Review Questions for Midterm 2 (Solution)

(1)

(a) (i) Normal Good: A good is a normal good if demand for the good increases as income increases.

(ii) Giffen Good: A good is a Giffen good if demand for the good increases as the price of this good increases.

(iii) Constant Returns to Scale: A given technology exhibits CRS if all inputs are increased by a common factor (α), output increased by the same factor (α)

(iv) Factor Demand Function relates the firm's profit-maximizing choice of an input as a function of all prices (p, w_1, w_2)

(b) Perfect Substitutes technology : $f(x_1, x_2) = \frac{1}{2}x_1 + \frac{1}{3}x_2$ Let x_1 = labor, x_2 = capital

2 units of labor
3 units of capital → produce same amt of output.

* compare

- cost of 2 units of labor = $2(2) = 4$
- cost of 3 units of capital = $3(1) = 3$

} use capital only.

$$f(x_1, x_2) = \tfrac{1}{2}(0) + \tfrac{1}{3}(x_2) = 90$$

$$x_1^* = 0 \qquad x_2^* = 270$$

$$\text{Cost} = w_1 x_1^* + w_2 x_2^* = 2(0) + (1)(270) = \underline{\underline{270}}$$

(c)

If $P=5$, $y=\frac{100}{5}=20$

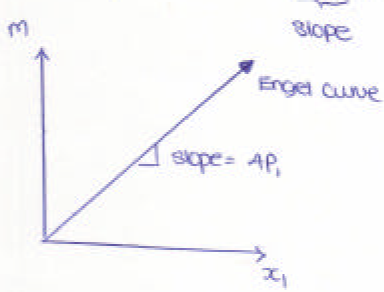
(d) All true

(2)

(a) $\frac{dx_1}{dm}=\frac{1}{4P_1}>0$ Normal

(b) $\frac{dx_2}{dP_2}=-\frac{3m}{4P_2^2}<0$ Ordinary

(c) $x_1=\frac{m}{4P_1} \rightarrow m=\underbrace{(4P_1)}_{\text{slope}}x_1$

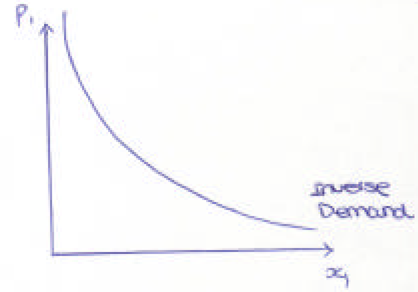


(d)

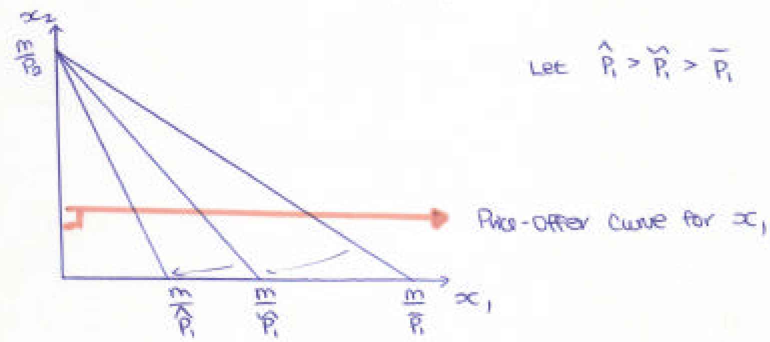
Demand: $x_1 = \frac{m}{4P_1}$

$4P_1 = \frac{m}{x_1}$

Inverse Demand: $P_1 = \frac{m}{4x_1}$



(e) Price-Offer Curve for Ice Cream (ΔP_1)



Let $\hat{P}_1 > \check{P}_1 > \tilde{P}_1$

(3)

(a) $f(x_1, x_2) = \text{Min}\{\frac{1}{5}x_1, x_2\} = 20$

At min: $\frac{1}{5}x_1 = 20 \rightarrow x_1 = 100$

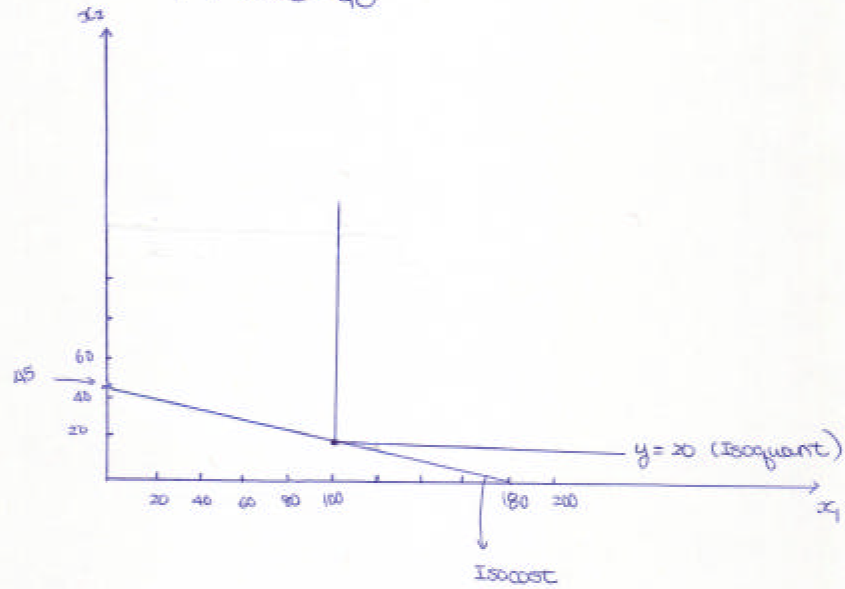
$x_2 = 20$

(b) $C(w_1, w_2, 20) = 0.5(100) + (2)(20) = 50 + 40 = 90$

(c)

