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Efficient Environmental Policy with Imperfect Compliance

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Abstract. Discussions of efficient environmental policy tend to recommend taxes rather than quotas on grounds of efficiency; a uniform tax will equalize marginal abatement cost between polluters. When polluters' actions are imperfectly observable, the distinction between taxes and quotas becomes less clear. Taxes may be evaded by underreporting of emissions, while quota violations will not always be discovered. This paper explores the conditions under which the efficiency properties of taxes continue to hold even when evasion is possible, and the extent to which the fine for quota violations plays the same role as a tax on emissions with similar efficiency properties.

Key words: compliance, efficiency, pollution, quotas, taxes

JEL classification: D62, H23, H26

1. Introduction

A classic issue in the design of policies for environmental protection is whether taxes or quotas should be the preferred tools of such policies. While the use of quotas or regulations may be the more important policy in practice, economists have as a rule favoured the use of taxes. The basic argument in favour of taxes is that they lead to socially efficient outcomes. There are two aspects of efficiency that are important in this connection. One is the question of the optimal amount of overall pollution, where the main issue is the assessment of consumers' marginal willingness to pay for an improved environment, to be set against the marginal cost of pollution abatement; this is the issue of *consumption efficiency.* The other is the efficient allocation of emissions between polluters, or equivalently, the efficient allocation of emission reductions; this issue is one of *production efficiency*, ensuring that the socially relevant abatement cost is as low as possible. Both aspects must be taken into account in the choice between alternative policy instruments, but the latter is arguably the more important. The choice of how high the Pigouvian tax rate should be raises essentially the same informational problem as the choice of the overall size of the quotas allocated to the polluters; the requirement of consumption efficiency alone does not, therefore, lead to a clear preference for either taxes or quotas.[1] As regards production efficiency, there is, however, a strong economic argument in favour of taxes as the best instruments. The reason is that a tax which is levied at a uniform rate on all polluters leads to equality of the marginal cost of emission across polluters, and this is exactly the condition for production efficiency (for the case of convex costs). Quotas, on the other hand, have to be set for each individual polluter, and the efficient allocation of quotas among them requires an unrealistic degree of detailed knowledge of individual cost conditions on the part of the environmental authority.

The conventional argument is usually set up in such a way that it neglects the issue of polluters' *compliance* with the policies; rather, the assumption is that whatever policy is implemented the polluters meet their legal obligations to pay taxes on emissions or stay within the limit on emissions fixed by the quota. But it is by no means clear that this is a realistic assumption. Taxes can be evaded by underreporting emissions, and quota violations may not be reported to the government. This becomes possible when government monitoring is imperfect and costly, so that inspection can only be made randomly. In this case, the polluter who decides to underreport his emissions or to violate his quota limitation faces a decision under uncertainty, where the outcome of his decision depends on whether he is inspected or not.

This extension of the standard analysis towards greater realism has potentially interesting consequences for one's view of taxes and quotas in terms of production efficiency. In the tax case, if the polluters are firms which maximise expected profits, what they save by cutting down on emissions may not be the Pigouvian tax, but some more complex parameter involving penalties and probabilities of detection. The argument that efficiency results because of the uniformity of taxes across polluters then becomes more doubtful. In the case of quotas, if violations are discovered, there will be a fine to pay, and this fine has some of the characteristics of a (non-linear) tax, although one that is imposed only with a certain probability. These considerations indicate that the distinction between the two sets of policy instruments may not in fact be as sharp as implied by standard arguments. However, in order to see more precisely the consequences that follow from tax evasion and quota violations, it is necessary to formulate the problem in more rigorous analytical terms, and to do so is the objective of the present paper.

This is not the first effort to study this problem. The first to set it up in a formal model and to derive some important results was Harford (1978), whose focus, however, was on the comparative statics of firm behaviour and less on the efficiency properties of alternative policy instruments. Later contributions include Harford (1987), which extends his previous model to take account of self-reporting by firms.[2] Two excellent surveys of the issues that arise in the choice of policy instruments are Bohm and Russell (1985) and Cropper and Oates (1992), while Hoel (1998) examines the choice between taxes and quotas in a broader framework which includes not only the standard issues related to efficiency but also effects on employment.

The approach taken in the present paper takes its point of departure from the theory of the economics of crime as developed by Becker (1968), and more specifically from the theory of tax evasion and compliance as originally formulated by Allingham and Sandmo (1972) and Srinivasan (1973) for the case of income taxation and later by Marrelli (1984) for indirect taxes. While the analysis is formally concerned with output, pollution and compliance decisions of a single firm, its aim is to study the social efficiency of alternative policy regimes. Its main contribution is to identify the efficiency properties of taxes and quotas within a unified framework which allows one to see more clearly than in previous studies both the similarities and differences between the two policy regimes.

The exposition is organised around the presentation of two different but related sets of models. The first is concerned with tax evasion, while the second set of models analyses quota violations. Within each set of models I distinguish between the cases where the probability of detection is exogeneous and where it is influenced by the pollution and compliance decisions of the firm. The role of risk aversion is studied separately for each set of models. For the case of quotas separate attention is also given to the implications of self-reporting, which has interesting consequences for the efficiency of the quota regime.

2. The Standard Argument

For simplicity the analysis will be built around the case of a single competitive firm which produces a single output. These assumptions are inessential to the arguments presented here and are made solely in order not to introduce unnecessary analytical complications. It should be stressed that the focus of the paper is on the ability of different policy regimes to achieve an efficient allocation of emissions among several polluting firms; however, this focus can be achieved without formally introducing the more complex notation required for the case of many firms.

We assume that the firm maximises profit, which is given as

$$\pi = px - c(x, e) - te \tag{1}$$

Here p and x are price and output, and e is the level of emissions, which is taxed at the rate t. We assume that the cost function is increasing in output and decreasing in emissions, and that it is strictly convex. It may seem somewhat artificial to have emissions in the cost function, but to have costs decreasing with emissions is of course the same as having the cost function increasing in emission *reduction*, which is a very natural assumption.

The first order conditions for profit maximisation are

$$c_x = p, \tag{2}$$

$$-c_e = t. \tag{3}$$

Here subscripts denote partial derivatives. Condition (3) implies that the firm will carry the cost of emission reduction – the abatement cost – to the point where the marginal cost is equal to the tax saved. When the tax rate on emission is the same for all firms which emit this particular kind of pollutant, the allocation of emissions between them will satisfy the condition for production efficiency, viz. that the marginal cost of emission reduction be the same for all firms[3]. The firm's optimum will be written as (x^*, e^*).

Alternatively, let us assume that a quota on emissions is imposed on the firm in the amount q. The problem of the firm is now to maximise profit with respect to the single choice variable x. Since, with perfect compliance, $e = q$, profit is now

$$\pi = px - c(x, q), \tag{4}$$

and the first order condition is

$$c_x(x^0, q) = p, \tag{5}$$

where x^0 denotes the solution value. Comparing the solutions to the two problems, it is now straightforward to establish the equivalence of taxes and quotas: If $q = e^*$, then $x^0 = x^*$. When the quota is fixed as the amount of emission chosen by the firm under the tax regime, the two regimes will yield exactly the same output-emission vector for the firm. It is then always possible to find a vector of quotas for all firms that will replicate the efficient allocation of emissions which emerges from the tax regime.

However, there is fairly general agreement that this equivalence result is of little practical importance. In order to achieve efficiency with quotas, each firm must be allocated its own individual quota, and this requires information about each individual polluter which the government is unlikely to possess. The tax solution is appealing because every firm will in its own interest equate the marginal cost of emission reduction to the tax rate. Hence there is no need for the government to collect information about individual cost functions. All it has to do is to set the tax rate at a level which induces the right reduction in *aggregate* pollution; the allocation of reductions between polluters will then be determined as a result of the profit-maximising decisions of the firms themselves.

3. Complying with Environmental Taxes: The Case of Risk-neutral Firms

A NON-LINEAR PENALTY SCHEME

We now assume that the government carries out its environmental policy by means of taxes, but in contrast to the standard treatment we allow for tax evasion on the part of firms. The assumption is that the government cannot monitor emissions perfectly and without cost. Instead it has to rely on a system of random inspections. It will be taken for granted that once an inspection is carried out, the government will discover the true amount of emissions. This will be denoted e as before, while the reported level of emissions is z. In the case of discovery a penalty will be imposed according to the function $\theta = \theta(e - z)$, which should have the properties that $\theta(0) = 0$ and $\theta'(e - z) > 0$ for $e \geq z$.

The firm perceives the probability that an inspection will be carried out to be equal to α. Its objective is to maximise expected profits, which can be written as

$$E[\pi] = \alpha\pi^0 + (1 - \alpha)\pi^1, \tag{6}$$

where

$$\pi^0 = px - c(x, e) - te - \theta(e - z),$$

which is profit in the case of discovery, and

$$\pi^1 = px - c(x, e) - tz,$$

which is profit in the case of non-discovery, i.e. when the government accepts the firm's own statement about emissions.

If the probability of discovery is seen by the firm as completely exogenous and the marginal penalty rate θ' is constant, the decision on whether or not to report will be of the all-or-nothing type. If $t < \alpha\theta'$ the firm will report all of its emissions, while if the opposite inequality holds it will report nothing. Only if the expected penalty rate $\alpha\theta'$ is equal to the regular tax rate t will the firm be indifferent between reporting and not reporting, and the smallest perturbation of the parameters will make it move directly into either of the two corner solutions.

For most applications this is not a very satisfactory predictive model. In order to make the firm choose a well-defined interior solution one has to introduce a form of non-linearity into the tax and penalty scheme. It is natural to assume that the penalty on undeclared emissions is a progressive one, so that large undeclared amounts imply a penalty per unreported unit which is higher than for small amounts; accordingly, we have that $\theta''(e - z) > 0$. With this assumption, which, together with the convexity of the cost function, also guarantees that the second-order conditions are satisfied, the maximisation of expected profit with respect to x, e and z leads to the first order conditions

$$c_x = p, \tag{7}$$

$$-c_e = \alpha(t + \theta'), \tag{8}$$

$$t = \alpha(t + \theta'). \tag{9}$$

(8) and (9) together imply that

$$-c_e = t. \tag{8*}$$

But (7) and (8*) taken together are obviously identical to the first order conditions (2) and (3) for the standard case. Hence the solution values for output and emission must be the same, i.e. (x^*, e^*). *The optimal amounts of production and emission are the same as in the case with perfect monitoring and no evasion*.[4]

The economic explanation for this result is that (9) is a tax arbitrage condition; the firm will engage in an amount of underreporting of emissions until the point where the expected penalty rate equals the regular tax rate. For this to represent an interior solution, i.e. a situation where $0 < z^* < e^*$, it is necessary to impose additional conditions on the penalty function. Necessary and sufficient conditions to ensure an interior solution are

$$\alpha\theta'(0) < (1-\alpha)t < \alpha\theta'(e^*). \tag{10}$$

From the separability property it follows that since x and e have been determined by (7) and (8*), z can be taken as determined by (9) alone, with e taken as fixed. It follows that changes in the probability of detection and the penalty function will only affect the amount of emission reported, and not actual emission and output. Reported emission will, however, vary positively both with the probability of detection and a positive shift in the marginal penalty function.

From the present perspective the crucial point to note is that the separability property allows us to conclude that *the efficiency of tax policy in the standard model carries over to the case with tax evasion*. At the margin the Pigouvian tax rate continues to represent the savings to be set against the cost of reducing emissions. Because this tax rate is the same for all polluters, it does not matter for efficiency if polluters have different subjective assessments of the probability of detection or are at different points on their expected penalty schedules. The mechanism that achieves this is the tax arbitrage of polluters which ensures that the expected marginal tax rate is the same for the two states of detection and non-detection.

AN ENDOGENOUS PROBABILITY OF DETECTION

The assumption that the polluting firm looks upon the probability of detection as being completely exogenous is evidently a simplification. A step in the direction of more realism is to assume that the firm perceives this probability as being to some extent influenced by its own actions. It would e.g. be reasonable to think that the probability depended negatively on the amount of emissions reported; although the government has imperfect knowledge of the technology and cost conditions of the firm, it could reasonably be assumed to have some a priori notion about the normal range of emissions in the industry. An unusually low level of *reported* emissions would therefore be a good reason for inspection, while the case for frequent inspections would diminish with the amount reported. But one could also make a case for having the probability depend positively on *actual* emissions. Although the government has no direct evidence of these, unusually high emission levels might give rise to observations (dead fish floating ashore, high incidence of a particular disease near the production site etc.) that would in turn give cause for frequent inspections. Both these hypotheses[5] may be captured by writing the probability of detection as

$$\alpha = \alpha(e, z), \quad \alpha_e > 0, \alpha_z < 0. \tag{11}$$

It is assumed that α is a strictly convex function of e and z.

Expected profit is now[6]

$$E[\pi] = \alpha(e, z)\pi^0 + [1 - \alpha(e, z)]\pi^1, \tag{12}$$

and the first order conditions with respect to *x, e* and *z* become

$$c_x = p, \tag{13}$$
$$-c_e = \alpha(t + \theta') + \alpha_e(\pi^1 - \pi^0), \tag{14}$$
$$t = \alpha(t + \theta') - \alpha_z(\pi^1 - \pi^0). \tag{15}$$

It is clear that this model does not have the separability property of the previous case. However, there is an interesting special case for which the separability result does hold. (14) and (15) can be combined to yield

$$-c_e = t + (\alpha_e + \alpha_z)(\pi^1 - \pi^0). \tag{14*}$$

The right-hand side of this equation can be regarded as the implicit tax rate on emissions. In general, this tax rate is different from the Pigouvian tax. Whether it is higher or lower depends on the properties of the probability function, i.e. on whether e or z is the dominating influence on the probability of detection. If the probability is mainly affected by e (in the upward direction), the implicit tax rate is higher than the Pigouvian tax, while if the main influence is through z, it is lower. If their effects are close to being symmetrical, they would tend to cancel out, and the second term on the right of (14*) would become insignificant. In particular, if the probability of detection is seen as depending only on the amount of underreporting, so that

$$\alpha(e, z) = \alpha(e - z) \quad \alpha' > 0, \alpha'' > 0, \tag{16}$$

then (14*) would become identical to the first order condition for the standard case and for the case where α is constant.

