Exercise 3

1) Consider the Bivariate series y_t generated by the VAR(2) system

$$y_t = \begin{pmatrix} 0.02 \\ 0.03 \end{pmatrix} + \begin{pmatrix} 0.5 & 0.1 \\ 0.4 & 0.5 \end{pmatrix} y_{t-1} + \begin{pmatrix} 0 & 0 \\ 0.25 & 0 \end{pmatrix} y_{t-2} + u_t$$

where $[u_t]$ is an i.i.d WN vector

a) Show that $[y_t]$ is weakly stationary

b) Determine the mean vector of y_t.

c) Calculate the impulse responses ψ_s for s= 1,2, as weel as the matrix long run multipliers.

2) Assume y_t follows an AR(3) process.

a1) Write y_t as an ADF equation.

a2) Which order of augmentation should capture the dynamics of y_t (i.e. the ADF equation should have WN errors)

a3) Which restriction the null hypothesis of y_t having a unit root impose on the parameters of the AR(3)

b) Which order of augmentation would you suggest when y_t follows an ARMA(2,1) process.

3) Using the file pv.wf1 (Shiller‘s 1987 data)

i) Check the order of integration of Real prices and dividens.

ii) Regresss Real Stock Prices on Real Dividends and a Constant. Check the order of integration of the residuals.

Solution

1) For the $VAR(2)$ presented in the exercise we note that the peculiarity is that the matrix of the second lag has three elements equal to zero. This implies that with these restrictions the relevant polynomial to determine stationarity is of order 3 and not 4.

For

$$y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + \varepsilon_t$$

where $c = \begin{pmatrix} 0.02 \\ 0.03 \end{pmatrix}$, $A_1 = \begin{pmatrix} 0.5 & 0.1 \\ 0.4 & 0.5 \end{pmatrix}$, $A_2 = \begin{pmatrix} 0 & 0 \\ 0.25 & 0 \end{pmatrix}$

a) The VAR is weakly stationary if all the roots of

$$Det(I - A_1 L - A_2 L^2) = 0$$

or

$$Det\begin{pmatrix} 1 - 0.5L & -0.1L \\ -.0.4L - 0.25L^2 & 1 - 0.5L \end{pmatrix}) = 0$$

lie outside the unit circle.

This gives a polynomial of order 3 in L and all the roots are outside the unit circle.

b) If the process is weakly stationary it has a mean, that is,

$$\mu = c + A_1\mu + A_2\mu$$

or

$$\mu = (I - A_1 - A_2)^{-1} c = \begin{pmatrix} 0.5 & -0.1 \\ -.0.65 & 0.5 \end{pmatrix}^{-1} \begin{pmatrix} 0.02 \\ 0.03 \end{pmatrix}$$

c) To calculate the impulse response functions we need to find the MA representation using the estimated coefficients of the VAR.In oreder to do that let us first define the non deterministic part of y_t as $y'_t = y_t - \mu$, and then write the moving average representation.

Remember that the definition of an impulse response function is

$$\frac{\partial y'_{t+s}}{\partial \varepsilon_t'} = \psi_s$$

where ψ_s is the matrix of the ε_{t-s} element of the Moving average representation:

$$y'_t = \psi(L)\varepsilon_t.$$

where $\psi(L) = (\psi_0 + \psi_1 L + \psi_2 L^2 + ...\psi_s L^s + ...)$

On the other hand since the process is stationary the moving average representation can be written as

$$.y'_t = (I - A_1 L - A_2 L^2)^{-1}\varepsilon_t.$$

Then it implies that

$(I - A_1 L - A_2 L^2)(\psi_0 + \psi_1 L + \psi_2 L^2 + ...\psi_s L^s + ...) = I$

This can be thought as an equality of two polynomials, then to find the values of ψ_s, for $s = 1, 2$, we only need to equate the terms in L^i for i = 0,1,2, that is .

$I.\psi_0 = I$ for terms in L^0 →this implies $\boxed{\psi_0 = I}$

$-A_1.\psi_0 + I\psi_1 = I$ for terms in L^1 →this implies $\boxed{\psi_1 = A_1}$

$-A_1.\psi_1 - A_2.\psi_0 + I\psi_2 = I$ for terms in L^2 →this implies $\boxed{\psi_2 = A_2 + A_1^2.}$

To find the long run Multiplier remember that the definition is $\sum\limits_{s=0}^{\infty} \psi_s. = \psi(1)$

Now recall that $\psi(1) = (I - A_1 - A_2)^{-1} = \begin{pmatrix} 0.5 & -0.1 \\ -.0.65 & 0.5 \end{pmatrix}^{-1}.$

2) Consider the following ARMA(3)

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + \varepsilon_t$$

We need to write this AR(3) as an augmented Dickey-Fuller regression, i.e.,

$$\Delta y_t = \lambda y_{t-1} + \sum_{i=1}^{k} \gamma_i \Delta y_{t-i} + \varepsilon_t$$

Parts *a) i) and ii)* involve doing a reparameterization of the AR(3).

Add in both sides of the equation y_{t-1}

$$\Delta y_t = (\phi_1 - 1)y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + \varepsilon_t$$

If we add and subtract $\phi_2\ y_{t-1}$ on the right hand side we get

$$\Delta y_t = (\phi_1 + \phi_2 - 1)y_{t-1} - \phi_2 \Delta y_{t-1} + \phi_3 y_{t-3} + \varepsilon_t$$

If we add and subtract$\phi_3\ y_{t-1}$ on the right hand side we get

$$\begin{aligned}
\Delta y_t &= (\phi_1 + \phi_2 + \phi_3 - 1)y_{t-1} - \phi_2 \Delta y_{t-1} - \phi_3 y_{t-1} + \phi_3 y_{t-3} + \varepsilon_t \\
&= (\phi_1 + \phi_2 + \phi_3 - 1)y_{t-1} - \phi_2 \Delta y_{t-1} - \phi_3 y_{t-1} + \phi_3 y_{t-2} - \phi_3 y_{t-2} + \phi_3 y_{t-3} + \varepsilon_t \\
&= (\phi_1 + \phi_2 + \phi_3 - 1)y_{t-1} - (\phi_2 + \phi_3)\Delta y_{t-1} - \phi_3 \Delta y_{t-2} + \varepsilon_t
\end{aligned}$$

This expression is the answer to a) i). The answer to a) ii) is simply to notice that $k = 2$. In general an $AR(p)$ can be written as an $ADF(p-1)$.

a)iii) The restriction is that under a unit root (the null hypothesis for a unit root test) $\phi_1 + \phi_2 + \phi_3 = 1$. Remember that $\phi_1 + \phi_2 + \phi_3 < 1$ is a necessary condition for stationarity of an AR(3). This is why when we test $H0)\ \lambda = 1$ against $H1)\ \lambda < 1$. we test unit the existence of a unit root against stationarity.

b) An ARMA(2,1) if it is invertible can be written as an infinite moving autoregressive. Therefore the ADF would need infinite lags.

3 i) For this exercise we use the sample 1871-1945. To check whether the "rdiv" series has a unit root. we execute the following Eviews commands: "Quick", "Series Statistcis", "Unit Root", and then type: rdiv. Then you have to choose the lags that you will include in the estimation. We are advisesd to chose the order of lags from "general to specific". Using this selection criteria we end with 0 lags. The results are presented below:

ADF Test Statistic	-3.119018	1% Critical Value*	-4.0853
		5% Critical Value	-3.4704
		10% Critical Value	-3.1620

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RDIV(-1)	-0.253034	0.081126	-3.119018	0.0026
C	0.001999	0.000689	2.900474	0.0050
@TREND(1871)	2.55E-05	1.37E-05	1.857603	0.0674

R-squared	0.123673	Mean dependent var	8.46E-05
Adjusted R-squared	0.098988	S.D. dependent var	0.001853
S.E. of regression	0.001759	Akaike info criterion	-9.808601
Sum squared resid	0.000220	Schwarz criterion	-9.715193
Log likelihood	365.9182	F-statistic	5.009985
Durbin-Watson stat	1.704169	Prob(F-statistic)	0.009218

With the critical values estimate for an ADF, we can not reject the null hipothesis of existence of unit root neither at a 5% nor at a 10%. Since the trend does not seem to be significant you are advised to delete the trend and perform the same type of test.

We carry out the same procedure for "rsp". We estimate select a DF model (no augmentation) and obtain the following results:

ADF Test Statistic	-2.912559	1% Critical Value*		-4.0853
		5% Critical Value		-3.4704
		10% Critical Value		-3.1620
Variable	Coefficient	Std. Error	t-Statistic	Prob.
RSP(-1)	-0.214348	0.073594	-2.912559	0.0048
C	0.036744	0.014853	2.473802	0.0158
@TREND(1871)	0.000350	0.000289	1.208597	0.2308
R-squared	0.108988	Mean dependent var		0.001997
Adjusted R-squared	0.083889	S.D. dependent var		0.046615
S.E. of regression	0.044617	Akaike info criterion		-3.341689
Sum squared resid	0.141341	Schwarz criterion		-3.248281
Log likelihood	126.6425	F-statistic		4.342329
Durbin-Watson stat	1.711044	Prob(F-statistic)		0.016629

We cannot reject the null hypothesis that rsp has a unit root.

