Lecture 4

Family Production Function

In class, we will cover new theories of demand. Readings will be used as the basis for overviewing general theory.

The structure of the lecture is as follows:

1. Criticism of existing demand theory.

2. Alternative consumer theories.

   (a) Lancaster

   (b) Becker

   Synthesis and implication and potential use.

3. Hedonic prices.

4.1 Demand Analysis

Main results of demand analysis:

1. Slutski equation:

$$\frac{dq_i}{dP_i} = \left.\frac{dq_i}{dP_I}\right|_{u=\bar{u}} x_i \frac{dq_i}{dI}, \qquad \left.\frac{dq}{dP}\right|_{u=\bar{u}} < 0$$

2. Symmetry of cross–price effects:

$$\left.\frac{dq_i}{dP_j}\right|_{u=\bar{u}} = \left.\frac{dq_j}{dP_i}\right|_{u=\bar{u}}$$

3. Definitions:

$$\eta_I = \frac{dq}{dI}\frac{I}{q} \equiv \text{income elasticity;} \quad \begin{cases} \eta_I > 1 & \text{luxury good} \\ 0 < \eta_I < 1 & \text{normal good} \\ \eta_I < 0 & \text{inferior good} \end{cases}$$

$$\eta_P = \frac{dP}{dq}\frac{q}{P} \equiv \text{price elasticity;} \quad \begin{cases} \eta_P > 0 & \text{Giffen good} \\ 0 > \eta_P > 1 & \text{inelastic demand} \\ \eta_P > 1 & \text{elastic demand} \end{cases}$$

4.1.1 Criticism of Existing Demand Theory

Lancaster Criticism

Lancaster criticized the standard economic theory of demand on several points.

1. The theory is elegant–general and vacuous. Pursuit of generality resulted in a minimal set of assumptions and a minimal set of results.
   Assumptions –Utility maximization, income constraint, quasi-concave utility function. *Only prediction* –Compensated demand curves are negatively sloped.

2. Because of pursuit of generality, all goods are treated alike. Everything that people want more of are goods. No attention is given to intrinsic properties of goods properties that separate bread from bicycles.

3. The theory assumes that preferences are given and does not try to explain variation in preferences.

4. The theory can explain observed patterns in a static primitive economy. It cannot deal with modern interesting issues such as prediction of demand for new products and understanding effects of product quality differences.

Is Lancaster right? Almost. Lancaster criticisms are somewhat harsh. The notions of substitutes and complements and the notions of separability were all attempts to enrich the power of traditional demand theory. These attempts, however, are weak. They do not take into account completely intrinsic properties. They are not useful for explaining product innovation and quality changes.

Michael and Becker Critiques

Demand theories use income and prices as the main explanatory variables for consumption patterns. Whatever is not explained by income and prices is explained by tastes. However,

1. (a) Income and price variations are found to explain only a small part of the variation of family consumption pattern.
   (b) Even in group studies, prices and income are not good explanatory variables and taste has an important role.
   (c) There is no theory of formation of taste.

2. A traditional demand theory explains consumption patterns of goods purchased in market and that imposes an important limitation on phenomenon investigation.
   (a) Applications are restricted to market segments or monetary segments of economy. A lot of activities which are not market activities are ignored. This limits the usefulness of theory to developed nations and reduces its effectiveness in developing nations, in which markets are incomplete or absent.
   (b) According to Becker, economics is a science which explains behavioral choices with limited resources among competing ends and includes, even in modem economy, non–market choices which includes
      i. Family size decisions.
      ii. Lifestyles and occupation choices.
      iii. Political choices.
      iv. Leisure choices.

Becker expanded the range of issues analyzed by economists to include some of life's basic choices: marriage, crime, suicides, migration, etc. His works are insightful but show the limitation of the notion of "economic man." A better understanding of choices and behavior may need to incorporate more from psychology and sociology into economic models.

Becker was a student of T.W. Schultz. Schultz coined the term "human capital" and started work on economics of education and learning. He emphasizes the study of behavior outside the market and in situations of disequilibrium. Another student of Schultz, Griliches, started formal economic research on technology adoption.

Conclusion of Critiques of traditional demand theory

Theory is limited because:

- it ignores intrinsic properties of goods;
- leaves too much explanation to "taste"

- cannot deal with important phenomenon such as

    1. Nonmarket activities.
    2. Product innovation.
    3. Product quality differences.

4.2 Alternative Approaches to Demand Analysis

Assume:

1. Purchased goods are not sources of utility *per se*. Other things –call them characteristics, attributes, needs, and commodities– are source of happiness. These items may be unobservable and abstract.

2. There is a production process within the consumption unit. This process transforms goods and other inputs into sources of utility.

There are several differences between the models proposed by Lancaster and by Michael and Becker:

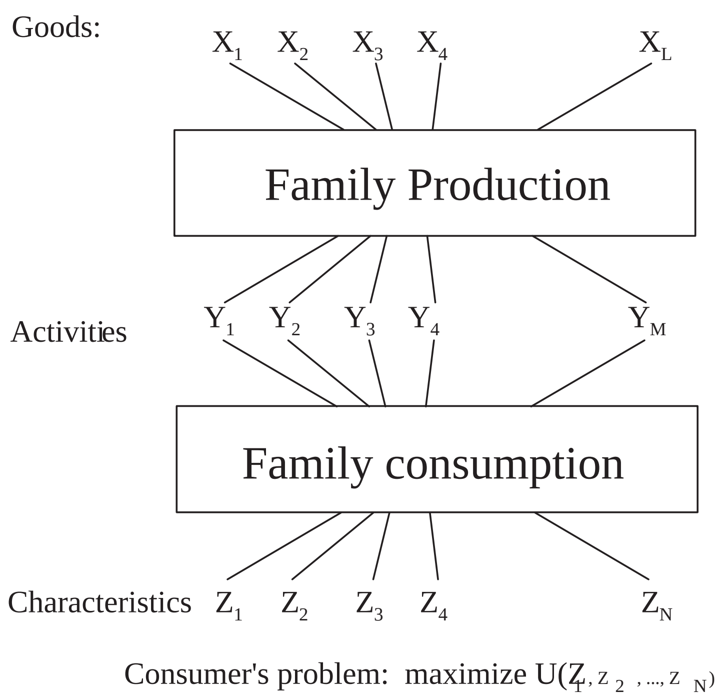
- The sources of utility are treated and defined differently:

    - Lancaster's *characteristics* are abstract: brightness, sweetness, beauty.
    - Becker's *commodities* are more concrete and physical. They distinguish between:
        - An omelet cooked at home and an omelet at a restaurant.
        - A salad vs. just tomato and lettuce.

- The models are different in the nature of production technology.

    1. - Lancaster technologies are linear. The production process includes three elements: *activities* which combine *goods* to yield *characteristics*.
       - In Becker, production process assumes a neoclassical production function. However, the production process is in two stages: Time + goods produce commodities.
    2. - In Lancaster, the emphasis is on intrinsic, atomistic elements in goods and on the actual enjoyment of production.
       - Becker cares more about family production and emphasizes the role of time.

Figure 4.1: Lancaster Model



Consumer problem

$$\max_{X,Y} U(Z_1, Z_N)$$

s.t.

$$Z \equiv \begin{pmatrix} Z_1 \\ \vdots \\ Z_N \end{pmatrix} = BY \equiv \begin{pmatrix} b_{11} & \cdots & b_{1M} \\ \vdots & \ddots & \vdots \\ b_{N1} & \cdots & b_{NM} \end{pmatrix} \begin{pmatrix} Y_1 \\ \vdots \\ Y_M \end{pmatrix} \tag{4.1}$$

where b_{ij} = amount of characteristic i in activity j

$$X \equiv \begin{pmatrix} X_1 \\ \vdots \\ X_L \end{pmatrix} = AY \equiv \begin{pmatrix} a_{11} & \cdots & a_{1M} \\ \vdots & \ddots & \vdots \\ a_{L1} & \cdots & a_{LM} \end{pmatrix} \begin{pmatrix} Y_1 \\ \vdots \\ Y_M \end{pmatrix} \tag{4.2}$$

where a_{lm} = amount of input l in activity m

$$\sum_{l=1}^{L} P_l X_l = \sum_{l=1}^{L} P_l \sum_{m=1}^{M} a_{lm} Y_m \leq K, \tag{4.3}$$

where K = income.

Some Results

1. The relations between characteristics and goods depend on their numbers.

   (a) There are N characteristics, M activities and L goods.
   Only when $M = N = L$ is there a one-to-one relationship and then

   $$Y = A^{-1}X$$
   $$L = BA^{-1}X$$
   $$U(Z) = U(BA^{-1}X) = U(X)$$

   and we have the traditional analysis.

   (b) Generally, $N \neq M \neq L$. There are many paths connecting goods to characteristics.

2. Assume that goods and activities are identical. Thus, we have

   $$\max U(Z) \qquad \text{s.t. } Z = BX, \qquad PX \leq K$$

   or

   $$\max U(BX) \qquad \text{s.t. } PX \leq K$$

   B is the consumption technique.

(a) If $N > M$ –more characteristics than goods– we have a *primitive society.* In this case, not all consumption combinations are reachable. There is a relatively small number of goods that cannot meet many needs, independent of income constraint.

(b) In any advanced economy, $M > N$: there are more goods than characteristics. Consumers can attain any characteristic combination in many ways. Here the consumer has to make two choices.

   i. Efficiency choices: Objective determination of the best way to produce a characteristic combination. This is a linear programming decision dependent on income and prices. For a given Z^*, it yields

   $$\min PX \qquad \text{s.t. } Z^* = BX, \qquad X > 0$$

   Repetition of this choice can yield the characteristic frontier (figure 5.2)

   ii. Private consumption choice: A selection of consumption point on the characteristic frontier. This choice depends on taste and is subjective.

Extension of Comparative Static Results

Change in price may have two effects: *objective* effects (changes in characteristic frontier) and *subjective* effects.

Suppose X_2 becomes cheaper. Then we move from $ABCD$ to AED. Before the reduction in price of X_2, good X_3 was consumed; now it is not in the optimal solution and it is not used. Thus, the efficient substitution effect eliminates the consumption of X_3 in our case and causes a switch. Thus, negative demand is not only a result of concave utility function but also of a certain consumption and production technology set. It is not only justified by one's taste but by all the people's consumption technology. Of course, in our case the switching effect eliminates the use of good 3 in cases when we have many goods and many activities –activity 3 will be eliminated but consumption of all goods only decreases. To summarize, one has to distinguish between:

- *Efficiency substitution*: A bundle of characteristics may be produced better by a different set of techniques. This is an objective substitution.

- *Private substitution*: A change in prices changes the characteristic frontier, and then there is a change in consumption within a given technology reflecting utility and preference.

These changes are complementing one another, and they strengthen the notion of negatively sloped demand.

Figure 4.2: The Characteristic Frontier

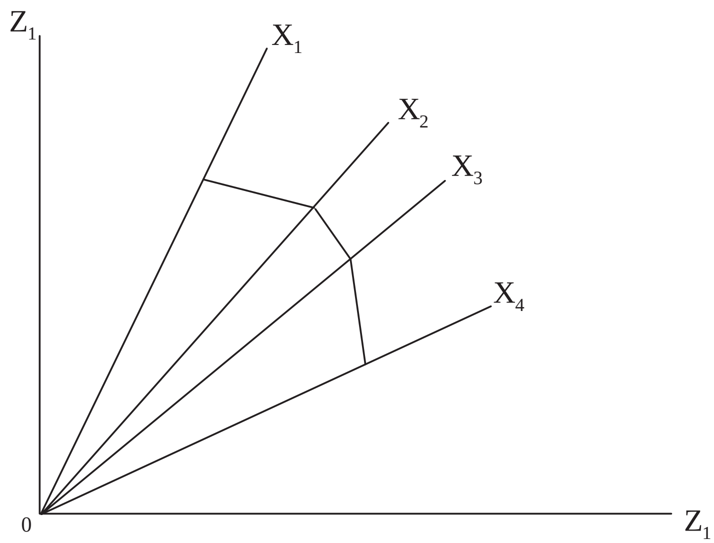
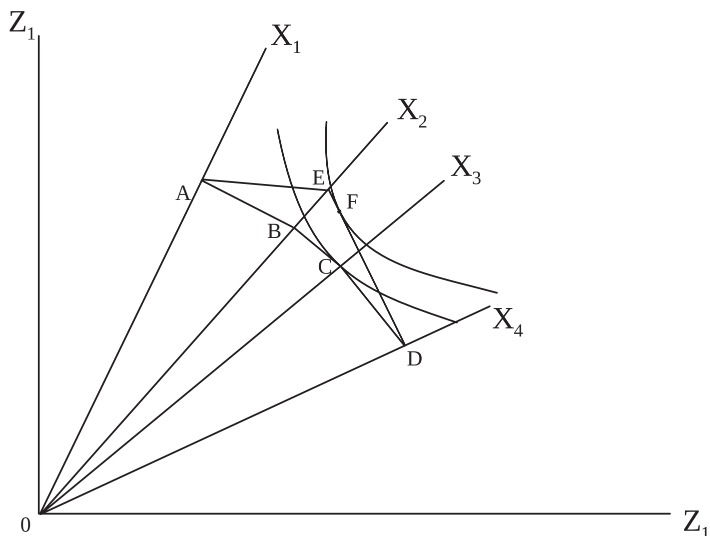


Figure 4.3: Effect of a price change



Use of the Model

The model can be used to forecast the demand for a new product. A new product is a new vector in the production matrix and changes the feasible characteristic set. It can be easily integrated within the framework.

4.3 Becker Model

Flaws in traditional demand theory.

1. It assumes that utility is derived from raw market goods which ignore the processing of these goods at the household level.
2. It ignores the time and effort allocated to activities within the household.
3. It ignores economic transactions within families.
4. It has very general assumptions but leads to one very general result – compensated demand is negatively sloped.
5. All goods are treated as generic-with no attention to their unique characteristics
6. It does not allow appropriate ways to assess the role of durable goods in consumption. (What is the utility of record players without records?)
7. Only quantifiable explanations to changes in consumption behavior are in terms of price and income effects. These effects count for less than 50% of variability in observed behavior.
8. Theory cannot be used to predict demand to new products. You can estimate demand only to existing goods.
9. Theory is difficult to apply to explain product differentiation and role of quality.

Response to Criticism

1. Theory allows relationships between goods. Some are substitutes and others are complements. Identification of relationships is part of the empirical effort.
2. Empirical models develop function forms that group commodities to allow analysis at different levels of aggregation.

Alternative Frameworks: Lancaster-Becker The Lancaster utility is derived from characteristics. Characteristics are results of activities with the household. Activities use market and non–market goods. Becker distinguishes between goods and commodities. Utility is derived from commodities and leisure:

$$Y = \text{commodity vector}; \qquad y_n = \text{quantity of commodity } n$$

where $n = 1, \ldots, N$. Commodities are generated by the family production function:

$$y_n = f_n(X_n, t_n)$$

where $X_n = (x_{n,1}, \ldots, x_{n,m})$ vector of levels of m inputs used in producing commodity n and t_n is the amount of time used in producing n.