Isoquant: $\text{Min}\{\frac{1}{5}x_1, x_2\} = 20$

Isocost: $0.5x_1 + 2x_2 = 90$



(4)(a) SR production function: $f(x_1, 1) = x_1^{1/2}$

$$MP_1 = \tfrac{1}{2} x_1^{-1/2}$$

$$P \cdot MP_1 = W_1 \Rightarrow P \cdot \left(\tfrac{1}{2} x_1^{-1/2}\right) = W_1$$

$$x_1^{-1/2} = \frac{2W_1}{P}$$

$$x_1^{1/2} = \frac{P}{2W_1}$$

$$x_1^* = \left(\frac{P}{2W_1}\right)^2$$

(b) $f(2x_1, 2x_2) = (2x_1)^{1/2}(2x_2)^{1/3} = 2^{1/2} x_1^{1/2} 2^{1/3} x_2^{1/3}$

$$= 2^{5/6} x_1^{1/2} x_2^{1/3}$$

$$= 2^{5/6} y < 2y$$

Decreasing Returns to Scale

(c) Profit-max conditions.

① $P \cdot MP_1 = W_1 \rightarrow P \cdot \left(\tfrac{1}{2} x_1^{-1/2} x_2^{1/3}\right) = W_1 \rightarrow P x_1^{1/2} x_2^{1/3} = 2W_1 x_1$

$\rightarrow Py = 2W_1 x_1$

$\rightarrow x_1 = \frac{Py}{2W_1}$

② $P \cdot MP_2 = W_2 \rightarrow P\left(\tfrac{1}{3} x_1^{1/2} x_2^{-2/3}\right) = W_2 \rightarrow P x_1^{1/2} x_2^{1/3} = 3W_2 x_2$

$\rightarrow Py = 3W_2 x_2$

$\rightarrow x_2 = \frac{Py}{3W_2}$

$$y = \left(\frac{Py}{2W_1}\right)^{1/2} \left(\frac{Py}{3W_2}\right)^{1/3}$$

$$= \left(\frac{P}{2W_1}\right)^{1/2} \left(\frac{P}{3W_2}\right)^{1/3} y^{5/6}$$

$$y^{1/6} = \left(\frac{P}{2W_1}\right)^{1/2} \left(\frac{P}{3W_2}\right)^{1/3}$$

$$y^* = \left(\frac{P}{2W_1}\right)^3 \left(\frac{P}{3W_2}\right)^2$$

$$\begin{cases} x_1^* = \dfrac{P}{2W_1} \left(\dfrac{P}{2W_1}\right)^3 \left(\dfrac{P}{3W_2}\right)^2 \\ x_2^* = \dfrac{P}{3W_2} \left(\dfrac{P}{2W_1}\right)^3 \left(\dfrac{P}{3W_2}\right)^2 \end{cases}$$

(d) Cost-Minimization Conditions.

① $TRS = -\frac{W_1}{W_2}$

② $x_1^{1/2} x_2^{1/3} = \bar{y}$

$$TRS = -\frac{MP_1}{MP_2} = -\frac{\frac{1}{2} x_1^{-1/2} x_2^{1/3}}{\frac{1}{3} x_1^{1/2} x_2^{-2/3}} = \boxed{-\frac{3x_2}{2x_1} = -\frac{W_1}{W_2}}$$

$$x_2 = \frac{2W_1 x_1}{3W_2}$$

Now,

$$x_1^{1/2}\left(\frac{2W_1 x_1}{3W_2}\right)^{1/3} = \bar{y}$$

$$\left(\frac{2W_1}{3W_2}\right)^{1/3} x_1^{5/6} = \bar{y}$$

$$x_1^{5/6} = \bar{y}\left(\frac{2W_1}{3W_2}\right)^{-1/3} = \bar{y}\left(\frac{3W_2}{2W_1}\right)^{1/3}$$

$$x_1^* = \bar{y}^{6/5}\left(\frac{3W_2}{2W_1}\right)^{2/5}$$

$$x_2^* = \frac{2W_1}{3W_2}\left[\bar{y}^{6/5}\left(\frac{3W_2}{2W_1}\right)^{2/5}\right]$$

(5)

(a) $C_v(y) = y^4 - 3y^2 + 9y$

$F = 10$

$MC(y) = \frac{dC(y)}{dy} = 4y^3 - 6y + 9$

(b) $AVC(y) = \frac{C_v(y)}{y} = y^3 - 3y + 9$

$\frac{d\,AVC(y)}{dy} = 3y^2 - 3 = 0 \rightarrow 3y^2 = 3$

$y^2 = 1$

$\boxed{y = 1}$ → where AVC is at its minimum.

$AVC(1) = (1)^3 - 3(1) + 9 = 7$ ✓

$MC(1) = 4(1)^3 - 6(1) + 9 = 7$ ✓

(c) Firm would shut down if $\boxed{P < \text{min } AVC.}$

From part (b): Minimum $AVC = 7$

⇒ Firm would shut down if $\boxed{P < 7}$

(d) Firm's supply: $P = MC(y) \rightarrow P = 4y^3 - 6y + 9$

$P = 4(2)^3 - 6(2) + 9$

$= 29$

Profit $= Py - C(y)$

$= (29)(2) - [(2)^4 - 3(2)^2 + 9(2) + 10]$

$= 58 - [32]$

$= 26$

Producer Surplus $= Py - C_v(y) = (29)(2) - [(2)^4 - 3(2)^2 + 9(2)]$

$= 58 - 22$

$= 36$