Review Questions for Midterm I (Solution)

(1)

(a) The "Best" bundle the consumer can afford.

(b) As consumer obtain more and more x_1 (or x_2), her willingness to substitute x_2 (or x_1) for an additional unit of x_1 (or x_2) diminishes.

(c) No, utility is an ordinal measure, therefore the utility difference between any two bundles are meaningless. All we know is that this consumer likes A more than B, but not by how much

(d) Given that at X^*: $\frac{MU_1}{MU_2} > \frac{P_1}{P_2} \rightarrow \frac{MU_1}{P_1} > \frac{MU_2}{P_2}$

The consumer would receive more utility per $ if spent on good one than on good two. Since she is already spending all of her income, she can $\uparrow x_1$ by $\downarrow x_2$ to make herself better off.

(e) ① B is a weighted average of A and D.

→ $B = \frac{1}{2}A + \frac{1}{2}D$.

→ By "strict convexity": $B \succ A, D$

② C offers at least as much in both goods as B.

But C offers strictly more good one compared to B.

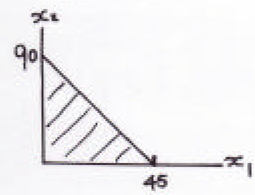
→ By "~~strong~~ monotonicity": $C \succ B$

Taken all together: $C \succ B \succ A, D$.

* The consumer will choose C

(2)

(a) $2x_1 + x_2 \leq 90$



(b) 2 bottles of milk

(c) $MRS = -\frac{MU_1}{MU_2} = -\frac{1}{2}$

(d) No, given that Tim has perfect substitutes preferences, his MRS is fixed (at $-\frac{1}{2}$). Increasing consumption of either good does not diminish his willingness to trade. (It's constant)

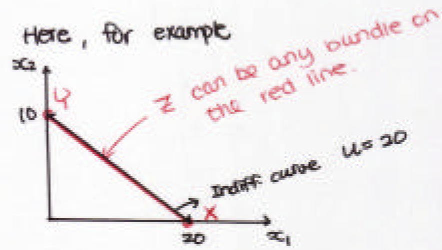
(e) <u>" Strict Convexity "</u>

Def: Given $X, Y : X \sim Y$

Let Z = Any weighted avg of X, Y

$\Rightarrow Z \succ X, Y$

Here, for example



Let $X = (20, 0)$: $U(X) = 20$

$Y = (0, 10)$: $U(Y) = 20$

$Z = (10, 5)$: $U(Z) = 20$

$\Rightarrow Z \sim X \sim Y$

$\Rightarrow$ Not $Z \succ X, Y$

(f) Given MRS = $-\frac{1}{2}$

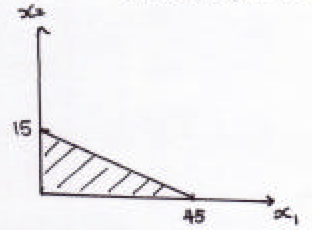
- $(\frac{1}{2})$ units of milk $\backsim$ (1) unit of juice
- $\frac{1}{2}P_2 = \frac{1}{2}$, $P_1 = 2$ $\Rightarrow$ $\boxed{\frac{1}{2}P_2 < P_1}$

Tim will choose to buy milk only: $x_1^* = 0$; $x_2^* = \frac{m}{P_2} = \frac{90}{1} = 90$

(g) $\hat{P_2} = P_2 + 5 = 6$

$\hat{P_2}$: price Tim pays; 5: tax

Budget Set: $2x_1 + 6x_2 \leq 90$



(h) Given MRS = $-\frac{1}{2}$

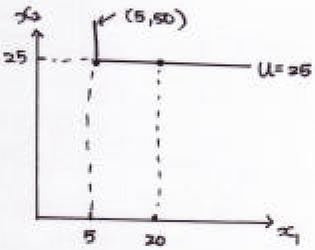
- $(\frac{1}{2})$ units of milk $\backsim$ (1) unit of juice.
- $\frac{1}{2}P_2 = 3$, $P_1 = 2$ $\Rightarrow$ $\boxed{\frac{1}{2}P_2 > P_1}$

Tim will choose to buy juice only: $x_1^* = \frac{m}{P_1} = 45$; $x_2^* = 0$

(3)

(a) $U(5,50) = \text{Min}\{5(5), (50)\} = \text{Min}\{25, 50\} = 25$

Any bundle that gives him an utility of (25) is equally preferred as (5,50) : Any (x_1, x_2) such that $\text{Min}\{5x_1, x_2\} = 25$



* consumer always want 5 times as much good two as good one.

