Exercise 2

1) Consider two AR(1) processes, say x_t and y_t.

i) Show that the sum of the processes is an ARMA(2,1) process.

2) Use the data in the file var.wf1 (three returns of individual stocks) to estimate a VAR.

i) Find the order of the VAR.

ii) Test for Granger Causality for each pair of series.

iii) Show the different impulse response functions

iv) Show the Variance Decomposition of the Forecasts.

3) Read the paper "Use of (time-domain) Vector Autoregressions to test uncovered interest parity".

Solution

1) Consider

$y_t = \phi_1 y_{t-1} + \varepsilon_{1t}$ and $x_t = \phi_2 x_{t-1} + \varepsilon_{2t}$

We need to show that $w_t = y_t + x_t$ is in general an $ARMA(2,1)$ process.

We will proceed to develop the proof in two stages. First we are going to develop the proof *assuming* that the sum of two moving averages (of orders q_1 and q_2)is also a moving average of order equal to maximum(q_1, q_2). Then we will prove that this statement is actually true.

Proof:

(a) Write the AR(1) processes with using lags operators.

$$(1 - \phi_1 L)y_t = \varepsilon_{1t}$$

$$(1 - \phi_2 L)x_t = \varepsilon_{2t}$$

Multiply the first equation by $(1 - \phi_2 L)$ and the second equation by $(1 - \phi_1 L)$, that is

$$(1 - \phi_2 L)(1 - \phi_1 L)y_t = (1 - \phi_2 L)\varepsilon_{1t}$$

$$(1 - \phi_2 L)(1 - \phi_1 L)x_t = (1 - \phi_1 L)\varepsilon_{2t}$$

Adding we get $(1 - \phi_2 L)(1 - \phi_1 L)(x_t + y_t) = (1 - \phi_2 L)\varepsilon_{1t} + (1 - \phi_1 L)\varepsilon_{2t}$ which is an ARMA(2,1) *if* the sum of moving averages is a moving average of order equal to the Max of each MA.

Notice that if $\phi = \phi_1 = \phi_2$, then $(x_t + y_t)$ is an AR(1).

(b) $MA(q_1) + MA(q_2) = MA(\max(q_1, q_2))$.

Consider

$f_t = (1 + \theta_1 L + ... + \theta_{q1} L^{q1})\varepsilon_{1t}$ and $g_t = (1 + \theta_1 L + ... + \theta_{q2} L^{q2})\varepsilon_{2t}$ with $\text{cor}(\varepsilon_{1t}, \varepsilon_{2z}) = 0$ for all z.

Then the autocovariance function for f_t is

$$\begin{aligned} \gamma^f(k) &= a \text{ scalar } \neq 0 \text{ for the first } q_1 \text{ values of k.} \\ &= 0 \text{ otherwise.} \end{aligned}$$

and the autocovariance function for g_t is

$$\begin{aligned} \gamma^g(k) &= a \text{ scalar } \neq 0 \text{ for the first } q_2 \text{ values of k.} \\ &= 0 \text{ otherwise.} \end{aligned}$$

Then defining $u_t = f_t + g_t$ it can be easily seen that

$$\gamma^u(k) = E(u_t u_{t-k}) = E(f_t + g_t)(f_{t-k} + g_{t-k}) = \gamma^f(k) + \gamma^g(k)$$

since the shocks were assumed to be orthogonal.

Then this implies that

$$\begin{aligned} \gamma^u(k) &= a \text{ scalar } \neq 0 \text{ for the first } \max(q_1, q_2) \text{ values of k.} \\ &= 0 \text{ otherwise.} \end{aligned}$$

2 i) To find the order of the VAR we begin guessing a VAR of order one.

To estimate this VAR, go to "Quick", "Estimate VAR", then type 1 in the "lags intervals" and then specify the "endogenous" series which in this case are: ALLD BCI ASDA.

