Economics 101A Notes on Externalities

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Abstract

This note discusses examples of externalities in consumption and production

Consider the following externalities problem:

Suppse that $A's$ utility depends upon her consumption of goods 1 and 2 while $B's$ depends upon his consumption of goods 1 and 2 along with $A's$ consumption of good 1. This latter dependence may be either positive or negative. That is,

$$U^A = U^A(x_1^A, x_2^A) \text{ and } U^B = U^B(x_1^B, x_2^B, x_1^A)$$

Our problem of efficiency is to find conditions under which a given bundle (x_1, x_2) iz divided efficienctly between the two parties.

The appropriate Lagrangian condition looks like

$$L = U^A(x_1^A, x_2^A) + \lambda_B(U^B(x_1^B, x_2^B, x_1^A) - \overline{U}^B) + \lambda_1(x_1 - x_1^A - x_1^B) + \lambda_2(x_2 - x_2^A - x_2^B)$$

and the corresponding FOC's become

$$\begin{aligned}
U_1^A(x_1^A, x_2^A) + \lambda_B U_3^B(x_1^B, x_2^B, x_1^A)) - \lambda_1 &= 0 \\
U_2^A(x_1^A, x_2^A) - \lambda_2 &= 0 \\
\lambda_B U_1^B(x_1^B, x_2^B, x_1^A)) - \lambda_1 &= 0 \\
\lambda_B U_2^B(x_1^B, x_2^B, x_1^A)) - \lambda_2 &= 0 \\
& \text{and of course} \\
x_1^A + x_1^B &= x_1 \\
x_2^A + x_2^B &= x_2
\end{aligned}$$

Thus

$$\frac{U_1^B(x_1^B, x_2^B, x_1^A)}{U_2^B(x_1^B, x_2^B, x_1^A)} = \frac{\lambda_1}{\lambda_2} = \frac{U_1^A(x_1^A, x_2^A) + \lambda_B U_3^B(x_1^B, x_2^B, x_1^A))}{U_2^A(x_1^A, x_2^A)}$$

If $U_3^B(x_1^B, x_2^B, x_1^A) > 0$ then the RHS of the above is greater than $A's$ MRS between goods 1 and 2 and efficiency requires that A value good 1 more than B. But since both A and B face the same prices in competition, this effieincy condition would fail under the usual market conditions.

Efficiency requires that an added unit of the consumption of good 1 take into account both the direct benefit to A **and** the added benefit it brings to B as well.

The following example depicts the inefficiency of equal MRS's (the market solution) when B cares positively about $A's$ consumption of good 1. In the figure below, consider A with an initial allocation at M yielding a utility of $\overline{U}^A$ and B at P with an initial utility of $\overline{U}^B$. Suppose that $A's$ allocation is changed by a small movement in the direction of her MRS to N. $B's$ allocation is changed in the opposite direction from P to O. If the movements are small, the utility levels are virtually unchanged. But since $B's$ utility depends upon $A's$ consumption of good 1, $B's$ indifference curve at the level $\overline{U}^B$ is shifted towards the origin and the movement to O would actually move above the shifted indifference curve *even for small changes!*.

Of course, a similar story could be told with two firms and two factors of production, where the use of one of the inputs by firm A changed the isoquants of firm B.

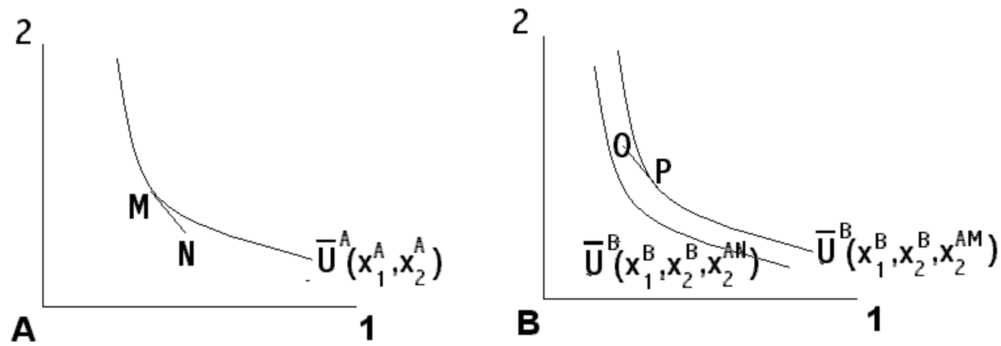
2
M
N
$\overline{U}^A(x_1^A, x_2^A)$
A
1
2
O
P
$\overline{U}^B(x_1^B, x_2^B, x_2^{AN})$
$\overline{U}^B(x_1^B, x_2^B, x_2^{AM})$
B
1

Figure 0.1: