Economics 302 EF
Microeconomic Theory
Fall 2005, Dr. Shirley Liu

Midterm Exam 2

To receive FULL CREDIT on any question, you MUST show ALL WORK

Math Review: Let a, b, and c be constants,

- $x^a x^b = x^{a+b}$
- $\frac{x^a}{x^b} = x^{a-b}$
- $x^0 = 1$
- $x^{-a} = \frac{1}{x^a}$
- $(x^a)^b = x^{ab}$
- $x^a y^a = (xy)^a$
- If $f(x) = ax^b$, then $f'(x) = abx^{b-1}$
- If $f(x) = c$, then $f'(x) = 0$
- If $f(x) = g(x) \pm h(x)$, then $f'(x) = g'(x) \pm h'(x)$
- If $f(x) = g(x)h(x)$, then $f'(x) = g'(x)h(x) + g(x)h'(x)$
- If $f(x) = \frac{g(x)}{h(x)}$, then $f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{[h(x)]^2}$
- If $f(x) = g(h(x))$, then $f'(x) = g'(h)h'(x)$

1. [18] Short Answer Questions:

(a) [4] Clearly define the difference between the "Short Run" and the "Long Run".

SR : At least one input is fixed

LR : All inputs can be varied

(b) [5] A firm faces the following technology: $f(x_1, x_2) = 5x_1 + 2x_2$. Is the production plan $(x_1, x_2, y) = (2, 4, 20)$ technologically feasible? *Show all work.*

$f(2,4) = 5(2) + 2(4) = 18 < 20$

NO

(c) [5] Consider a firm facing the technology represented by: $f(x_1, x_2) = x_1^{\frac{1}{4}} x_2$. Does the technology facing the firm exhibit increasing, decreasing, or constant returns to scale? *Show your answer analytically, do not plug in numbers.*

$$f(2x_1, 2x_2) = (2x_1)^{1/4}(2x_2)$$
$$= 2^{5/4} x_1^{1/4} x_2$$
$$= 2^{5/4} y > 2y$$

* Increasing

(d) [4] A profit-maximizing firm uses two inputs (x_1, x_2) to produce one output (y). Suppose this firm is operating in the short run, such that the level of input two is fixed. Suppose at the current production level: $p \times MP_1 < w_1$. What can the firm do to increase profits? Explain.

Reduce the amount of input 1 used in the production process.

2. [18] A consumer's demand functions for two goods are: $x_1 = 0$ and $x_2 = \frac{m}{p_2}$.

(a) [5] Clearly define what it means for a good to be an "Ordinary Good".

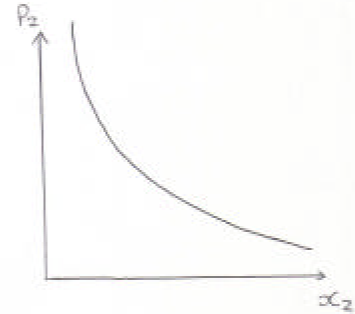
If the price of this good increases, demand for this good would decrease

(b) [8] Graphically illustrate the consumer's <u>Inverse Demand Curve for Good TWO</u>. What is the slope of this curve? *Keep your answer general, do not plug in numbers. Make sure you label all axis.*

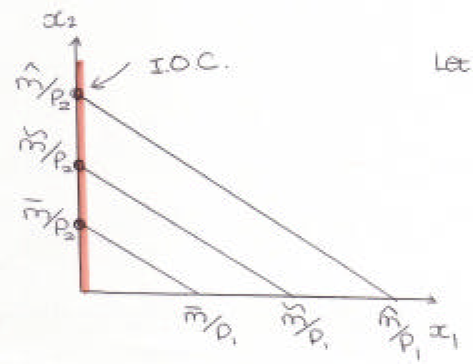
Demand: $x_2 = \frac{m}{P_2}$

Inv. Demand: $P_2 = \frac{m}{x_2}$

slope: $\frac{dP_2}{dx_2} = -\frac{m}{x_2^2}$



(c) [5] Graphically illustrate the consumer's <u>Income Offer Curve</u>. *Keep your answer general, do not plug in numbers. Make sure you label all axis.*



Let $\hat{m} > \check{m} > \bar{m}$

3. [14] A firm uses two inputs, x_1 = (labor) and x_2 = (capital), to produce one output (Y). The technology facing the firm is: $f(x_1, x_2) = 4x_1^{\frac{1}{2}}x_2^{\frac{1}{4}}$. Let (p, w_1, w_2) be the per unit price of labor, capital, and output, respectively.

(a) [6] Suppose the firm is operating in the SHORT RUN, such that Captial is fixed at 1 units. To maximize profits, how many units of labor would this firm choose to use? *Keep your answer general.*

SR prod. function: $f(x_1, 1) = 4x_1^{1/2}$

$MP_1 = 2x_1^{-1/2}$

$MP_1 = \frac{w_1}{p} \rightarrow 2x_1^{-1/2} = \frac{w_1}{p}$

$x_1^{-1/2} = \frac{w_1}{2p}$

$x_1^* = \left(\frac{w_1}{2p}\right)^{-2} = \left(\frac{2p}{w_1}\right)^2$

(b) [8] Suppose now the firm is operating in the LONG RUN. To maximize profits, how many units of labor and capital would this firm choose to use? *Show all work.* [If needed: $MP_1 = 2x_1^{-\frac{1}{2}}x_2^{\frac{1}{4}}$; $MP_2 = x_1^{\frac{1}{2}}x_2^{-\frac{3}{4}}$; and $TRS = -\frac{2x_2}{x_1}$]

- $p \cdot MP_1 = w_1 \rightarrow p \cdot 2x_1^{-1/2}x_2^{1/4} = w_1 \rightarrow py = 2w_1x_1 \rightarrow x_1 = \frac{py}{2w_1}$
- $p \cdot MP_2 = w_2 \rightarrow p \cdot x_1^{1/2}x_2^{-3/4} = w_2 \rightarrow py = 4w_2x_2 \rightarrow x_2 = \frac{py}{4w_2}$

$y = 4\left(\frac{py}{2w_1}\right)^{1/2}\left(\frac{py}{4w_2}\right)^{1/4}$

$= 4\left(\frac{p}{2w_1}\right)^{1/2}\left(\frac{p}{4w_2}\right)^{1/4}y^{3/4}$

$y^{1/4} = 4\left(\frac{p}{2w_1}\right)^{1/2}\left(\frac{p}{4w_2}\right)^{1/4}$

$y = 4^4\left(\frac{p}{2w_1}\right)^2\left(\frac{p}{4w_2}\right)$

$= \frac{16p^3}{w_1^2w_2}$

$$\begin{cases} x_1^* = \frac{p}{2w_1}\left[\frac{16p^3}{w_1^2w_2}\right] \\ x_2^* = \frac{p}{4w_2}\left[\frac{16p^3}{w_1^2w_2}\right] \end{cases}$$

4. [22] Starbucks uses x_1 = (shots of expresso) and x_2 = (cups of milk) to make y = (cups of lattes). Suppose the technology Starbucks uses is represented by: $f(x_1, x_2) = \text{Min}\{\frac{1}{2}x_1, x_2\}$. Each shot of expresso costs \$1, each cup of milk costs \$2, and each cup of latte costs \$5.

(a) [12] To produce 40 lattes in the least costly way, how many shots of expresso (x_1^*) and how many cups of milk (x_2^*) should Starbucks use?

$x_1^* = 80$

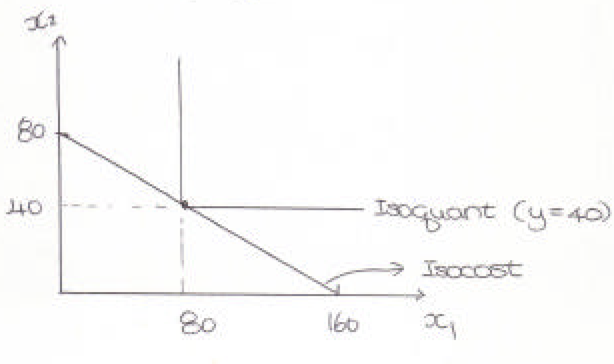
$x_2^* = 40$

(b) [10] On the same graph, graphically illustrate the associated <u>isoquant</u> and <u>isocost</u> lines that pass through Starbucks' cost-minimizing choice of inputs (x_1^*, x_2^*) in producing 40 lattes. *Make sure you label which line is the Isoquant, which line is the Isocost, all axis, intercepts, kinks, if applicable.*

cost of $(80, 40) = (1)(80) + (2)(40) = 160$

Isoquant: $\text{Min}\{\frac{1}{2}x_1, x_2\} = 40$

Isocost: $x_1 + 2x_2 = 160$.



5. [28] A perfectly competitive, profit-maximizing firm faces the following total cost function: $C(y) = y^4 - 27y^2 + 100y + 80$.

(a) [12] Derive the firm's Average Variable Cost function ($AVC(y)$), Average Fixed Cost function ($AFC(y)$), and the Marginal Cost Function ($MC(y)$).

$AVC(y) = y^3 - 27y + 100$

$AFC(y) = 80/y$

$MC(y) = 4y^3 - 54y + 100$

(b) [10] Suppose the market price for each unit of output is \$140. Calculate the firm's profits and producer's surplus.

$P = 4(6)^3 - 54(6) + 100 = 640$

Profit $= Py - C(y) = 3840 - 1004 = 2836$

Producer's Surplus $= Py - C_v(y) = 3840 - 924 = 2916$

(c) [3] If the firm is supplying 5 units of output. What is the current market price?

$P = 4(5)^3 - 54(5) + 100$

$= 330$

(d) [3] Over what price range should this firm choose to shut down? *Numeric Range.*

$P < \min AVC \rightarrow \frac{dAVC}{dy} = 3y^2 - 27 = 0$

$y = 3$

$AVC(3) = (3)^3 - 27(3) + 100$

$= 46$

Shut Down if

$P < 46$

Extra Credit Question [10 **Points**]:

It is not possible for a good to be observed as an "Inferior Good" over every possible income level. Clearly explain why.

For a good to be observed as an "Inferior Good" over some income level, it must have been a normal good over some lower range of income.

Otherwise, suppose a good is an inferior good over all income levels:

At $m = 0$: $x = 0$

At $m > 0$: $x < 0$

$x \downarrow$ as $m \uparrow$

* This is NOT possible.