You get the following results:

Vector Autoregression Estimates
Sample(adjusted): 2 1500
Included observations: 1499 after adjusting endpoints
Standard errors in ( ) & t-statistics in [ ]

	$ALLD$	$ASDA$	BCI
$ALLD(-1)$	0.082680	0.060850	0.005032
	(0.02904)	(0.03356)	(0.02279)
	[2.84689]	[1.81338]	[0.22079]
$ASDA(-1)$	−0.028847	0.051185	0.012260
	(0.02479)	(0.02864)	(0.01945)
	[−1.16376]	[1.78715]	[0.63021]
$BCI(-1)$	−0.023206	−0.026752	0.179633
	(0.03561)	(0.04114)	(0.02795)
	[−0.65171]	[−0.65022]	[6.42789]
C	0.081916	0.076758	0.047208
	(0.04095)	(0.04731)	(0.03214)
	[2.00042]	[1.62232]	[1.46895]

Determinant Residual Covariance	9.011146
LogLikelihood(d.f.adjusted)	−8028.714
Akaike Information Criteria	10.72810
Schwarz Criteria	10.77063

VAR Lag Order Selection Criteria

The order of the VAR has to be obtained using the selection criteria presented in the theory. In Eviews 4. you should type (from the output menu), *"view";"lag structure"; lag length criteria".*

Endogenous variables: ALLD ASDA BCI
Exogenous variables: C
Sample: 1 1500
Included observations: 1492

Lag	*LogL*	*LR*	*FPE*	*AIC*	*SC*	*HQ*
0	−8021.395	*NA*	9.420893	10.75656	10.76723	10.76054
1	−7980.815	80.94094*	9.030417*	10.71423*	10.75692*	10.73014*
2	−7973.653	14.25735	9.052691	10.71669	10.79140	10.74453

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

All the criteria show that the preferred model is a VAR(1).

2 ii) The Granger Causality test inquires whether a variable improves the forecast over a given information set. The null hypothesis takes the form of :

H_0 = y does not cause x , which implies the null of some coefficient of the VAR being equal to zero.

In order to do that, go from the output menu to "lag structure" and "Pairwise Granger Causality Test" (specified for the VAR(1)).

VAR Pairwise Granger Causality/Block Exogeneity Wald Tests

Date: 01/14/02 Time: 15:52

Sample: 1 1500

Included observations: 1499.

Dependent variable: ALLD

Exclude	**Chi-sq**	**df**	**Prob.**
ASDA	1.354326	1	0.2445
BCI	0.424728	1	0.5146
All	2.227880	2	0.3283

Dependent variable: ASDA

Exclude	**Chi-sq**	**df**	**Prob.**
ALLD	3.288359	1	0.0698
BCI	0.422780	1	0.5156
All	3.316090	2	0.1905

Dependent variable: BCI

Exclude	**Chi-sq**	**df**	**Prob.**
ALLD	0.048750	1	0.8253

ASDA	0.397168	1	0.5286
All	0.582536	2	0.7473

We do not reject the null hypothesis of Granger Non Causality for any of the variables under scrutiny.

