Econ 301, Solutions to Homework 6

10.9

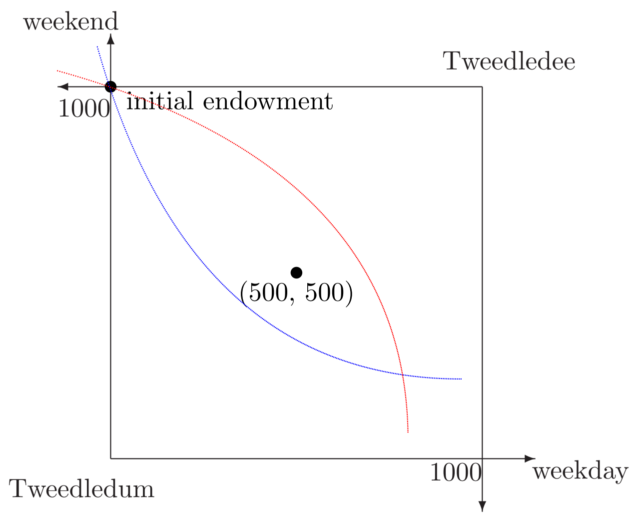
(a) $1.21 \times c_1 + c_2 = 3,520$.

(b) He picks $c_1 = c_2$. Substitute into the budget to find $c_1 = \frac{3,520 \times \frac{1}{2}}{1.21} = 1,592.8$. Then, he saves $2000 - 1592.8 = 407.2$

(c) In period 2, the price of bread will be \$1.1 per a loaf. Then his budget constraint is $1.21 \times c_1 + 1.1c_2 = 3,520$.

Problem X

(a) There are 1,000 units of each good available for consumption. The initial endowment and the preferred bundle $(500, 500)$ are indicated in the graph.

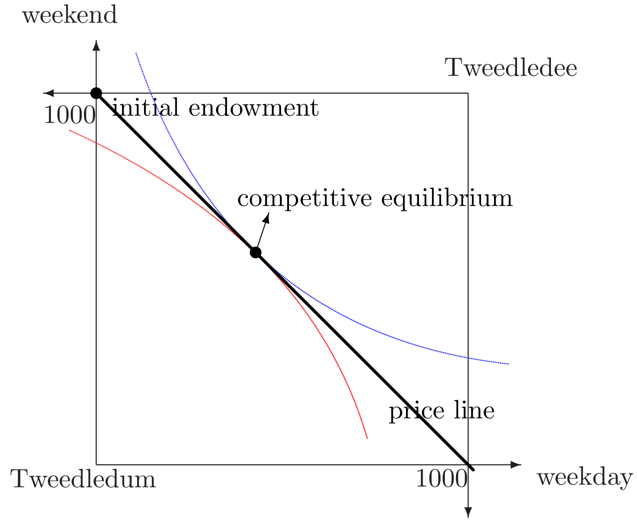


(b) A competitive equilibrium for this exchange economy consists of prices (p_1, p_2) and an allocation such that:

(i) given the prices, both Tweedledum and Tweedledee are choosing the best bundle available

(ii) markets for both weekday and weekend minutes clear (i.e. aggregate demand for each good equals aggregate supply of each good).

Graphically, a competitive equilibrium is a point in the Edgeworth box where the two indifference curves are tangent to each other at that point, and the tangent line passes through the initial endowment. Note that this tangent line divides the Edgeworth box into two areas: the one below the line is the budget set for Tweedledum, and the above one is the budget set for Tweedledee. Both agents move along the budget line, from the initial endowment to the competitive allocation.



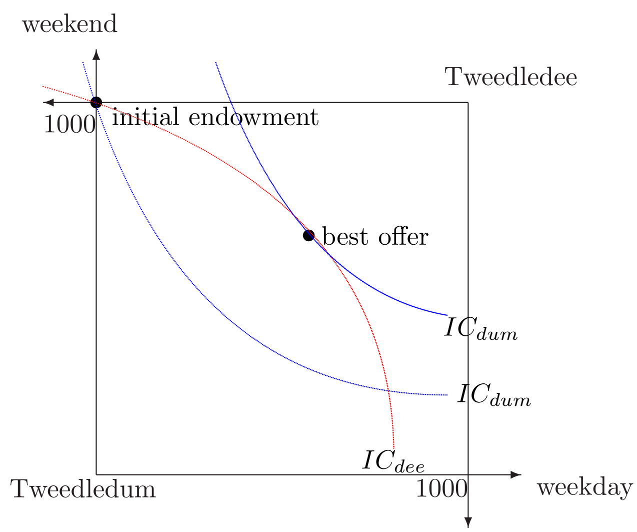
(c) Tweedledum's best offer should satisfy the following conditions:

(i) Tweedledum must be at least as well off as at the initial endowment also so that he is willing to make the offer

(ii) Tweedledee must be at least as well off as at the initial endowment (when there is no trade) so that he is willing to accept the offer

(iii) Tweedledum should be maximizing his utility given (i) and (ii).

