Lecture 1: Public Goods

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Public Goods: Some Characterizations

Institutional Characterizations

- Marketable good: a good whose production and allocation is freely determined in a market (via the functioning of the price system), subject to neither public intervention nor discriminatory taxation.

- Non-marketable good: a good whose production and allocation is publicly financed and decided

- Impure marketable good: a good whose production and allocation is determined in a market (via the functioning of the price system) but subject to public intervention or discriminatory taxation.

  Sometimes called "Publicly provided private goods"

  Why an institutional characterization? Because a simple political decision can make a given good to change from one to another category.

Economic Characterizations

- Non-rivalry: if the consumption by one person does not reduce the quantity available for consumption by any other.

  This is related to non-divisibilities.

- Non-excludability: if no one can be excluded from consumption

- Obligation of use vs. disposal

- A *pure public good* has these properties

  Each person consumes, not a part, but the quantity of the pure public good.

  If z_i is the consumption of the public good by individual i, then

$$z_1 = ... = z_i = ...z_I = g$$

- By contrast, private goods are rivals. Hence, if x_i^l is the consumption of the private good l by individual i, then

$$\sum_{i=1}^{I} x_i = y^l$$

- An *impure public good* is a public good that has the following characteristics
  - exclusion is possible
  - suffers from some congestion: reduction in the return the public good gives to each user as the use of a given supply increases.

Optimal Provision of Pure Public Goods

- An economy with two goods

  private good: x

  pure public good: g.

- $X =$ initial resources of the private good

- Each individual i has utility $U_i(x_i, g)$

  x_i : consumption of the private good by individual i.

- U_i is increasing in both arguments

- The public good is produced from the private good according to a technology given by

$$g = f(x)$$

where $f(\ )$ is increasing and concave.

- Denote $x = h(g)$ the cost, measured in the private good, of providing the quantity g of the public good.

  $h(\ )$ is increasing and convex.

- Formal derivation of the Pareto optimum

$$\begin{cases} \underset{x_1,g}{Max} & U_1(x_1,g) \\ s.t. & \forall j = 2, ..., I \quad U_j(x_j,g) \geq k \quad \lambda_j \\ & \sum_{i=1}^{I} x_i \leq X - h(g) \qquad \mu \end{cases}$$

$$\Rightarrow \sum_{i=1}^{I} \frac{\partial U_i/\partial g}{\partial U_i/\partial x_i} = h'(g) = \frac{1}{f'(x)} \qquad \text{Bowen-Lindhal-Samuelson Condition}$$

where x is the amount of private good needed to provide the public good

- Recall that $\frac{\partial U_i/\partial g}{\partial U_i/\partial x_i}$ is the individual i's willingness to sacrifice his consumption of private good in order to increase "his consumption" of public good. It's a measure of it's marginal benefit of another unit of the public good. But as increase in the provision of the public good benefits all individuals, the social benefit of this extra unit is found by summing the marginal benefits. Therefore optimality requires that the public good's marginal cost must be equal to the sum of all marginal rates of substitution.

- In contrast, as an extra unit of private good can only be given to one consumer or another, optimality requires that the marginal benefits of all individuals are equalized to the marginal cost of producing this extra unit of the private good.

How to Implement this Optimal Level of Public Good?

Which are the different modes of economic organization that may a priori lead to a Pareto-optimal allocation?

Assumption to what follows: the private good is the $numéraire$ so its price is normalized to 1.

The Subscription Equilibrium

- Each individual i is endowed with ω_i units of the *numéraire*

- Each individual is asked to subscribe an amount s_i, part of their wealth ω_i, to contribute to the production of the public good.

  "Private provision of the public good"

- The total quantity of public good produced is

$$g = f(\sum_{i=1}^{I} s_i)$$

- The choice of s_i is made according to the principles of the Nash equilibrium: individual i take the quantity subscribed by the others (denoted by s_{-i}) as given and solves

$$\begin{cases} \underset{x_i, s_i, g}{Max} & U_i(x_i, g) \\ s.t. & x_i + s_i = \omega_i \\ & g = f(\sum_{j=1}^{I} s_j) \end{cases}$$

$$\Rightarrow \frac{\partial U_i/\partial g}{\partial U_i/\partial x_i} = \frac{1}{f'(\sum_{j=1}^{I} s_j)}$$

- Summing up

$$\sum_{i=1}^{I} \frac{\partial U_i/\partial g}{\partial U_i/\partial x_i} = \frac{I}{f'(\sum_{j=1}^{I} s_j)} \qquad \neq \qquad \text{BLS}$$

- Result: non-optimal provision of the public good

Quantities under private provision

- Comparisons between the equilibrium quantities and the optimal quantities of the public good.

- Contrary to the common wisdom, there is no general result: both under-provision and over-provision can emerge

Case 1 Under-provision of the public good

- Under very reasonable conditions on cross second derivatives of utility functions U_i, this is the “normal” case
- Graphical representation.

- In this case, individuals do not take into account that their subscription increases the others utility

- Private provision creates a situation in which externalities are present

- So each individual strategically under-subscribes.

- The $free - rider$ phenomenon: each individual relies on others to provide the public good and thus avoids the need to subscribe himself.

“If the individual is to spend his money for private and public uses so that his satisfaction is maximized he will obviously pay nothing whatsoever for public purposes. ... Whether he pays much or little will affect the scope of public service so slightly, that for all practical purposes, he himself will not notice it at all. Of course, if everyone were to do the same, the State will soon cease to function”

Wicksell (1896)

- How under-provision of the public good is related with the number of individuals?

  Analytical example

  Let $U_i(x_i, g) = \gamma \ln g + \ln x_i$, with $\omega_i = \frac{X}{I}$ and $f(g) = g$.

  Egalitarian Optimum: $g^* = \frac{\gamma X}{1+\gamma}$

  Subscription equilibrium: $\widehat{g} = \frac{\gamma X}{I+\gamma}$

  Explanation: when the number of individuals I increases, the gap between the private MRS and the sum of MRS's increases.

Case 2 Over-provision of the public good

- The result described above, namely that the subscription equilibrium is Pareto dominated by allocations with a higher level of public good, has often been interpreted as demostrating that private provision leads to an undersupply relative to the socially optimal level.

- However, a global optimum of a Paretian Social Welfare Function may lie anywhere on the Utility Possibility Frontier (UPF), not necessarily on the part of the UPF that Pareto dominates $(\widehat{s_1}, \widehat{s_2})$

- Graphical representation

- The failure of the subscription mechanism to provide public goods efficiently suggests that alternative allocation mechanisms should be considered.

- In the next two sections, we will see two mechanisms that correct some of the drawbacks of the subscription mechanism, but that also have issues to worry about.

The Voting Equilibrium

- In practice, the level for public goods is frequently determined by the political process with competing parties differing in the level of public good provision they promise.

- $I > 2$ and I is odd

- Each individual has a different wealth ω_i

- Assume $\omega_1 < \omega_2 < ... < \underbrace{\omega_m}_{median\ wealth} < ... < \omega_{I-1} < \omega_I$

- Each individual is taxed with the same tax rate τ on his wealth ω_i

- Public budget constraint: $\sum_{i=1}^{I} \tau.\omega_i = h(g)$

- Utility $U_i(x_i, g) = x_i + v(g)$, where $v(\ )$ is strictly increasing and concave

- Utilities are concave in the level of public good g

  Proof: Using the public budget constraint,

$$\begin{aligned} U_i(x_i, g) &= \omega_i(1 - \tau) + v(g) \\ &= \omega_i\left(1 - \frac{h(g)}{\sum_{j=1}^{I} \omega_j}\right) + v(g) \end{aligned}$$

  one can compute

$$\frac{\partial^2 U_i}{\partial g^2} = v''(g) - \frac{\omega_i}{\sum_{j=1}^{I} \omega_j} h''(g) < 0$$

- $\widetilde{g}_i$: preferred level of public good of individual with wealth ω_i, verifies the following FOC

$$v'(\widetilde{g}_i) - \frac{\omega_i}{\sum_{j=1}^{I} \omega_j} h'(\widetilde{g}_i) = 0 \qquad (1)$$

- Looking at (1), $\widetilde{g}_i$ depends upon ω_i

- Moreover, $\widetilde{g}_i$ decreases with ω_i : richer individuals want a lower level of public good because, with proportional taxation, they pay a large share of the tax burden.

- Order the different $\widetilde{g}_i$, from the richest to the poorest, and plot the utilities.

- Graphical representation

- The State organizes a referendum on pairs of different levels of the public good.

- The level of public good $\widehat{g}$ is a Condorcet winner if it beats any other level of public good g' in a pairwise vote.

- The Median Voter Theorem ensures that the individual with the median wealth ω_m is decisive and his preferred level of public good $\widetilde{g}_m$ is the Condorcet winner.

- Property of the voting equilibrium: non-manipulability

  Proof: Consider an individual k such that $\widetilde{g}_k < \widetilde{g}_m$. The only way that such individual can manipulate a referendum is to vote against $\widetilde{g}_m$.

  If the other proposed level of public good is at the left of $\widetilde{g}_m$, nothing changes

  If the other pair is to the right of $\widetilde{g}_m$, opposing $\widetilde{g}_m$ will yield an outcome that is worse for individual k.

- Is the voting equilibrium efficient? Generally not.

- Necessary condition for the voting equilibrium to yield the efficient solution

$$\omega_m = \frac{\sum_{i=1}^{I} \omega_i}{I}$$

- If, as it is empirically observed, $\omega_m < \frac{\sum_{i=1}^{I} \omega_i}{I}$, then voting yields to over-provision of the public good.

The Lindahl Equilibrium

- The previous two allocation mechanisms lead to inefficient outcomes

- Why? Because individuals face incorrect incentives and so private and social benefits do not coincide

- The mechanism that we will study now uses an extended price system for the public good

  different "personalized" prices: so that each individual pays its valuation for the public good

- Why different prices?

- Equilibrium with private goods: same price paid by everyone, different quantities bought by everyone

- “Equilibrium” with public goods: same quantity for everyone

  How to implement this efficiently? If consumers wish to buy the same quantity

  Different consumers can be induced to do this if they face the correct price

- Two individuals: 1,2

- Mechanism

  1. The State announces the share of the cost of the public good that each individual will bear: $\{\tau_1, \tau_2\}$, with $\tau_1 + \tau_2 = 1$. These shares are the "personalized" prices.

  2. Each individual announces $\widetilde{g}_i$, the level of public good he wishes to have provided under that cost sharing regime

3. If both individuals announces the same level of public good, then that level is provided.

4. If their announcements differ, the shares are adjusted and the process is repeated until shares are reached at which both individuals announce the same level of public good. This level of public good is called the Lindahl Equilibrium.

- Formal derivation of the Lindahl Equilibrium

- Graphical representation

Main property of the Lindahl mechanism

- It yields the efficient allocation, for a given distribution of wealth

- Explanation: the “personalized” prices equate the individual valuation of the provision of public good with the cost of production.

Main drawback of the Lindahl mechanism

- The analysis implicitly assumed that individuals were honest in revealing their Lindahl reaction functions to the different announced cost sharing regimes

- However, there will in fact be a gain to any individual who attempts to manipulate the allocation mechanism

- How? By announcing a different Lindahl reaction function

- Graphical representation of the gain from misrepresentation

Preference Revelation Mechanisms

General setting

- Public good of fixed size (or project) $\Rightarrow g \in \{0, 1\}$

  Example: decision to build a bridge

- Simplification: the project is costless

- Quasi-linear utilities: $U_i(x_i, g) = x_i + u_i g \Rightarrow$ no income effects

  where u_i : willingness to pay for the public good

- u_i is only known by individual i (private information)

- Denote by $\mathbf{u} = (u_1, ..., u_i, ..., u_I)$

- Formalization of the decision to undertake the project: $d(\mathbf{u}) = 1$

- Efficient solution: $d(\mathbf{u}) = 1$ if and only if $\sum_{i=1}^{I} u_i \geq 0$

- The State needs to know the vector $\mathbf{u}$

Examples of misrepresentation

Example 1 False understatement

- $i = 1, 2$
- Cost $c = 1$
- $u_i \in \{0, 1\} \Rightarrow$ 4 states of Nature: $(0,0),(0,1),(1,0),(1,1)$
- Consider only the state $(1,1)$ and the fact that u_i are private information

