$\Delta z_t = \Gamma_1 \Delta z_{t-1} + ... + \Gamma_8 \Delta z_{t-8} + \Pi z_{t-1} + u_t$$

The results are presented below results of Wald tests to choose the order of the VECM are presented below:

VEC Lag Exclusion Wald Tests				
Date: 07/07/03 Time: 03:15				
Sample: 1 147				
Included observations: 138				
Chi-squared test statistics for lag exclusion:				
Numbers in [ ] are p-values				
	D(R3)	D(R6)	D(R12)	Joint
DLag 1	3.795556	5.046446	6.393459	72.44040
	[0.284403]	[0.168428]	[0.093960]	[5.05E-12]
DLag 2	3.669698	4.180503	4.242954	50.53832
	[0.299411]	[0.242621]	[0.236396]	[8.53E-08]
DLag 3	10.06537	7.097250	6.049544	41.82781
	[0.018019]	[0.068862]	[0.109225]	[3.53E-06]
DLag 4	3.669227	3.778101	2.023004	32.02195
	[0.299468]	[0.286444]	[0.567646]	[0.000197]
DLag 5	5.742318	3.410638	1.915220	24.67676
	[0.124843]	[0.332538]	[0.590188]	[0.003350]
DLag 6	8.867471	8.240781	5.744135	18.34101
	[0.031106]	[0.041289]	[0.124744]	[0.031417]
DLag 7	7.834670	7.428994	8.551366	12.14414
	[0.049555]	[0.059411]	[0.035890]	[0.205305]
DLag 8	2.682312	3.277464	4.956070	15.07768
	[0.443242]	[0.350792]	[0.175042]	[0.088824]
df	3	3	3	9

Based on the results presented below we choose a VECM(6):

$$\Delta z_t = \Gamma_1 \Delta z_{t-1} + ... + \Gamma_6 \Delta z_{t-6} + \Pi z_{t-1} + u_t$$

Where $\Pi = \alpha\beta'$, α representing the speed of adjustment and β' being the long-run cointegrating relations. As our variables are all assumed to be I(1), the difference terms are I(0) and the last term must be I(0) (to ensure that the dependent variable and the conditional mean are integrated of the same order). We must look at the Π rank to figure out if it exists any cointegration relationship. So, Π could have:

a) Full rank, then z_t have to be I(0) (these results will contradict our preliminary analysis which suggested that the variables where I(1)).

b) It is zero, there is no cointegration vector, (this will imply that relationships such as the term structure of the interest rates can not hold)

c) It is reduced $rank = r$, the number of cointegrated vector is equal to $Rank(\Pi) = r$.

To find the number of cointegrating vectors we said that is equivalent to find the number of linearly independent columns in Π or the number of $n - r$ columns of significantly small.

The approach amounts to a reduced rank regression which provides n eigenvalues $\hat{\lambda}_1 > \hat{\lambda}_2 > .....\hat{\lambda}_n$ and their corresponding eigen vectors,$\widehat{\beta} = (\widehat{\beta}_1 > \widehat{\beta}_2 > ... > \widehat{\beta}_n)$. Those r elements in $\widehat{\beta}$ which determine linear combinations of stationary relationships can be denoted $\beta = (\widehat{\beta}_1, \widehat{\beta}_2, ..., \widehat{\beta}_r)$ that is, the distinct $\widehat{\beta}_i' z_t$, which we will denote $\beta_i' z_t$, are correlated with the stationary part of the model. The last $n - r$ combinations obtained from the Johansen approach indicate the non stationary combinations, and theoretically these are uncorrelated with the stationary elements in the ECM. Consequently, for the eigenvectors corresponding to the non-stationary part of the model, $\hat{\lambda}_i = 0$ for $i = r + 1, ..., n$.

Thus to test the null hypothesis that there are at most r cointegration vectors amounts to test

$$H_0)\lambda_i = 0 \quad i = r + 1, ..., n.$$

where only the first r eigenvalues are non-zero. (This is tested against the alternative of n cointegrating vectors).

It can be shown (see Hamilton) that the likelihood test that corresponds to the ECM under the null that there are only r cointegrating vectors is

$$L^*(H_0) = -(Tn/2)log(2\pi) - (Tn/2) - (T/2)log\left|\widehat{S}_{00}\right| - (T/2)\sum_{i=1}^{r}\log(1 - \hat{\lambda}_i).$$

It can also be shown that the likelihood test that corresponds to the ECM without any restriction on the number of cointegrating vectors is

$$L^* = -(Tn/2)log(2\pi) - (Tn/2) - (T/2)log\left|\widehat{S}_{00}\right| - (T/2)\sum_{i=1}^{n}\log(1 - \hat{\lambda}_i).$$

Then a likelihood ratio test, *using a non standard distribution*, can be constructed, using what is known as the **Trace statistic.**

$$\lambda_{trace} = -T \sum_{i=r+1}^{n} \log(1 - \hat{\lambda}_i) \qquad r = 0, 1, ..., n-2, n-1$$

The results obtained when we include r3, r6 and r12 are somehow puzzling. We find that we reject the hypothesis of i) none against 3, ii) at most 1 against 3 and iii) at most 2 against 3. We should conclude from our analysis that the three series are stationary.

Date: 07/07/03 Time: 03:17				
Sample(adjusted): 8 147				
Included observations: 140 after adjusting endpoints				
Trend assumption: Linear deterministic trend				
Series: R3 R6 R12				
Lags interval (in first differences): 1 to 6				
Unrestricted Cointegration Rank Test				
Hypothesized		Trace	5 Percent	1 Percent
No. of CE(s)	Eigenvalue	Statistic	Critical Value	Critical Value
None **	0.219618	63.60823	29.68	35.65
At most 1 **	0.137887	28.89211	15.41	20.04
At most 2 **	0.056353	8.120480	3.76	6.65
(**) denotes rejection of the hypothesis at the 5%(1%) level				
Trace test indicates 3 cointegrating equation(s) at both 5% and 1% levels				

Another test of the significance of the largest λ_r is the so called maximal-eigenvalue or $\boldsymbol{\lambda} -$ **max statistic** :

$$\lambda_{\max} = -T \log(1 - \hat{\lambda}_{r+1}) \qquad r = 0, 1, ..., n-2, n-1.$$

This tests the existence of r cointegrating vectors against the alternative that $r+1$ exist and is derived in exactly same way.

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	5 Percent Critical Value	1 Percent Critical Value
None **	0.219618	34.71612	20.97	25.52
At most 1 **	0.137887	20.77163	14.07	18.63
At most 2 **	0.056353	8.120480	3.76	6.65
(**) denotes rejection of the hypothesis at the 5%(1%) level				
Max-eigenvalue test indicates 3 cointegrating equation(s) at both 5% and 1% levels				

This results are consistent with the results for the trace statistic.

This exercise show how fragile all this tests can be. The implications in this case are of great relevance since different results about the order of integration of the variables have different implications in terms of how we should parameterize the term structure model.