Going back to the general case, it is clear from equation (14*) that not only is the implicit tax rate different from the Pigouvian rate, it must also be expected to vary between firms. This is because the properties of the probability functions reflect the beliefs of the firms with respect to the nature of public environmental policies and these beliefs are essentially subjective (although to attempt to make them uniform might actually be a sensible objective of government policy). However, uniformity of probability beliefs would not be sufficient to ensure efficiency, since the term

$\pi^1 - \pi^0$ depends on properties of the cost function, which must also be expected to vary between firms.

Summing up the lessons to be learned from the analysis of a variable probability of detection, it is obvious that – unlike the case where α is constant – it does confirm the prior intuition that tax evasion may destroy the appealing efficiency property of the standard analysis. However, it is interesting to note that there is no clear bias as regards the magnitude of the implicit tax rate, which may be higher or lower than the Pigouvian tax. Moreover, there is an important special case in which the two rates coincide, and in which the efficiency property of the Pigouvian tax continues to hold.[7]

4. The Role of Risk Aversion

The assumption of risk neutrality which has been used so far is in some ways a natural one for a firm that operates in a setting of well-developed risk markets. It also makes it easy to distinguish conceptually between production efficiency on the one hand and consumption efficiency on the other. Nevertheless, there are clearly many contexts in which it seems realistic to assume risk averse behaviour on the part of firms, and the question then arises of whether the results derived so far carry over to this case. In the following, therefore, we reexamine the models of the previous section under the assumption that the firm has a utility function which is concave in profits, i.e. it is risk averse. As before, it is natural to begin with the case where α is constant.

The objective of the firm now has to be redefined as the maximisation of

$$E[U(\pi)] = \alpha U(\pi^0) + (1-\alpha)U(\pi^1), \tag{17}$$

with respect to *x, e* and *z*. The first order conditions can be written as

$$(1-\alpha)(p - c_x) + \alpha(1+r)(p - c_x) = 0, \tag{18}$$

$$(1-\alpha)(-c_e) + \alpha(1+r)(-c_e - t - \theta') = 0, \tag{19}$$

$$t = \alpha[t + (1+r)\theta'] \; or \; (1-\alpha)t = \alpha(1+r)\theta'. \tag{20}$$

In these conditions the parameter *r* represents the risk attitudes of the firm; it can be interpreted as the risk premium for transferring a unit of profit from state 0 to state 1. It is defined as

$$r = [U'(\pi^0) - U'(\pi^1)]/U'(\pi^1). \tag{21}$$

Since risk aversion implies that the marginal utility is decreasing, it is clear that the risk premium *r* is positive for $z < e$. (For $z = e$, we have $r = 0$.) The condition for an interior solution now becomes

$$\alpha\theta'(0) < (1-\alpha)t < \alpha(1 + r|_{z=0})\theta'(e^*).$$

Thus, the range of parameters allowing for an interior solution is a wider one than in the risk neutral case, in the sense that it is more unlikely that the firm will not report its emissions at all.

From (18) it immediately follows that

$$c_x = p. \tag{18*}$$

Combining (19) and (20), it follows after a few manipulations that

$$-c_e = t. \tag{19*}$$

We can thus conclude that the separability property that was established for the risk neutral case carries over to the case of risk aversion. It still holds that the Pigouvian tax rate is equal to the marginal cost of emission reduction, and *the allocation of emissions among polluters will accordingly be efficient in the same sense as before*.

What does change in this model is the determination of the amount of pollution reported by the firm. This will now have to satisfy condition (20), which can be rewritten more explicitly as

$$(1-\alpha)t = \alpha[1 + r(x^*, e^*, z)]\theta'(e^* - z). \tag{22}$$

Notice that the condition depends in principle on all three decision variables *x, e* and z. However, since the first two have already been determined by conditions (18*) and (19*), as indicated by the asterisks, (22) depends on z alone. For given values of t, α and the penalty function θ, risk aversion leads to a higher degree of compliance than risk neutrality.

The case of an endogenous probability of detection can now be treated somewhat more summarily. Deriving the first order conditions, it turns out that the equality of marginal production cost to output price still holds. However, the relationship between the Pigouvian tax and marginal abatement cost becomes less transparent, since the first order condition for e becomes

$$-c_e = t + (\alpha_z + \alpha_e)/(1 + \alpha r) \times \Delta U, \tag{23}$$

where

$$\Delta U = [U(\pi^1) - U(\pi^0)]/U'(\pi^1).$$

The second term on the right of equation (23) is the gain or loss from the change in the probability of detection due to the firm's own actions. Its sign will depend on whether the amount of pollutant emitted or reported is the most powerful influence on the probability of detection. However, it is easy to see from (23) that the important special case where $\alpha = \alpha(e - z)$ again makes the last term vanish, so that the efficiency condition is restored. The qualitative conclusion is therefore very much in line with the risk neutral case.