$5x_1 = x_2 = 25$

Kink* $\begin{cases} x_1 = 5 \\ x_2 = 25 \end{cases}$

(b) Given Sam has Perfect complements preferences:

At Sam's choice of optimal bundle (x_1^*, x_2^*)

① $5x_1 = x_2$

② $P_1x_1 + P_2x_2 = m$

Note: $P_2 = 1$ since x_2 = \$ spent on everything else (composite good)

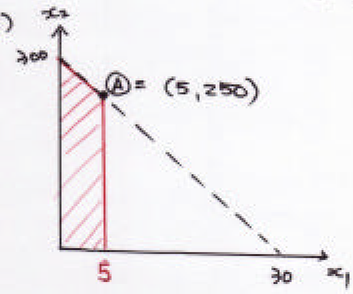
→ $P_1x_1 + P_2(5x_1) = m$

$x_1(P_1 + 5P_2) = m$

$x_1^* = \dfrac{m}{P_1 + 5P_2} = \dfrac{(300)}{(10)+5(1)} = \dfrac{300}{15} = \boxed{20}$ * Answer

$x_2^* = 5x_1^* = \dfrac{5m}{P_1 + 5P_2} = 5(20) = 100$

(c)



- - - - : Budget w/o rationing.

—— : Budget w/ rationing.

At Ⓐ :

$x_1 = 5$

$P_1(5) + P_2x_2 = m$

$(10)(5) + (1)x_2 = 300$

$50 + x_2 = 300$

$\boxed{x_2 = 250}$

(4)

(a) $MRS = -\frac{MU_1}{MU_2} = -\frac{x_2^3}{3x_1x_2^2} = -\frac{x_2}{3x_1}$

At (10, 60):

$MRS = -\frac{(60)}{3(10)} = -2$ → The consumer is willing to give up (2) units of good two for (1) unit of good one to stay equally well-off.

(b) Given preferences are "Well-Behaved".

At the optimal bundle (x_1^*, x_2^*)

① $MRS = -\frac{P_1}{P_2}$

② $P_1x_1 + P_2x_2 = m$

From part (a): $MRS = -\frac{x_2}{3x_1}$

① $MRS = -\frac{P_1}{P_2} \longrightarrow -\frac{x_2}{3x_1} = -\frac{P_1}{P_2} \longrightarrow x_2 = \frac{3P_1x_1}{P_2}$

② $P_1x_1 + \cancel{P_2}\left(\frac{3P_1x_1}{\cancel{P_2}}\right) = m$

$P_1x_1 + 3P_1x_1 = m$

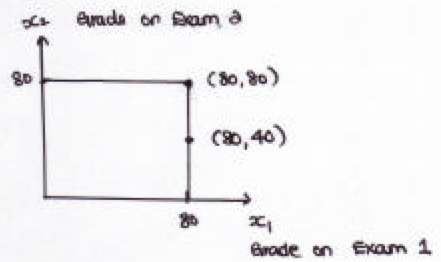
$$\begin{cases} x_1^* = \frac{m}{4P_1} \\ x_2^* = \frac{3P_1}{P_2}\left(\frac{m}{4P_1}\right) = \frac{3m}{4P_2} \end{cases}$$

(c) $\frac{3}{4}$

(5)

(a) Given (80, 40), the student's final grade = Max {80, 40} = $\underline{\underline{80}}$

- Any (x_1, x_2) that yield the same final grade of 80 would be equally preferred.



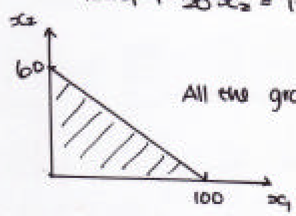
(b) . John spends a total of 1200 minutes studying for exam

$\rightarrow \quad m_1 + m_2 = 1200$

- Given $x_1 = \frac{m_1}{12} \rightarrow 12x_1 = m_1$

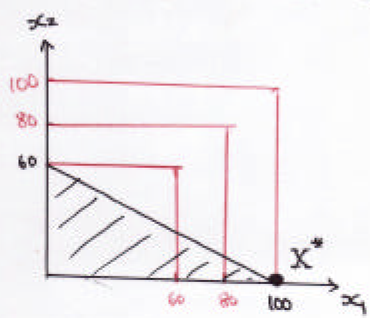
$x_2 = \frac{m_2}{20} \rightarrow 20x_2 = m_2$

Together: $12x_1 + 20x_2 = 1200$ (Budget)



All the grade combinations John can get.

(c)



: John's "budget set")

: John's " Indiff. curves"

X^*: Best choice for John.

Final Grade = Max {100, 0} = 100