- Efficiency: the project should be undertaken

- Mechanism

  1. Each individual reports $v_i \in \{0, 1\}$

  2. If $\sum_{i=1}^{I} v_i \geq 1$, the project is undertaken; not otherwise

  3. The cost of the project is shared proportionally to the announcements

$$
\begin{array}{ll}
c_i = 1 & \text{if } v_i = 1 \text{ and } v_j = 0 \\
c_i = \frac{1}{2} & \text{if } v_i = v_j = 1 \\
c_i = 0 & \text{if } v_i = 0 \text{ and } v_j \in \{0, 1\}
\end{array}
$$

- $v_1 = v_2 = 0$ are (weakly) dominant strategies $\Rightarrow$ inefficient under-provision

- Why? The proportional cost-sharing rule gives an incentive to under-report

Example 2 False overstatement

- $i = 1, 2$
- Cost $c = 1$
- $u_i \in \{0, 1\}$ and $u_2 \in \left\{\frac{3}{4}, 1\right\}$
- Consider the state $(0, \frac{3}{4})$
- Efficiency: the project should not be undertaken

- Mechanism

1. Each individual reports $v_1 \in \{0, 1\}$ and $v_2 \in \left\{\frac{3}{4}, 1\right\}$

2. If $\sum_{i=1}^{I} v_i \geq 1$, the project is undertaken; not otherwise

3. The cost of the project is uniformly charged if it is undertaken.

- $v_1 = 0$ and $v_2 = 1$ are (weakly) dominant strategies $\Rightarrow$ inefficient over-provision

- Why? Individual 2 can guarantee himself that the project is undertaken by misreporting, obtaining a gain of $\frac{1}{4}$, without taking into account the loss imposed on individual 1

Comment on these examples

The drawbacks of these two examples seem to come from

1. transfers directed linked to the announcements

2. individual welfare not linked to the welfare of others

Now we will analyze a class of mechanisms that correct these drawbacks.

Vickrey-Clarke-Groves mechanisms (VCG)

- The State demands the individuals to announce their u_i's
- Let $\mathbf{v} = (v_1, ..., v_I)$ be the vector of announcements

- The State commits to implement the following:

  1. $d(\mathbf{v}) = 1$ if and only if $\sum_{i=1}^{I} v_i \geq 0$

  2. impose to the individual i the transfer

  $$t_i(v_i) = -\sum_{j \neq i} v_j.d(\mathbf{v}) + h_i(\mathbf{v}_{-i})$$

  where $\mathbf{v}_{-i} = (v_1, ..., v_{i-1,} v_{i+1,} ..., v_I)$ and $h_i(\mathbf{v}_{-i})$ is any sum that depends only on statements of other individuals.

Fundamental properties of VCG mechanisms

- Groves and Loeb (1975): A VCG mechanism is strongly incentive compatible, i.e. individual truthtelling is a dominant strategy.

- A VCG mechanism implements the correct project decision

- Proof in a special case: the Clarke mechanism, where

$$h_i(v_{-i}) = \max\left\{\sum_{j\neq i} v_j, 0\right\}$$

- Another expression

$$t_i(v_i) = -\left|\sum_{j\neq i} v_j\right| \quad \text{if} \quad \left(\sum_{j\neq i} v_j\right) \cdot \left(\sum_{i=1}^{I} v_i\right) < 0$$

$$t_i(v_i) = 0 \qquad\qquad \text{otherwise}$$

1. $\sum_{j \neq i} v_j > 0$ and $\sum_{j \neq i} v_j + u_i > 0$

- $U_i(u_i, v_{-i}) = u_i$ no transfer and public project undertaken

- $U_i(v_i, v_{-i}) = u_i$ if $\sum_{i=1}^{I} v_i > 0$ no transfer and public project undertaken

- $U_i(v_i, v_{-i}) = -\left|\sum_{j \neq i} v_j\right| < 0 < u_i$ if $\sum_{i=1}^{I} v_i < 0$ negative transfer and public project not undertaken

2. $\sum_{j \neq i} v_j > 0$ and $\sum_{j \neq i} v_j + u_i < 0$

- $U_i(u_i, v_{-i}) = -\left|\sum_{j \neq i} v_j\right| < 0$ negative transfer and public project not undertaken

- $U_i(v_i, v_{-i}) = -\left|\sum_{j \neq i} v_j\right| < 0$ if $\sum_{i=1}^{I} v_i < 0$ negative transfer and public project not undertaken

- $U_i(v_i, v_{-i}) = u_i < -\sum_{j \neq i} v_j$ if $\sum_{i=1}^{I} v_i > 0$ no transfer and public project undertaken

3. $\sum_{j \neq i} v_j < 0$ and $\sum_{j \neq i} v_j + u_i < 0$

- $U_i(u_i, v_{-i}) = 0$ no transfer and public project not undertaken

- $U_i(v_i, v_{-i}) = 0$ if $\sum_{i=1}^{I} v_i < 0$ no transfer and public project not undertaken

- $U_i(v_i, v_{-i}) = u_i + \sum_{j \neq i} v_j < 0$ if $\sum_{i=1}^{I} v_i < 0$ negative transfer and public project undertaken

4. $\sum_{j \neq i} v_j < 0$ and $\sum_{j \neq i} v_j + u_i > 0$

- $U_i(u_i, v_{-i}) = u_i + \sum_{j \neq i} v_j > 0$ negative transfer and public project undertaken

- $U_i(v_i, v_{-i}) = u_i + \sum_{j \neq i} v_j > 0$ if $\sum_{i=1}^{I} v_i > 0$ negative transfer and public project undertaken

- $U_i(v_i, v_{-i}) = 0$ if $\sum_{i=1}^{I} v_i < 0$ no transfer and public project not undertaken

Comments

- In cases 1 and 4, the announcement of individual i does not change the decision to undertake the project. Therefore, he pays no transfer.

- But in cases 2 and 3, individual i modifies the decision with his announcement and so he pays a transfer. He is called a "pivot"

- There is no direct effect of individual i's strategy on transfers. Transfers are designed so that the only effect the strategy of an individual can have upon the size of the transfer is via the effect that the decision on the public project, based on that strategy, has upon other individuals welfare's.

- The mechanism internalizes the external consequences of the strategy choice of each individual, since the external consequence are given by the welfare effects on other individuals of the public decision.

Generality of VCG mechanisms

- Green and Laffont (1977): The VCG are the only strongly incentive compatible mechanisms that implement the correct project's decision

Drawback of VCG mechanisms

- Do the analyzed VCG mechanism yields the Pareto optimum?

  NO

- Why? Because the budget $\sum_{i=1}^{I} t_i$ is seldom balanced

  Individuals never receive money (i.e. no transfer is positive)

  The sum of transfers is strictly negative if there is at least one "pivot"

- What happens if the sum of transfers are redistributed, in a particular way, to individuals?

  The incentive properties of the mechanism would be modified.

- Money is taken out of the economy. It is the price to pay for the revelation of preferences.

- Can we find a VCG such that the budget is balanced for every vector of announcements $\mathbf{v} = (v_1, ..., v_I)$?

- Green and Laffont (1979): There exists no VCG mechanism whose budget is balanced for all possible vector of valuations $\mathbf{u} = (u_1, ..., u_I)$

- Proof with an example

Extensions

1. $I \to \infty$

   The Clarke mechanism is asymptotically balanced: the number of pivots becomes quite small, since it is improbable that an individual can change the project's decision on his own.

   If the mechanism is nevertheless not balanced and the surplus is redistributed, truthtelling is almost optimal for individuals.

2. Relax the concept of equilibrium

   d'Asprémont and Gérard-Varet (1979): if we only ask for Bayesian implementation, a fully Pareto optimal mechanism exists...

   but that violates *interim* individual rationality constraints.

3. Relax the search for fully efficiency: constrained/second-best Pareto optimality

Importance of free-riding: experimental evidence

- Bohm (1972): agents seem to announce their true preferences

- Marwell and Ames (1981), Isaac, Walker and Thomas (1984), Isaac, Mc Cue and Plott (1985): experiments about private provision of a public good

  - There is little evidence of free-riding in one-shot experiments.

  - But in repeated experiments, the contributions fall

  - Allowing communications before the subscription occurs increase (although not noticeable) contributions

- Fehr and Gächter (2000, 2002): same experiments, with subsequent costly punishments to free-riders
  - Contributions increase with punishments