We have seen that risk aversion does not change in any fundamental way the characterization of firm behaviour with regard to tax compliance. With α exogenous the predictions with regard to output and emissions are exactly the same as before, and the efficiency properties of the tax regime are the same as in the model with risk neutrality; the characterisation of optimal compliance becomes only slightly more complex. When α is allowed to depend on real and reported emissions, we get exactly the same ambiguity with respect to the effective tax rate as in the risk neutral case, and the same assumption (16) about the probability of detection is sufficient to restore the efficiency property.

It is worth taking a look at the condition for optimal reporting in the special case when assumption (16) holds, in which

$$(1-\alpha)t = \alpha(1+r)\theta'(e^* - z) + \alpha' \times \Delta U. \tag{24}$$

There are now two reasons why the effective penalty rate is higher in this case than in the constant probability, risk neutral case. One is the risk premium applied to the marginal penalty rate, which makes the firm perceive it as effectively larger than its expected value. The other is the marginal effect of evasion in increasing the probability of detection.

5. Complying with Quotas: Risk Neutrality

We now turn to the case where the authorities regulate emissions by means of a quota, q. We will assume that the quota is binding on the firms, so that the firm that we analyse will never find it optimal to emit less than the quota.[8] However, it might find it optimal to emit more. If it is not discovered, it will have saved on costs and increased its profit. If, on the other hand, it is discovered, it will have to pay a fine on the amount of violation equal to $\tau(e-q)$. Profit under the two outcomes will therefore be

$$\pi^0 = px - c(x,e) - \tau(e-q)$$

and

$$\pi^1 = px - c(x,e).$$

As in the treatment of tax evasion we first focus on the penalty function, taking the probability of detection as exogenously given. We then extend the analysis to the case of an endogenous probability.

A PROGRESSIVE PENALTY FUNCTION

In the first version of the model there is a convex penalty function with $\tau' > 0$ and $\tau'' > 0$. Expected profit maximisation then leads to the first order conditions

$$c_x = p, \tag{25}$$
$$-c_e = \alpha\tau'. \tag{26}$$

These conditions describe an interior solution with a positive amount of quota violation. It is of course not clear that this is the only interesting outcome, but it is easy to check that it will in fact be the case if

$$-c_e(x, q) < \alpha\tau'(0), \tag{27}$$

and this condition is assumed to hold in the following.

From equation (26) two implications of the model deserve to be stressed. The first is that *the expected marginal fine does have the property of an effective tax rate*, to which marginal abatement cost will be equated. The other is that this effective tax rate must in general be expected to vary between firms, so that *the efficiency property of the Pigouvian tax rate does not hold* in this model. There are two reasons for this variation between firms. The first is that they might have different assessments of the probability of detection. The other is that when the amounts of violation differ between firms, their marginal fines will also differ.

However, it deserves to be pointed out that these two effects to some extent counteract each other. This becomes apparent when we take the derivative of the effective tax rate – the right hand side of equation (26) – with respect to α:

$$\partial(\alpha\tau')/\partial\alpha = \tau' + \alpha\tau''(\partial e/\partial\alpha). \tag{28}$$

While the first term in the expression on the right is positive, the second is negative, since $\partial e/\partial\alpha$ can easily be shown to be negative. A higher probability of detection leads to less violation of the quota, and this decreases the marginal fine. The formal progressivity of the fine does not necessarily lead to progressivity of the *effective* fine; at least, this progressivity might be considerably less. Given the formal progressivity of the fine schedule, we thus arrive at the slightly paradoxical conclusion that attempts by the government to establish a uniform perception of the probability of detection among polluters – which on the face of it should be a step in the direction of efficiency – could possibly lead to greater variation in the effective marginal fine and thus to larger variations in the marginal costs of abatement.

A VARIABLE PROBABILITY OF DETECTION

It seems realistic to assume that the probability of getting caught for a violation of the quota should be seen by the firm as dependent on the amount of violation.

The reasons are basically the same as those already given for the corresponding assumption in the analysis of the tax regime. Here the framework is simpler, since the firm does not have to make a report on emissions.[9] The probability therefore depends only on the actual level of emissions, and it is reasonable to assume that the crucial variable is the level of emissions in excess of the quota. The probability can therefore be written as $\alpha(e - q)$ with $\alpha' > 0$ and $\alpha'' > 0$. The first order conditions for expected profit maximisation are then simply derived as

$$c_x = p, \tag{29}$$
$$-c_e = \alpha\tau' + \alpha'(\pi^1 - \pi^0), \tag{30}$$

where $\pi^1 - \pi^0 = \tau(e - q)$. Again, these are conditions for an interior solution. For this to be an optimum for the firm it must be the case that

$$-c_e(x, q) > \alpha\tau'(0) + \alpha'(0)\tau(0). \tag{31}$$

At the point where the amount of emission equals the quota, the marginal cost of abatement must exceed the sum of the expected marginal fine and the expected change in profit following from an increase in the probability.