$$\text{Total consumption of input m } \bar{x}_m = \sum_{n=1}^{N} x_{nm}$$

The consumer choice problem is

$$\max U\left(t_0, f_1(x_1, t_1), \cdots, f_n(x_n, t_n)\right) \tag{4.4}$$

s.t.

$$\sum_{m=1}^{M} \sum_{n=1}^{N} P_m x_{mn} = sum_{m=1}^{M} P_m \bar{x}_m = wt_l + I, \qquad \text{Income constraint} \tag{4.5}$$

$$\sum_{n=0}^{N} t_n + t_l = T, \qquad \text{Time constraint} \tag{4.6}$$

where T denotes total time constraint; t_l, labor time; and t_0, leisure time. The utility function is $U(t_0, y_1, \ldots, y_N)$; the price of good m is P_m; labor wage is W; and the consumer has non–labor income, I.

The two constraints (4.5) can be combined

$$\sum_{m=1}^{M} \left[P_m \sum_{n=1}^{N} x_{mn} + wt_n \right] + wt_o = I + wT \tag{4.7}$$

Equation (4.7) is the *full-income* constraint. Let λ be the shadow price of the full-income constraint. The F.O.C.'s are

$$\frac{\partial L}{\partial x_{mn}} = U_n f_{mn} - \lambda P_m = 0, \qquad m = 1, \ldots M, \quad n = 1, \ldots, N \tag{4.8}$$

$$\frac{\partial L}{\partial t_0} = U_{t_0} - \lambda w = 0 \tag{4.9}$$

$$\frac{\partial L}{\partial t_n} = U_n f_{nt} - \lambda w = 0, \qquad n = 1, \ldots, N \tag{4.10}$$

Some of the implications of the model are:

1. Solution of the system provides the demands for $y_n, \bar{x}_m, x_{mn}, t_n, t_0$ and supply of labor t_l. Demand for y_n can be presented as a function of P_m, W, I, and technology parameters or as a function of shadow price of y_n, ϕ_n where, for all m,
$$\phi_n/\lambda f_{nm} = P_m$$
and
$$\phi_n/\lambda f_{nt} = W$$
The conditions, in essence, state that the value of marginal product of inputs equals input prices.

2. Total demand for a good (seen as an input in commodity production) is the sum of demands in all commodities. The demand for goods depends on decomposition of income between wage and non–wage income. Wage affects the demand of goods which are substitutes or complements of labor in production of commodities.

3. The model can be expanded to include several family members with different wage rates and to address labor allocations within the family.

4.4 The Lancaster model when $M = N = L$

The model assumes $Z = BY; X = AY \Rightarrow Y = A^{-1}X$; and $Z = BA^{-1}X$ where $Z = N \times 1$ and BA^{-1} is a matrix of dimensions $N \times N$:

$$Z = \begin{pmatrix} b_{11} & \cdots & b_{1N} \\ \vdots & \ddots & \vdots \\ b_{N1} & \cdots & b_{NN} \end{pmatrix} \begin{pmatrix} a_{11} & \cdots & a_{1M} \\ \vdots & \ddots & \vdots \\ a_{N1} & \cdots & a_{NM} \end{pmatrix}^{-1} \begin{pmatrix} X_1 \\ \vdots \\ X_N \end{pmatrix}$$

or

$$Z_n = \sum_{j=1}^{N} \sum_{l=1}^{N} b_{nj} a_{jl}^{-1} X_l$$

The optimal resource allocation problem is

$$\max_X U(BA^{-1}X) \qquad \text{s.t. } P'X = I$$

It can be presented in Lagrangian form:

$$L = \max_X U(BA^{-1}X) + \lambda[I - P'X]$$

where λ is the shadow price of the income constraint. The first order conditions are

$$\frac{\partial L}{\partial X_k} = \sum_{n=1}^{N} \frac{\partial U}{\partial Z_n} \frac{\partial Z_n}{\partial X_k} - \lambda P_k = 0 \qquad k = 1, \ldots, N$$

$$\frac{\partial L}{\partial \lambda} = I - P'X = 0$$

Since $\frac{\partial Z_n}{\partial X_k} = \sum j = 1^N b_{nj} a_{jk}^{-1}$, the FOC is

$$\frac{\partial L}{\partial X_k} = \sum_{n=1}^{N} \frac{\partial U}{\partial Z_n} \sum j = 1^N b_{nj} a_{jk}^{-1} - \lambda P_k = 0 \qquad k = 1, \ldots, N$$

