Economics 101A Notes on the Efficiency of Competitive Equilibria

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Abstract. This note describes the conditions for Pareto Optimality and explains (briefly) why they are satisfied under perfect competition.

1. The Bator conditions for Efficiency

1. *Distributive Efficiency*: The Equality of MRS's among consumers for the same pair of goods. Were the marginal rates of substitution different among consumers (for any pair of goods) then the individual who valued one of the goods relatively less could trade with the individual who valued that good relatively more at terms which would be preference improving for both. This is reflected by, say, a point in an Edgeworth box where the indifference curves are not tangent. Note that such a trade would not require a change in total production or in the consumption of any other agent.

   The formal problem can be characterized as maximizing the utility of one agent given the total goods available and the utility level of the other: i.e.

$$\max U^B(x_1^B, x_2^B) \text{ s.t. } U^A(x_1^A, x_2^A) \geqq \overline{U^A}, x_1 \geqq x_1^B + x_1^A, x_2 \geqq x_2^B + x_2^A$$

   where x_j^i denotes $i's$ holding of commodity j and x_j denotes the total holding of j between the two parties. The first order conditions reveal that efficiency requires equating the marginal rates of substitution. Now

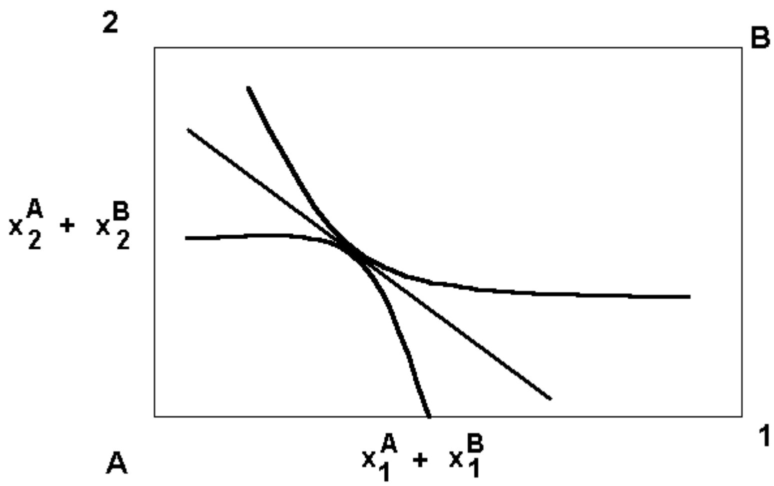
$$L(x,\lambda) = U^B(x_1^B, x_2^B) + \lambda_A(U^A(x_1^A, x_2^A) - \overline{U^A}) + \lambda_1(x_1 - x_1^B + x_1^A) + \lambda_2(x_2 - x_2^B + x_2^A)$$

   so the first order condition imply

$$U_i^B(x_1^B, x_2^B) = \lambda_i \text{ for } i = 1,2$$

$$\lambda_A U_i^A(x_1^A, x_2^A) = \lambda_i \text{ for } i = 1,2$$

   and therefore $\frac{U_i^j(x_1^j, x_2^j)}{U_k^j(x_1^j, x_2^j)} = \frac{\lambda_i}{\lambda_k}$ where j is any individual and i and k are any two goods. That is, $MRS_{i,k}^j = \frac{\lambda_i}{\lambda_k}$ and hence is independent of j.



2. *Productive Efficiency:* The Equality of the RTS's among different firms for the same pair of inputs. This is virtually identical in a formal sense to the above. Here, a difference in the RTS's would enable the firms to exchange some of their inputs so as to increase the production of both goods with the same total resources.

$$\max F^B(x_1^B, x_2^B) \text{ s.t. } F^A(x_1^A, x_2^A) \geqq Y_A, x_1 \geqq x_1^B + x_1^A, x_2 \geqq x_2^B + x_2^A$$

where x_j^i denotes firm $i's$ use of input j and x_j denotes the total use of j between the two firms. The first order conditions reveal that efficiency requires equating the marginal rates of substitution. Now

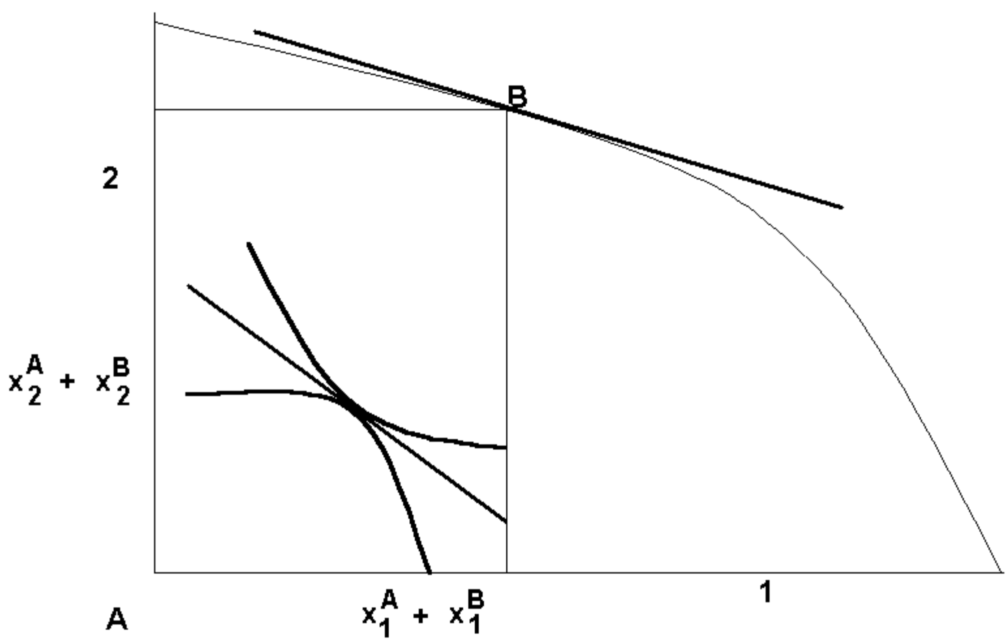
$$L(x, \lambda) = F^B(x_1^B, x_2^B) + \lambda_A(F^A(x_1^A, x_2^A) - Y_A) + \lambda_1(x_1 - x_1^B + x_1^A) + \lambda_2(x_2 - x_2^B + x_2^A)$$