From (30) it seems clear that this condition is almost certain to be inconsistent with the efficiency condition, since it depends both on first and second order properties of the probability and penalty functions. In order to check this we may proceed as in the previous case and look at how the effective tax rate τ^*, given by the right hand side of equation (30), varies with the perception of the probability of detection. However, one has to keep in mind that the magnitude of the probability is now determined endogenously, so that we have to consider a positive shift in the probability function, reflecting an increased probability of detection for any given amount of quota violation. For this purpose we may rewrite the probability function as $\alpha = \gamma\alpha(e - q)$, where $\gamma = 1$ initially, and take the derivatives of the system (29)–(30) with respect to γ. We then obtain

$$\partial e/\partial\gamma = -D^{-1}c_{xx}(\alpha\tau' + \alpha'\tau) < 0. \tag{32}$$

Here D is the second order determinant, which is positive from the convexity assumption, as is c_{xx}; the inequality sign then follows. The effect on the effective tax rate now follows as

$$\partial\tau^*/\partial\gamma = (\alpha\tau' + \alpha'\tau) + (2\alpha'\tau' + \alpha\tau'' + \alpha''\tau)\partial e/\partial\gamma. \tag{33}$$

The first term on the right of this equation is clearly positive. The second term is negative in view of (32), the progressivity of the penalty function, and the convexity of the probability function. The second term is therefore counteracting the effect of the first term. This is similar to the result derived for the previous model, and the interpretation is basically the same: A firm which perceives a higher probability of detection will reduce its emissions, thereby reducing the marginal fine and also – in a second round, as it were – the probability of detection. The effective tax rate will therefore not shift in the same proportion as the probability of detection.

A more detailed comparative statics analysis would show that under the assumptions already made the second effect will never outweigh the first, so that a positive shift in the probability function will indeed increase the effective tax rate. But these details are less interesting in the present context and will be left out.

6. Quotas and Risk Aversion

I now briefly consider what happens to the above analysis when the polluting firm is assumed to be risk averse. Let the utility function for profit be $U(\pi)$ as before. Under maximisation of expected utility the first order condition (26) for the progressive penalty case changes to

$$-c_e = \alpha^* \tau'. \tag{34}$$

α^* is the *effective probability*, i.e. a probability adjusted by the marginal utility of profit in the two states of detection and non-detection. It is defined as

$$\alpha^* = \alpha U'(\pi^0)/[\alpha U'(\pi^0) + (1-\alpha)U'(\pi^1)] = \alpha(1+r)/(1+\alpha r). \tag{35}$$

Under risk neutrality, when marginal utility is constant and there is no risk premium, this is simply α, as in the previous model. Under risk aversion, the effective probability is computed as a weighted average of marginal utilities, and the result is that the polluter uses an effective probability which is higher than the actual one, i.e. $\alpha^* > \alpha$. How much higher is then a question of the curvature of the utility function, i.e. of the magnitude of the risk premium. The effective tax rate, being now the product of the effective probability and the marginal penalty, will then depend on two individual characteristics of the polluter, viz. the perceived probability and the degree of risk aversion. This makes it more likely that there will be considerable variation in the effective tax rate among polluters[10].

Extending the model to allow for an endogenous probability of detection with $\alpha = \alpha(e-q)$, the condition for the profit-maximising level of emission is

$$-c_e = [\alpha(1+r)/(1+\alpha r)]\tau' + [\alpha'/(1+\alpha r)] \times \Delta U. \tag{36}$$

One sees immediately that this reduces to expression (30) for the risk neutral case, for then $r = 0$, and ΔU is simply the profit differential. The conclusion is essentially the same as for the previous model. The effective tax rate to which the marginal cost of abatement will be equated will depend on individual characteristics of the polluting firm, in particular on its perception of the function relating the probability of detection to the amount of violation, and its degree of risk aversion.

7. Self-reporting of Quota Violations

These models of the compliance with quotas have been built on the assumption that the firm does not report on the extent of its emissions and its possible violations of the quota; however, the environmental legislation in several countries actually requires firms to file such a report. This makes the decision problem of the firm slightly more complex. Not only does it have to make decisions on output and emission, it must also decide on how much violation, if any, to report. Two types of fine now enter the picture. The first is a fine related to the violation as such. The other is a fine for underreporting, which is similar to the penalty for tax evasion in the tax compliance models. This extension was first considered by Harford (1987). As in his earlier paper he focused on the implications for firm behaviour rather than on the conditions for productive efficiency, but he derived the interesting result that the introduction of self-reporting would, under certain conditions, introduce a kind of separability into the model of quota violations that is similar to the one derived for the case of tax evasion. I demonstrate this here for a model that includes both non-linear fines and a variable probability of detection. Rather than consider all possible specifications of the arguments of the fine and probability functions, I go directly to the case in which the possibility of an efficient regime arises.