Notice that given that the trend is not significant we should exclude it from the ADF regression. The results of the regression are:

ADF Test Statistic	-2.679183	1% Critical Value*	-3.5200
		5% Critical Value	-2.9006
		10% Critical Value	-2.5874

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RSP(-1)	-0.165935	0.061935	-2.679183	0.0091
C	0.039052	0.014777	2.642718	0.0101

R-squared	0.090657	Mean dependent var	0.001997
Adjusted R-squared	0.078027	S.D. dependent var	0.046615
S.E. of regression	0.044760	Akaike info criterion	-3.348352
Sum squared resid	0.144249	Schwarz criterion	-3.286080
Log likelihood	125.8890	F-statistic	7.178022
Durbin-Watson stat	1.756453	Prob(F-statistic)	0.009139

We also do not reject the null with this specification and colclude that "rsp" seems to be I(1).

3 ii) Since both series: "rdiv" and "rsp" are integrated of order 1, the residual of the estimation of the Real Stock Prices on Constant and Dividend could be either I(1) or I(0). The latter implies that both series are cointegrated so they follow a long run relationship.

Go to "quick", "Estimate equation", and type : RSP C RDIV. The results of this regression are:

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.012535	0.014506	-0.864143	0.3903
RDIV	20.80421	1.218307	17.07633	0.0000
R-squared	0.799781	Mean dependent var		0.223645
Adjusted R-squared	0.797038	S.D. dependent var		0.084062
S.E. of regression	0.037871	Akaike info criterion		-3.682968
Sum squared resid	0.104696	Schwarz criterion		-3.621169
Log likelihood	140.1113	F-statistic		291.6011
Durbin-Watson stat	0.727003	Prob(F-statistic)		0.000000

Then, we have to see if the residual of this regresion is I(1) or I(0). Therefore we should generate the residual, go to "Quick", "Generate Series", and type ECM=RESID.

Now, we perform the "unit root test" (using the general to specific methodology) to the variable ECM. . Go to "Quick", "Series Statistics" "URT" and specify 1 lag at first, and then we will check the residual. The results are:

$ADFTestStatistic$	-4.265652	$1\%CriticalValue*$		-4.0871
		$5\%CriticalValue$		-3.4713
		$10\%CriticalValue$		-3.1624
$Variable$	$Coefficient$	$Std.Error$	$t-Statistic$	$Prob.$
$ECM(-1)$	-0.442636	0.103767	-4.265652	0.0001
$D(ECM(-1))$	0.149666	0.118893	1.258826	0.2123
C	0.007474	0.007278	1.026985	0.3080
$@TREND(1871)$	-0.000187	0.000169	-1.108218	0.2716
$R-squared$	0.212572	$Meandependentvar$		0.000353
$AdjustedR-squared$	0.178336	$S.D.dependentvar$		0.032497
$S.E.ofregression$	0.029457	$Akaikeinfocriterion$		-4.158507
$Sumsquaredresid$	0.059874	$Schwarzcriterion$		-4.033002
$Loglikelihood$	155.7855	$F-statistic$		6.209028
$Durbin-Watsonstat$	1.935496	$Prob(F-statistic)$		0.000852

With this specification we can reject the null hipothesis, at a 5% or 1% and then we cen affirm that the residual is integrated of order cero, but the constant and the trend are not significant to explain the evolution of the residual, so we exclude them, and then we obtain:

$ADFTestStatistic$	-4.142031	$1\%CriticalValue*$		-2.5945
		$5\%CriticalValue$		-1.9448
		$10\%CriticalValue$		-1.6181
$Variable$	$Coefficient$	$Std.Error$	$t-Statistic$	$Prob.$
$ECM(-1)$	-0.414846	0.100155	-4.142031	0.0001
$D(ECM(-1))$	0.137926	0.117787	1.170979	0.2455
$R-squared$	0.198424	$Meandependentvar$		0.000353
$AdjustedR-squared$	0.187135	$S.D.dependentvar$		0.032497
$S.E.ofregression$	0.029299	$Akaikeinfocriterion$		-4.195493
$Sumsquaredresid$	0.060950	$Schwarzcriterion$		-4.132741
$Loglikelihood$	155.1355	$F-statistic$		17.57554
$Durbin-Watsonstat$	1.937775	$Prob(F-statistic)$		0.000078

We do reject the null Hypoteis that ECM has a unit root therefore Prices and Dividends seem to cointegrate. The long run relationship that is described by the coefficient of the regression of RSP on RDIV and the constant can be characterised using the following cointegrating vector:

$$\begin{pmatrix} 1 \\ +0.012 \\ -20.80 \end{pmatrix}$$