so the first order condition imply

$$F_i^B(x_1^B, x_2^B) = \lambda_i \text{ for } i = 1, 2$$
$$\lambda_A F_i^A(x_1^A, x_2^A) = \lambda_i \text{ for } i = 1, 2$$

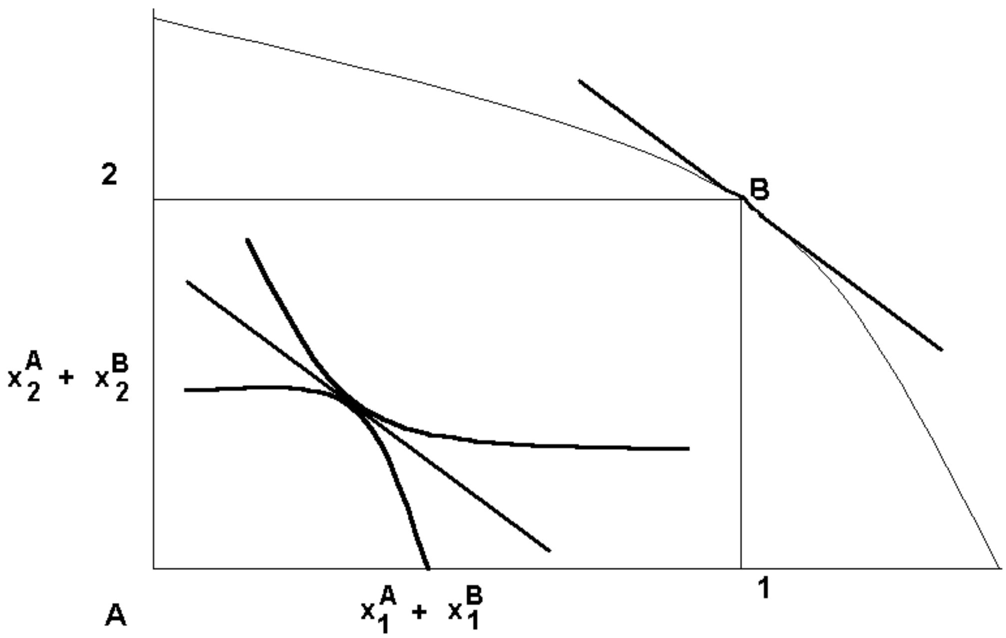
and therefore $\frac{F_i^j(x_1^j, x_2^j)}{F_k^j(x_1^j, x_2^j)} = \frac{\lambda_i}{\lambda_k}$ where j is any individual and i and k are any two goods. That is, $RTS_{i,k}^j = \frac{\lambda_i}{\lambda_k}$ and hence is independent of j.

3. *Allocative Efficiency:* The Equality of the RPT with the MRS. Satisfying Distibutive and Productive Efficiency do not guarantee Paretio Optimality. It may still be the case that by efficiently producing a different aggregative bundle, there would be a distibution of that bundle which could render everyone better off.



In the figure above, producing more of good 1 and less of good 2 woulkd allow both consumers to be better off. The rate at which the "economy" can turn one good into another must match the (common) rate at which all individuals are willing to make the conversion. Were it to differ, an individual could, in effect, trade with the economy. The RPT is described by the rate at which the output of one good can be increased with the resources freed by a decrease in the production of another good. That is, once again, consider the Lagrangian expression in the problem immediately above. The Lagrangian multiplier associated with the first constraint, λ_A, measures the additional amount of good B which could be produced were the output of good A to decline by a unit. Since $F_i^B(x_1^B, x_2^B) = \lambda_i = \lambda_A F_i^A(x_1^A, x_2^A)$ for $i = 1, 2$, λ_A is simply equal to $\frac{F_i^B(x_1^B, x_2^B)}{F_i^A(x_1^A, x_2^A)}$

for any factor i. For efficiency, this ratio must equal $\frac{U_B^j(x_1^j,x_2^j)}{U_A^j(x_1^j,x_2^j)}$ where A and B may be either 1 or 2.



2. Competition

Under competition, consumers equate their MRSs to the price same price ratio, thus satisfying condition 1. Firms equate their RTSs to the ratio of factor prices, satisfying condition 2. Finally, since firms equate the value of the marginal product of each factor to its factor price, the ratio of the marginal physical product of the same factor across firms must equal the ratio of the output prices, i.e.

$$p_A F_i^A(x_1^A, x_2^A) = p_i = p_B F_i^B(x_1^B, x_2^B) \text{ so } \frac{F_i^A(x_1^A, x_2^A)}{F_i^B(x_1^B, x_2^B)} = \frac{p_B}{p_A}$$

Since this is also equal to the individuals MRS between A and B, the third condition is satisfied as well.