Which can be rewritten as:

$$\sum_{n=1}^{N} \underbrace{\frac{1}{\lambda} \frac{\partial U}{\partial Z_n}}_{\substack{\text{marginal} \\ \text{value of} \\ \text{characteristic } n}} \times \underbrace{\sum_{j=1}^{N} b_{nj} a_{jk}^{-1}}_{\substack{\text{marginal productivity} \\ \text{of input } k \text{ summed} \\ \text{in producing characteristic} \\ n \text{ in all the activities}}} = P_k$$

An alternative solution. Find the optimal Z. If $Z = BA^{-1}X$, then $X = B^{-1}AZ$. The optimization problem becomes

$$\max_Z U(Z) \quad \text{s.t. } I = P'B^{-1}AZ$$

$$L_1 = \max_Z U(Z) + \lambda[I - P'B^{-1}AZ]$$

and the F.O.C. are:

$$\frac{\partial L_1}{\partial Z_n} = \frac{\partial U}{\partial Z_n} - \lambda \sum_{k=1}^{N} P_k \sum_{j=1}^{N} b_{nj}^{-1} a_{jk} \qquad n = 1, \ldots, N$$

which can be rewritten as:

$$\sum_{k=1}^{N} \underbrace{P_k \sum_{j=1}^{N} b_{nj}^{-1} a_{jk}}_{\substack{\text{Implied price of} \\ \text{characteristic } j \\ \text{reflecting the value} \\ \text{of input contributing} \\ \text{to its production}}} = \underbrace{\frac{1}{\lambda} \frac{\partial U}{\partial Z_n}}_{\substack{\text{marginal} \\ \text{value of} \\ \text{characteristic } n}}$$

4.5 Strauss Model

This model is relevant for peasants who consume what they produce. Emphasize the role of time and ignore the difference between goods and commodities. Let $U(Y_1, Y_2, t_0)$ be utility. $Q_1 = f(t_l)$ is the quantity of good 1 produced by family. The budget constraint is

$$P_1 Y_1 + P_2 Y_2 = I + P_1 Q_1$$

and the time constraint is

$$t_0 + t_l = T$$

Therefore, the consumer problem is

$$L = \max U[Y_1, Y_2, T - t_l] + \lambda[I + P_1 f(t_l) - P_1 Y_1 - P_2 Y_2] \tag{4.11}$$

The F.O.C.s are

$$\begin{aligned} L_\lambda &= I + P_1 f(t_l) - P_1 Y_1 - P_2 Y_2 = & (4.12)\\ L_{y1} &= U_1 - \lambda P_1 = 0 & (4.13)\\ L_{y2} &= U_2 - \lambda P_2 = 0 & (4.14)\\ L_{tl} &= -U_{t_0} + \lambda P_1 f_t = 0 & (4.15) \end{aligned}$$

This model yields: output supply Q_1, demand for Y_1, Y_2, and leisure t_0, and labor use t_l.

To obtain properties of these relationships, total differentiation of (1) to (4) is required. It will yield

$$\begin{bmatrix} \text{matrix} \\ \text{of second} \\ \text{order} \\ \text{conditions} \end{bmatrix} \begin{bmatrix} dY_1 \\ dY_2 \\ dt_l \\ d\lambda \end{bmatrix} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ \lambda f_1 & 0 & 0 \\ Y_1 - f(t_l) & Y_2 & -1 \end{bmatrix} \begin{bmatrix} dP_1 \\ dP_2 \\ dI \end{bmatrix}$$

which implies

$$\begin{bmatrix} dY_1 \\ dY_2 \\ dt_l \\ d\lambda \end{bmatrix} = \begin{bmatrix} \text{matrix} \\ \text{of second} \\ \text{order} \\ \text{conditions} \end{bmatrix}^{-1} \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ \lambda f_1 & 0 & 0 \\ Y_1 - f(t_l) & Y_2 & -1 \end{bmatrix} \begin{bmatrix} dP_1 \\ dP_2 \\ dI \end{bmatrix}$$

The matrix H^{-1} is 4x4 with elements H_{ij}^{-1}. Note that

$$\frac{dY_2}{dP_2} = \underbrace{\lambda H_{22}^{-1}}_{\text{price effect}} + \underbrace{Y_2 H_{24}^{-1}}_{\text{income effect}}$$

and

$$\frac{dY_1}{dP_1} = \underbrace{\lambda H_{11}^{-1}}_{\text{price effect}} + \underbrace{Y_1 H_{14}^{-1}}_{\text{income effect}} - \underbrace{f(t_l) H_{14}^{-1}}_{\text{revenue effect}} - \underbrace{\lambda f_t H_{13}^{-1}}_{\text{productivity effect}}$$

The increase of price of good 1 will result in two additional elements when it is produced by family. The revenue effect operates in the opposite direction of income effect and productivity effect.

4.6 Hedonic Prices

According to Lancaster, each individual has a shadow price of characteristics reflecting his family production function, preference, and wealth. Rosen argues that characteristics are purchased in goods and sometimes one cannot construct a combination of characteristics by mixing goods. There are indivisibilities and one has to buy a small number of packages (goods) which determine the characteristic levels. Rosen spoke in terms of *product quality*, $P = P(Z)$, equivalent to a hedonic price function relating output to quality level Z. This function is called a **hedonic price function** and is a "reduced form" relationship combining supply and demand information. For example, when one buys a house, he has to choose one quality. His utility function is

$$\max U(Z, E)$$

where $E = I - P(Z)$ is expenditure on all other goods consumed with $P_E = 1$ and $P(Z)$ is the hedonic function. The consumer chooses Z_i so that

$$U_{Z_i} = U_E P_{Z_i} \qquad \text{or} \qquad P_{Z_i} = \frac{U_{Z_i}}{U_E} \tag{4.16}$$

It is reasonable that $P_{Z_i} > 0$. Rosen argues that $P_{Z_i Z_i}$ is likely to be positive. It is also likely that richer people will choose higher quality Z. To see the last point, note that total differentiation of (4.16) yields

$$X P_{Z_i} C_{Z_i} = 0$$

If each production unit produces only one quality with the cost function $C(X, Z)$, then its outcome is determined by

$$\max_{Z,X} X P(Z) - C(X, Z) \tag{4.17}$$

whose FOC are

$$X P_Z - C_Z = 0 \tag{4.18}$$
$$P(Z) - C_X = 0 \tag{4.19}$$

Thus, for a given $P(Z)$ function, there are quantities demanded and quantity supplied at every Z and when, $Q^D(Z, P(Z)) = Q^S(Z, P(Z))$, we have an equilibrium.

There is much literature on quality –both theoretical and empirical. One area of emphasis in theoretical literature is uncertainty about qualities and the role of the market as conveyers of signals regarding quality. Much of the literature on search addresses quality issues. Brand names are conveyers of quality. Another line of research addresses the condition for existence of equilibrium in the case of products with varying quality. Obviously, when products have many qualities, the market structure may be of monopolistic competition.

Empirical studies wrestle with definition and measurement of quality. First, quality data rarely exist. Second, quality grading, whenever it exists, may vary over time. Third, it is not clear what the relevant quality characteristics are –observed ones (color of fruits) or inherit ones (sugar content).

Most frequent applications: housing values and property values.

Others: Recreation demand, grading of agricultural commodity, and policy problems.