I concentrate to begin with on the case of an interior solution in which the firm violates the quota but underreports the extent of its violation. Letting z be the amount reported, the ordinary fine for violation will be $\tau(z-q)$, while the penalty for underreporting is $\theta(e-z)$. Profit in the states of discovery and non-discovery will then be

$$\pi^0 = px - c(x,e) - \tau(z-q) - \theta(e-z)$$

and

$$\pi^1 = px - c(x,e) - \tau(z-q),$$

with $\alpha = \alpha(e-z)$. That the probability of detection depends on the extent of underreporting is a reasonable assumption, but is certainly not the only one; one could e.g. imagine it to depend on the amount of emission. But my interest here is not in exploring all possible formulations but rather to concentrate on a set of assumptions that yields a particularly interesting result.

Maximising expected profit now leads to the following first-order conditions:

$$c_x = p, \tag{37}$$

$$-c_e = \alpha\theta' + \alpha'(\pi^1 - \pi^0), \tag{38}$$

$$\alpha\theta' + \alpha'(\pi^1 - \pi^0) = \tau'. \tag{39}$$

From (38) and (39) it follows that

$$-c_e = \tau'. \tag{40}$$

It follows immediately that if the function τ is linear so that τ' is constant, then all polluters will select the same level of marginal abatement cost, and production efficiency will result. In other words, *the marginal penalty for quota violation, τ', becomes the analogue of the Pigouvian tax in this model*. Moreover, *the penalty for underreporting affects only the degree of underreporting and not the amounts of output and emission*. The economic intuition behind this result is the same as for the corresponding result in the tax evasion case: Condition (39) can be interpreted as one of optimal tax-penalty arbitrage, where underreporting is carried out to the point where the expected marginal tax or penalty rate is the same in the two states. It is a remarkable feature of this analysis that when the firm is allowed not only the choice of violating the quota, but in addition to lie about the extent of its violation, this additional option helps to bring about a socially efficient allocation of emissions.

It is a simple extension of this model to show that the central efficiency result (40) continues to hold if the firm is risk averse; the only feature of the solution which changes is the precise form of the arbitrage condition. As in the tax evasion model, penalties, probabilities and the firm's attitude to risk influences the extent of underreporting, but not the output-emission decision.[11]

8. Concluding Remarks

Environmental taxes levied at the same rate on the emissions of individual firms have efficiency properties that are well understood for the case of perfect tax compliance. The present paper has studied the problem for the case of imperfect compliance, including tax evasion and quota violations. For the case of tax evasion, the chief conclusion is that the production efficiency property of taxes in the main carries over to the imperfect compliance case. This is definitely true for the case of an exogenous probability of detection, and when this is endogenous, the property still holds for the central case where the probability is perceived to depend only on the extent of underreporting.

In the case of quota violations, one has to distinguish between the cases where firms are required to file a report on their own violations and the case where they are not. In the latter case, the expected fine does play the role of an effective tax rate, but its magnitude depends in an essential way on the perceived probability of detection and the degree of risk aversion, which are individual characteristics of each particular firm. This regime cannot therefore be expected to result in an efficient allocation of emissions among the polluting firms. The case of self reporting, on the other hand, can be designed in such a way that tax arbitrage leads to an efficient distribution of emissions.

It is important to emphasize that these conclusions are virtually unchanged when the analysis is extended from risk neutrality to risk aversion. One would perhaps have expected that risk aversion would significantly have influenced emission decisions in the tax regime, but this turned out not to be the case; the predictions of the model are hardly influenced by the introduction of risk averse behaviour. This is because the separability between the production and emission decisions on the one hand, and the reporting decision on the other, carries over from the risk neutral case. However, risk aversion makes much more of a difference for the decision to violate quotas, where, as expected, risk aversion makes firms more cautious in exposing themselves to the risk of penalties.

What are the general conclusions that one can draw on the relative merits of taxes and quotas under imperfect compliance? The Introduction made the point that intuition would lead us to the conjecture that the two cases would be much more like each other than they are made out to be in the standard model. To some extent this has turned out to be true, but it is also fair to say that the properties of the quota model have moved closer to the tax model than the other way around. The effective penalty rate for quota violations can be designed in such a way that it has many of the properties of a Pigouvian tax, and this is an important insight that emerges from the analysis.

One apparent omission from the analysis is the case of transferable quotas. However, a study of this regime does not add much to the analysis. As in the standard analysis, it turns out that the efficiency properties of transferable quotas are the same as for taxes, with the quota price playing essentially the same role as the Pigouvian tax.[12]

In this comparison between taxes and quotas I have abstracted from some of the considerations that are relevant for the choice between them as the preferred policy regime. In establishing the standard case for taxes, I used convexity conditions for cost functions that will not necessarily obtain in practice, and with non-convexities the standard case for taxes breaks down. It has also been assumed that there are no redistributional concerns that modify the desirability of efficiency, although this is likely to be important in a number of practical applications. Also, the analysis has been static, and there has been no attempt to study the dynamic effects on the technology of production and pollution abatement. Another limitation of the study is that we have only considered firm behaviour as a response to a given set of policy parameters, and in a wider perspective it would be natural to look also at the optimum choice of policy from the government's point of view, taking account of the administrative costs of taxes and quotas.

Moreover, there are non-economic arguments that are relevant for policy choice and which are not captured by the present models. In the words of Schelling (1983, p. 7)

> . . . a fee entitles one to what one has paid for. It is not levied in anger, it does not tarnish one's record, and even if paid to cover damages it is not expected to be paid grudgingly and received resentfully. But a fine that is paid upon conviction for an offense does not erase the offense.

This is an important reminder of the limitations of studies of the present type. If the prevailing moral attitude is that some types of pollution fall in the class of offences that should not be erased by a simple payment, quotas combined with fines may turn out to be the politically preferred instruments even if they do not satisfy the conditions for production efficiency.

These limitations should not lead us to underestimate taxes as instruments of environmental policy; indeed, the analysis has demonstrated that the efficiency case for taxes remains strong, even with imperfect compliance.

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Notes

1. A preference for taxes may, however, be derived from the desire to minimize the deadweight loss from taxation in general through the substitution of environmental for other, distortionary, taxes. This consideration, which is related to recent discussions of the "double dividend" from environmental tax reform, lies outside the framework of the present paper. For surveys and discussions of this issue see Bovenberg (1999) and Sandmo (2000).
2. Self-reporting is also examined in a more general economics of crime context by Kaplow and Shavell (1994).
3. There is no need to assume that the value of t reflects the marginal social damage from pollution, i.e. t need not necessarily satisfy the conditions for consumption efficiency. The government might have set a standard for aggregate emissions and decided to use the tax to achieve this standard in an efficient way. The standards-and-taxes approach to pollution control was first analysed by Baumol and Oates (1971).
4. This result was derived by Harford (1978) in a slightly more complicated model. But Harford does not explicitly conclude that the separability result implies that the efficiency property of the tax regime continues to hold even in the presence of tax evasion. A similar separability property obtains in Marrelli's (1984) model, where the extent of the monopolist's tax evasion is decided separately from the price markup decision.
5. In addition, as a referee has pointed out, one could also imagine that the firm would perceive the probability as depending on output, x. This dependence could come about because the government has some notion about the normal level of emissions per unit of output. For any given level of z, the probability would then be an increasing function of x. Underlying the present treatment is the assumption that the cost function defines a certain size of the firm, and that the government's assessment of the significance of z is relative to this size, being little affected by variations in output within the firm's normal range of operations.
6. The following analysis holds also for the case where the penalty function is linear; the non-linearity required for the model to yield an interior solution is taken care of by the assumptions made about the probability function. However, since it does not complicate the analysis I continue to assume a progressive penalty function, although with linearity as a special case.
7. This discussion of the properties of the probability function, as perceived by the firm, does not rule out that public policy is based on a comparison of the firm's reported emission with e.g. an estimate of the industrial average. As long as the individual firm takes this average as exogenous, the results continue to hold.

8. This is not an innocuous assumption. First, if quotas are fixed more or less in ignorance of the cost conditions of individual firms, it is not inconceivable that some quotas might be set in excess of what would have been the firm's level of emissions in the absence of any regulation. A more general problem is that firms are frequently observed to be overcomplying with quotas in the sense of reducing emissions by more than the quotas require. The present models do not attempt to explain this observation, which would presumably require a richer set of assumptions about firms' objectives or the technology of pollution. See the discussion in Cropper and Oates (1992), pp. 696–697.
9. Self-reporting is considered in Section 7 below.
10. How much variation there would be, depends on the correlation between r and α; a negative correlation would make for less variation. But intuition would suggest that the correlation is more likely to be positive, since those who are strongly averse to risk, would probably tend to have a high assessment of the probability of getting caught.
11. The result (40) was also derived by Harford (1987), but he does not emphasize the significance of the result for the production efficiency of environmental policy. Moreover, he does not note the fact that the result continues to hold under risk aversion. The focus in the article by Kaplow and Shavell (1994) is on optimal enforcement, and they do not discuss the particular aspect of self-reporting that has been stressed in the present paper. Malik (1993) analyses the relationship between a regulatory agency and a single polluter and the costs of monitoring the polluter with and without self-reporting. Innes (1999) extends the analysis by taking account of the social benefits that occur through polluters' remedial activities, an aspect of the problem which is neglected in the present paper.
12. Formally, the case of transferable quotas can be interpreted as our model of tax evasion with t being the unit price of quotas and θ the penalty function for excess emissions. As long as firms take the price t as given, the two regimes are equivalent from a production efficiency point of view. This does not imply that the two regimes are otherwise completely equivalent; for an analysis that takes account of the effects of quota violations on the equilibrium price see Andréasson-Gren (1992).

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