Economics 101A Notes - A Search Model of Unemployment

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1. An individual while searching for a job receives unemployment insurance of ε.
2. The individual samples one job offer per period from a distribution with density function $f(w)$.
3. The optimal strategy consists of choosing a reservation wage, μ, where the hunter accepts job offers for $w \geq \mu$ and rejects all others to search again in the next period.
4. Jobs are forever.
5. The interest rate if given by r.

The expected present value of the individual's lifetime earnings are given by

$$V(\mu) = \varepsilon + (1/r)\int_{\mu}^{\infty} wf(w)dw + [V(\mu)/(1+r)]\int_{0}^{\mu} f(w)dw.$$

The first term denotes the receipt of unemployment insurance while searching, the second is the present value of the expected wage income from locating a job at a wage of greater than μ in the first period. The last term is the probability that a job isn't found in the first period times the discounted value of beginning search again in the second period.

Solving for $V(\mu)$ we find

$$V(\mu) = [(1+r)/r]\frac{[r\varepsilon + \int_{\mu}^{\infty} wf(w)dw]}{[1+r-\int_{0}^{\mu} f(w)dw]} = [(1+r)/r]\frac{[r\varepsilon + \int_{\mu}^{\infty} wf(w)dw]}{[r+\int_{\mu}^{\infty} f(w)dw]}$$

Maximizing V with respect to μ by taking the natural log of both sides, differentiating and resolving terms yields

$$\frac{V'(\mu)}{V(\mu)} = f(\mu)\frac{[r(\varepsilon-\mu) + \int_{\mu}^{\infty}(w-\mu)f(w)dw]}{[r+\int_{0}^{\mu} f(w)dw][r\varepsilon + \int_{\mu}^{\infty} wf(w)dw]} = 0$$

and therefore the optimal reservation wage μ is determined implicitly by $r(\varepsilon - \mu) + \int_{\mu}^{\infty}(w-\mu)f(w)dw = 0$. Clearly, the reservation wage will be set in excess of the unemployment payment so the first term is negative and, of course, the second is positive. Let

$$G(r, \varepsilon, \mu(r, \varepsilon)) \equiv r(\varepsilon - \mu) + \int_{\mu}^{\infty}(w-\mu)f(w)dw \equiv 0$$

define the optimal reservation wage $\mu(r, \varepsilon)$. It us readily seen that $G_r < 0$, $G_\varepsilon > 0$ and

$$G_\mu = -r - \int_{\mu}^{\infty} f(w)dw < 0$$

for positive r. Since $\mu_\varepsilon = -\frac{G_\varepsilon}{G_\mu} > 0$ and $\mu_r = -\frac{G_r}{G_\mu} < 0$ we conclude that an increase in unemployment payments will raise the reservation wage while a rise in the interest rate will lower it.

The average duration of search:

The probability of accepting employment in a given period is

$$p = \int_{\mu}^{\infty} f(w)dwM.$$

The expected time til a job is found is then given by

$$p + 2p(1-p) + 3p(1-p)^2 + ...$$

$$= \sum_{1}^{\infty} np(1-p)^{n-1} = p\sum_{1}^{\infty} n(1-p)^{n-1}$$

Let $h(q) = \sum_{1}^{\infty} nq^{n-1}$.Then

$$\int h(q)dq = \sum_{1}^{\infty}\int nq^{n-1}dq = \sum_{1}^{\infty} q^n = \frac{q}{1-q}.$$

Differentiating w.r.t. q to recover $h(q)$ yields

$$h(q) = (1-q)^{-2}$$

So for $q = 1-p$, $\sum_{1}^{\infty} n(1-p)^{n-1} = h(1-p) = p^{-2}$ and

$$p + 2p(1-p) + 3p(1-p)^2 + ... = p\sum_{1}^{\infty} n(1-p)^{n-1} = \frac{1}{p}$$

Thus, an increase in the μ, say resulting from an increase in ε, would lower p and raise the average duration of unemployment and an increase in the rate of interest, r would lower μ, and reduce unemployment duration.