# Economics 101A Lecture Notes on Convexity and Competitive Equilibrium

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Abstract. Convexity is closely connected to continuity of the excess demand function - a condition used in establishing the existence of competitive equilibrium.

The proof of the existence of competitive equilibrium required that the excess demand function be continuous in prices. The proof did not require a description of where the excess demand function came from, but for a competitive economy, this condition for continuity is closely tied to convexity.

If the individuals' indifference curves are not quasi-concave (i.e. exhibit diminishing marginal rates of substitution) then a small change in prices in the neighborhood of the nonconvexity could result in a discontinuous change in demand. In the example below, a change of the budget line from I to II, brought about by a fall in the price of good 1, cause demand to shift from x^1 to x^2.
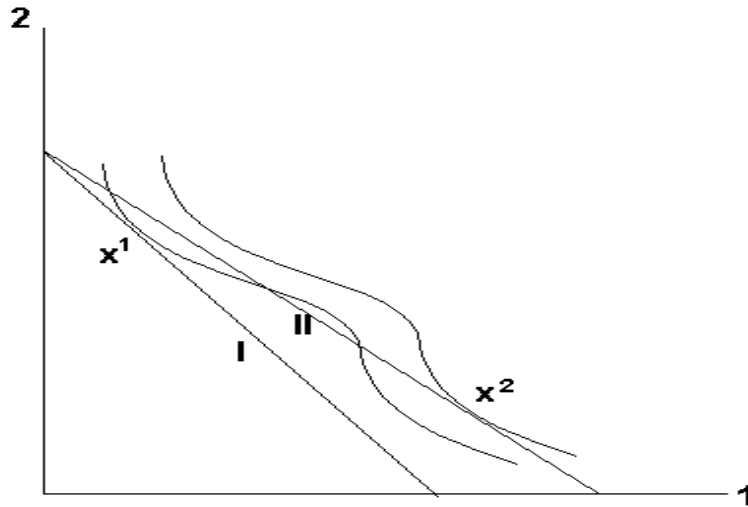
If the firms' production sets are not convex, then a small change in prices, again near the nonconvexity, could result in a discontinuous change in supply. A fall in the ratio of the price of factor x to output y from I to II results in a change of the net output vector from $(-x^1, y^1)$ to $(-x^2, y^2)$.
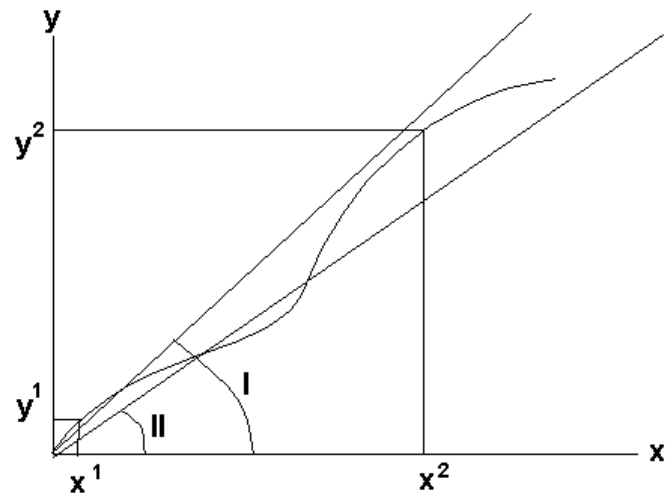
In the Robinson Crusoe model below, a competitive equilibrium does not exist. Convexity insures that such discontinuities will not arise.
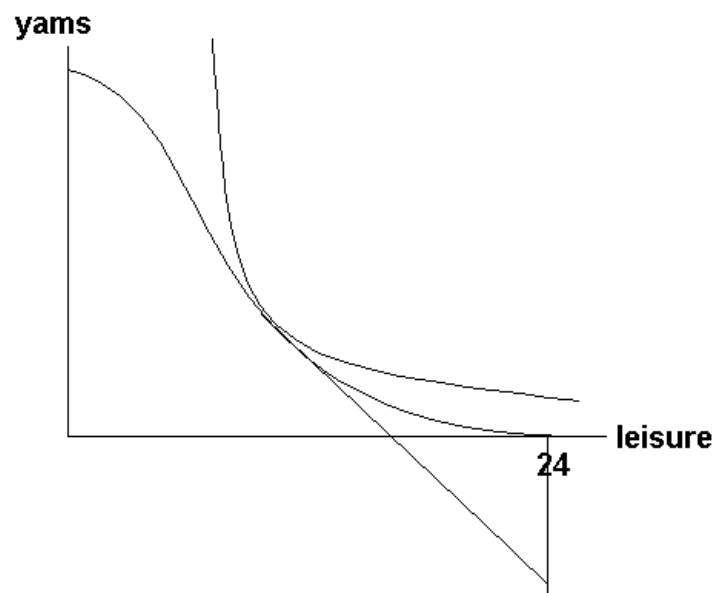
The "competitive equilibrium" at the point of tangency of Robinson's indifference curve and production possibility set would entail negative profits and the firm would simply shut down.