On the graph, the best offer is where Tweedledum's indifference curve is tangent to Tweedledee's initial indifferent curve. At this point, Tweedledum is extracting all profit opportunities since he has all the bargaining power (as he is the one who gives the offer), and Tweedledee is not worse off (he is just as indifferent between the offer and and his initial endowment).

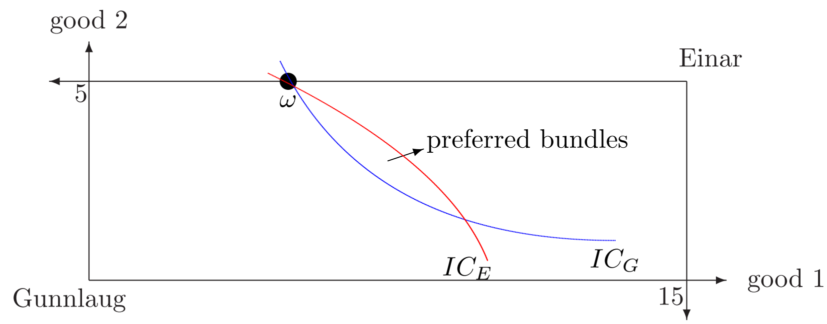


Problem Y

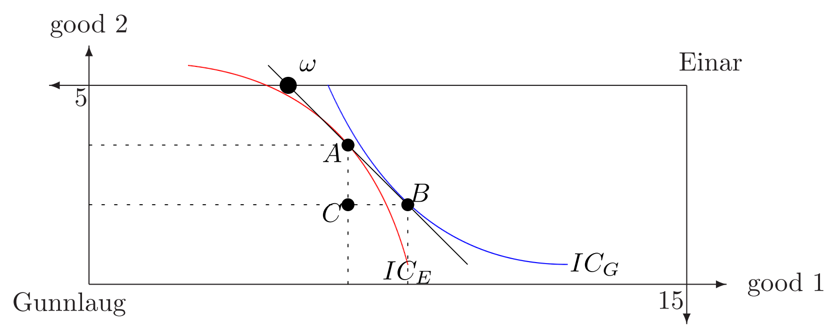
(a) Total amount of good 1: $\omega_1^G + \omega_1^E = 5 + 10 = 15$

Total amount of good 2: $\omega_2^G + \omega_2^E = 5 + 0 = 5$

ω is the endowment point. Any allocation in the lens shaped area is preferred to ω by both agents.



(b) In the picture below, Gunnlaug is maximizing utility at point B and Einar at point A. The vertical distance $A - C$ is the excess supply of good 2 and horizontal distance $B - C$ is the excess demand for good 1. The market is not cleared in this exchange economy. Therefore we do not have a competitive equilibrium.

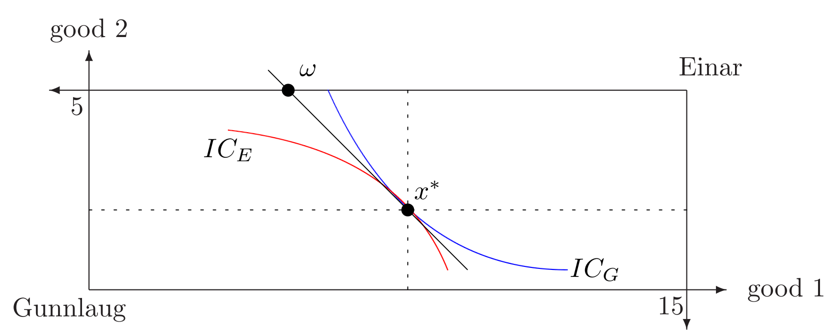


(c) Conditions for a competitive equilibrium

(i) Allocation must be feasible $\Rightarrow$ Graphically this means that both agents choose the same point in the Edgeworth box. Both are choosing x^* here.

(ii) Both agents must choose the best bundle available $\Rightarrow$ Graphically this means at the competitive equilibrium both agents' indifference curves are tangent to their budget line (which is the line passing through the endowment). Both agents maximize utility at bundle x^* the Edgeworth box..

So x^* is a competitive equilibrium allocation.

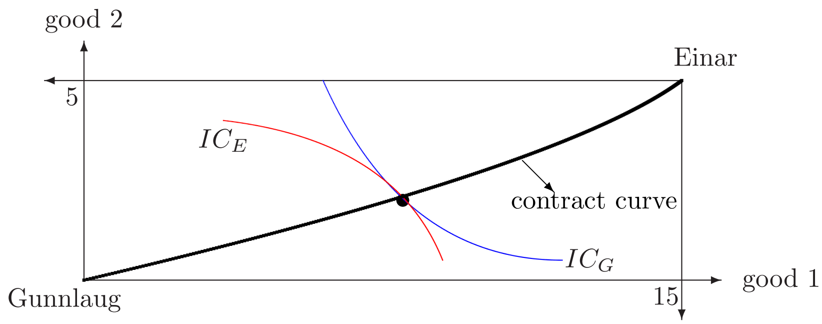


(d) A Pareto efficient allocation is an allocation such that:

(i) the allocation is feasible

(ii) no one can be made better off without making the other worse off.

Any competitive equilibrium is in fact Pareto efficient. This result is well known as the "First Fundamental Theorem of Welfare Economics". Graphically, this means that any competitive equilibrium allocation should be located on the contract curve.



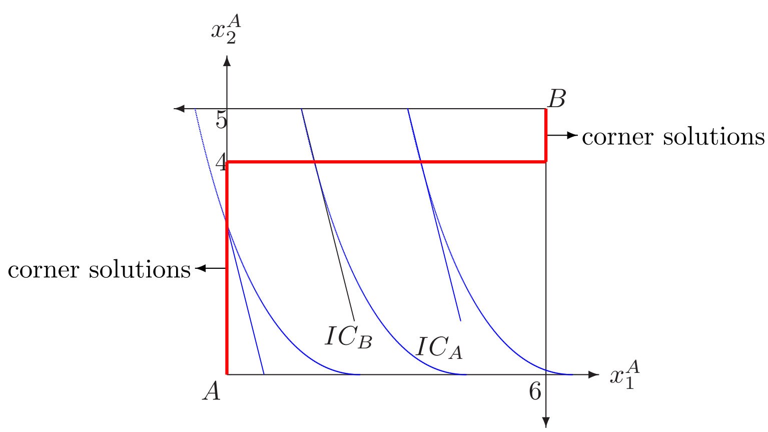
Problem Z.1

I will change the initial endowment of agent B to $\omega_B = (0, 5)$. Otherwise the answer might be misleading. At the end of the question I will explain why I made this change.

Since the preferences are smooth, we shall first equate the marginal rate of substitutions for both agents:

$$MRS_A = MRS_B \;\Rightarrow\; \frac{\partial u_A/\partial x_1^A}{\partial u_A/\partial x_2^A} = \frac{\partial u_B/\partial x_1^B}{\partial u_B/\partial x_2^B} \;\Rightarrow\; \frac{1}{1/(2\sqrt{x_2^A})} = \frac{4}{1} \;\Rightarrow\; 2\sqrt{x_2^A} = 4 \;\Rightarrow\; x_2^A = 4$$

Since this curve does not pass through the two origins, there are some corner solutions as well. The following picture gives the set of Pareto efficient allocations, i.e. the contract curve.



If we keep agent B's endowment at (0,4) then $x_2^A = 4$ corresponds to the upper side of the Edgeworth box. Since that includes agent B's origin, we would need to include the entire left side as corner solutions. I first solved the case with $\omega_B = (0, 5)$ since it corresponds to a more general case.

Problem Z.2

The first important observation is that for agent B, the utility function $\upsilon_B = x_1^B x_2^B$ is a monotonic transformation of u_B. That is because $\upsilon_B = \sqrt{u_B - 5}$ which is a monotonic increasing function. (Always do that whenever you argue that a function is a monotonic transformation of another. In other words, indicate how the utility functions are related to each other.) Therefore we can as well use the Cobb-Douglas utility function υ_B for agent B since it gives precisely the same demand functions. Now let us find the competitive equilibrium.

Step 1: Finding the demand functions

Let $p_1 = 1$ and $p_2 = p$. Then agent A's income is $m_A = 2$ and agent B's income is $m_B = 3p$. Since both agents have Cobb-Douglas utility functions, their demands are:

$$x_1^A = \tfrac{1}{2}\tfrac{m_A}{p_1} = 1, \quad x_2^A = \tfrac{1}{2}\tfrac{m_A}{p_2} = \tfrac{1}{p}, \qquad x_1^B = \tfrac{1}{2}\tfrac{m_B}{p_1} = \tfrac{3p}{2}, \quad x_2^B = \tfrac{1}{2}\tfrac{m_B}{p_2} = \tfrac{3}{2}$$

Step 2: Clearing the markets

Clearing either one of the markets will give us the competitive prices.

$$x_1^A + x_1^B = \omega_1^A + \omega_1^B \;\Rightarrow\; 1 + \tfrac{3p}{2} = 2 \Rightarrow p = \tfrac{2}{3}$$

Therefore the competitive prices are $(p_1, p_2) = (1, \frac{2}{3})$.

Step 3: Finding the competitive allocation

Using Step 1 and Step 3,

$$x_1^A = 1, \quad x_2^A = \tfrac{1}{p} = \tfrac{3}{2}, \qquad x_1^B = \tfrac{3p}{2} = 1, \quad x_2^B = \tfrac{3}{2}$$

Therefore the competitive allocation is $(x^A, x^B) = [(1, \frac{3}{2}), (1, \frac{3}{2})]$