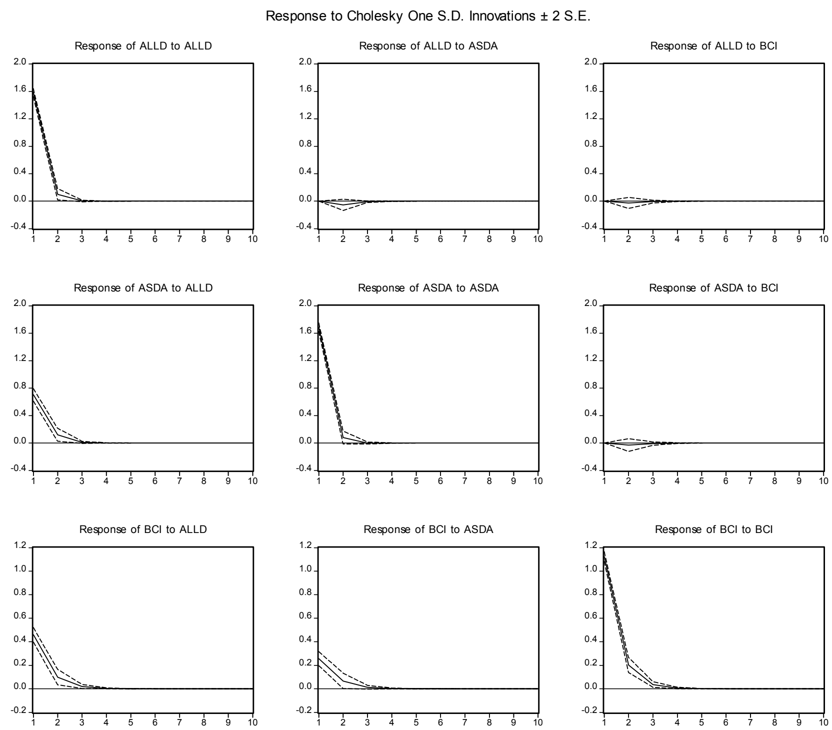

Figure 1:

2 iii) To estimate the impulse-response functions we go to "View" "Impulse-Response", choose impulse-response and then specify the innovations to the variable and the ordering of the VAR. We will type ALLD ASDA BCI (the default specification)

The impulse-response function shows the effect of a one unit increase in the j th variable´s innovation (where the legend shows the shocks to the different variables) at time t, for the value of the i th variable at time t+s (in the graphic you can see the response of the variable p.e ALLD for t+1, t+2....t+s, etc.).

The I-R functions are orthogonalized because $E(\varepsilon_t \varepsilon_t^{`})$ is not diagonal and as a result a shock in one variable innovation's has to be accompanied by a shock in other variables at the same time. To avoid this we obtain the orthogonalized impulse-response functions of the form:

$\psi^* = \psi A$

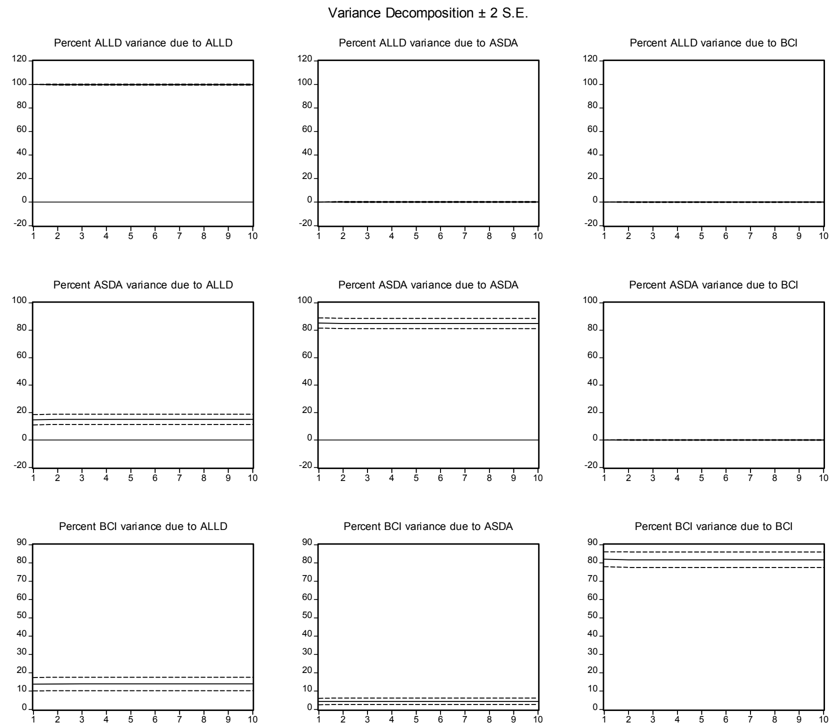

Figure 2:

Where A is a lower triangular matrix, therefore W_{1t} is the only one with potential immediate impact on all others variable, that is the case of ALLD.

Because of our specification of the VAR (ALLD ASDA BCI), an innovation to ALLD has a potential INSTANTANEOUS impact in ALLD, ASDA and BCI, and innovation in ASDA has a potential INSTANTANEOUS impact in ASDA and BCI and an innovation in BCI has a potential INSTANTANEOUS impact in BCI only.

You should check whether changing the order of the VAR qualitatively change the plots

2 iv The Variance decomposition explains how useful is each variable in the forecast of the dependent variable. The highest is the percentage, the more useful is that variable to forecast. This measure also depends on the orthogonalization. For the series considered above we get: