Economics 101A Notes on Scitovsky Contours

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ABSTRACT.   This note describes the construction of a Scitovsky Contour

## 1.   The Construction of Scitovsky Contours

Suppose that we have two individuals, Joe and Percy, with respective utility levels $\overline{U^j}$ for Joe and $\overline{U^p}$ for Percy. We wish to find the aggregate amounts of commodities which could be distributed in such a way as to allow these individuals to achieve these utility levels. In other words $S(\overline{U^j}, \overline{U^p})$ equals the minimal elements of $\{x | \exists x^j, x^p \geq 0, x^j + x^p = x, U^j(x^j) = \overline{U^j}, U^p(x^p) = \overline{U^p}\}$. To construct the locus, first draw the indifference cure at utility level $\overline{U^j}$ for Joe (see figure 1). Then, taking an arbitrary point on that indifference curve, *add on* the Percy's indifference curve (as in the figure 2). That is, any point on the second curve (viewed with respect to the lower left origin) would have enough stuff to give Joe the arbitrarily chosen point on his indifference curve (x^A) and have enough left over to place Percy at the utility level $\overline{U^p}$.

   Of course, this would be true for *any* point chosen on the $\overline{U^j}$ indifference curve, not merely (x^A). We could have chosen (x^B). Now any point reachable by adding Percy's indifference curve to *either* (x^A) or (x^B) would have enough stuff to give Joe a utility level of $\overline{U^j}$ and Percy a level of $\overline{U^p}$. Continuing in this fashion for any starting point on Joe's $\overline{U^j}$ indifference curve, and taking the lower hull (or the efficient combinations) we have created the Scitovsky contour $S(\overline{U^j}, \overline{U^p})$.
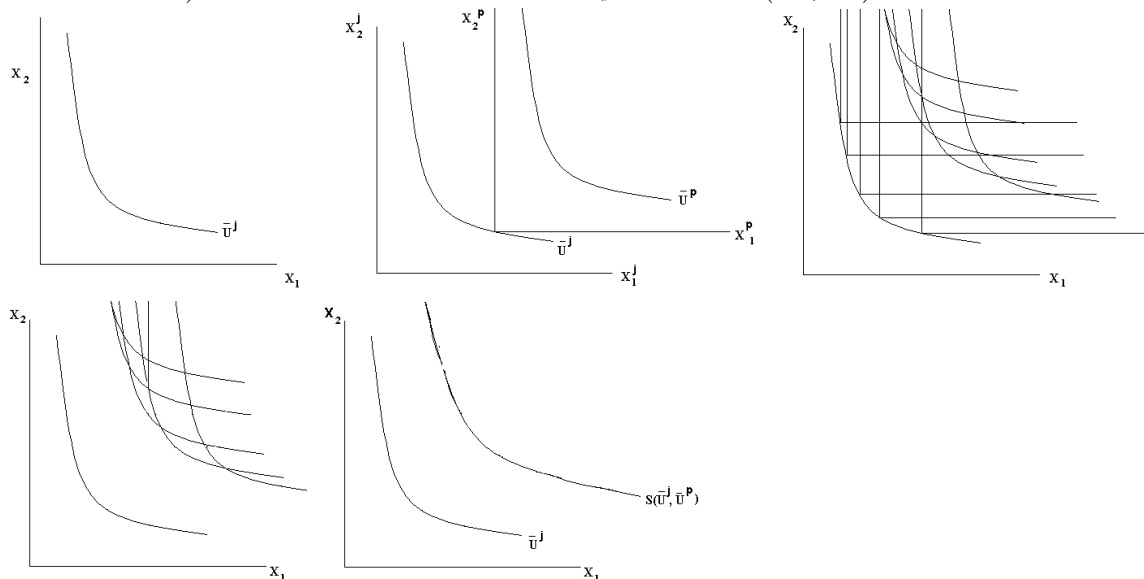
1.1.   Community Indifference Curves , NOT!.   It is important to note, that the contour is dependent upon the distribution of utility. That is, taking a point on $S(\overline{U^j}, \overline{U^p})$ and distributing the aggregate bundle (efficiently) to produce different utility levels, say $\widetilde{U^j}, \widetilde{U^p}$ by making Joe better off and Percy worse off, will produce a new Scitovsky contour $S(\widetilde{U^j}, \widetilde{U^p})$ which will go through the same aggregate bundle but, in general, have a different slope!

1.2.   Cost Benefit Analysis.   Consider now a project z which alters the aggregate bundle. Since z is in a direction above the Scitovsky contour $S(\overline{U}^j, \overline{U}^p)$, there is a redistribution of the original aggregate bundle plus z which is Pareto Superior to the orginal bundle alone.
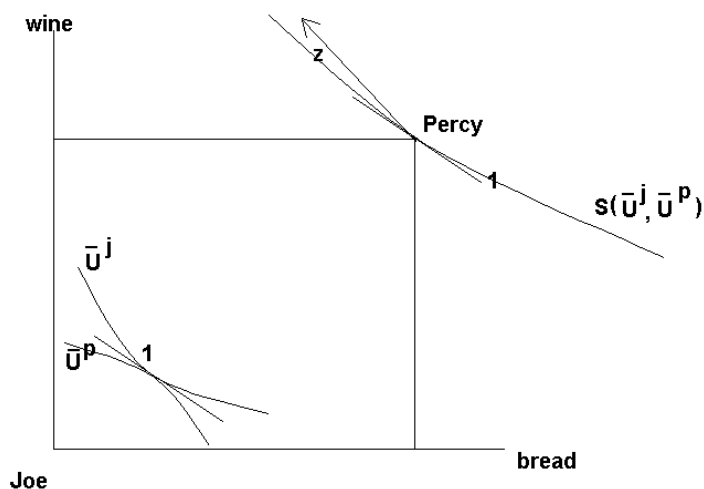
But note that had the original distribution been at 2 instead of at 1, then the slope of the Scitovsky contour through the aggregate bundle would be altered to 2 and z would no longer be Pareto improving. That is, the potential desirability of a change in the aggregate bundle depends upon the original distribution of utility.
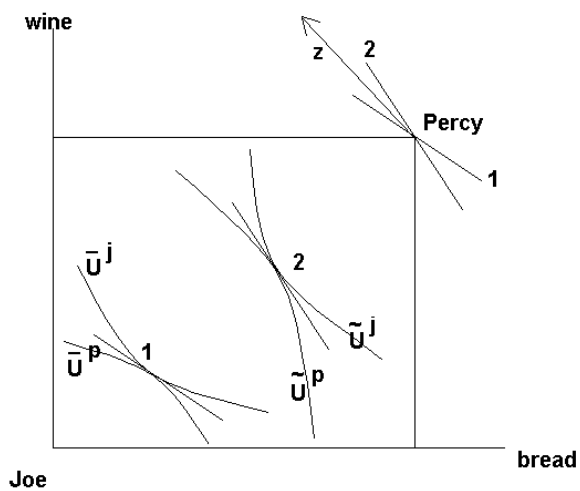
Where market prices reflect a common marginal rate of substitution, z is in a direction above the Scitovsky contour if the inner product of the price vector and z is positive, that is, if benefits minus costs for the project (measured at market prices) is positive. But with a different underlying distribution of utility, market prices would (in general) be different and consequently the sign of the difference between benefits and costs could change.