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Economics 302 EF
Microeconomic Theory
Fall 2005, Dr. Shirley Liu

Midterm Exam 2

To receive FULL CREDIT on any question, you MUST show ALL WORK

Math Review: Let a, b, and c be constants,

- $x^a x^b = x^{a+b}$
- $\frac{x^a}{x^b} = x^{a-b}$
- $x^0 = 1$
- $x^{-a} = \frac{1}{x^a}$
- $(x^a)^b = x^{ab}$
- $x^a y^a = (xy)^a$
- If $f(x) = ax^b$, then $f'(x) = abx^{b-1}$
- If $f(x) = c$, then $f'(x) = 0$
- If $f(x) = g(x) \pm h(x)$, then $f'(x) = g'(x) \pm h'(x)$
- If $f(x) = g(x)h(x)$, then $f'(x) = g'(x)h(x) + g(x)h'(x)$
- If $f(x) = \frac{g(x)}{h(x)}$, then $f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{[h(x)]^2}$
- If $f(x) = g(h(x))$, then $f'(x) = g'(h)h'(x)$

1. [18] Short Answer Questions:

(a) [4] Clearly define the difference between the "Short Run" and the "Long Run".

SR: At least one of the inputs are fixed

LR: All inputs can be varied

(b) [5] A firm faces the following technology: $f(x_1, x_2) = 2x_1 + 5x_2$. Is the production plan $(x_1, x_2, y) = (2, 4, 20)$ technologically feasible? *Show all work.*

$f(2,4) = 2(2) + 5(4) = 24 > 20$

yes.

(c) [5] Consider a firm facing the technology represented by: $f(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{1}{4}}$. Does the technology facing the firm exhibit increasing, decreasing, or constant returns to scale? *Show your answer analytically, do not plug in numbers.*

$$f(2x_1, 2x_2) = (2x_1)^{1/4}(2x_2)^{1/4}$$
$$= 2^{1/2} x_1^{1/4} x_2^{1/4}$$
$$= 2^{1/2} y < 2y$$

* Decreasing

(d) [4] A profit-maximizing firm uses two inputs (x_1, x_2) to produce one output (y). Suppose this firm is operating in the short run, such that the level of input two is fixed. Suppose at the current production level: $p \times MP_1 > w_1$. What can the firm do to increase profits? Explain.

Increase the amount of x_1 in the production process.

2. [18] A consumer's demand functions for two goods are: $x_1 = 0$ and $x_2 = \frac{m}{p_2}$.

(a) [5] Clearly define what it means for a good to be an "Inferior Good".

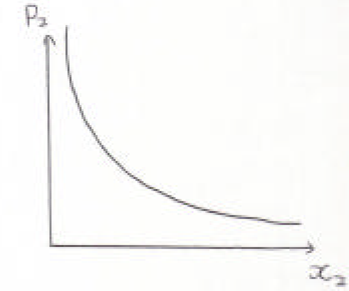
If income increases, the demand for this good decreases

(b) [8] Graphically illustrate the consumer's <u>Inverse Demand Curve for Good TWO</u>. What is the slope of this curve? *Keep your answer general, do not plug in numbers. Make sure you label all axis.*

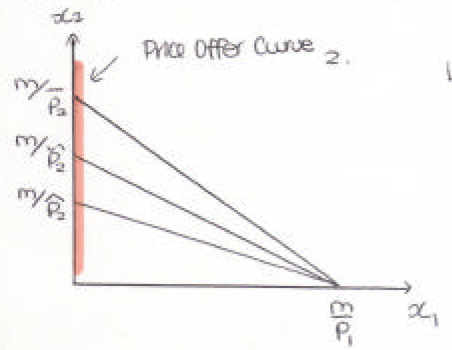
Demand : $x_2 = \frac{m}{P_2}$

Inv. Demand : $P_2 = \frac{m}{x_2}$

slope : $\frac{dP_2}{dx_2} = -\frac{m}{x_2^2}$



(c) [5] Graphically illustrate the consumer's <u>Price Offer Curve for Good TWO</u>. *Keep your answer general, do not plug in numbers. Make sure you label all axis.*



Let $\hat{P}_2 > \breve{P}_2 > \bar{P}_2$

3. [14] A firm uses two inputs, x_1 = (labor) and x_2 = (capital), to produce one output (Y). The technology facing the firm is: $f(x_1, x_2) = 4x_1^{\frac{1}{4}}x_2^{\frac{1}{2}}$. Let (p, w_1, w_2) be the per unit price of labor, capital, and output, respectively.

(a) [6] Suppose the firm is operating in the SHORT RUN, such that Captial is fixed at 1 units. To maximize profits, how many units of labor would this firm choose to use? *Keep your answer general.*

SR prod. fcn: $f(x_1, 1) = 4x_1^{1/4}$

$MP_1 = x_1^{-3/4}$

$P \cdot MP_1 = W_1 \rightarrow P \cdot x_1^{-3/4} = W_1$

$x_1^{-3/4} = \frac{W_1}{P}$

$x_1^* = \left(\frac{W_1}{P}\right)^{-4/3} = \left(\frac{P}{W_1}\right)^{4/3}$

(b) [8] Suppose now the firm is operating in the LONG RUN. To maximize profits, how many units of labor and capital would this firm choose to use? *Show all work.* [If needed: $MP_1 = x_1^{-\frac{3}{4}}x_2^{\frac{1}{2}}$; $MP_2 = 2x_1^{\frac{1}{4}}x_2^{-\frac{1}{2}}$; and $TRS = -\frac{x_2}{2x_1}$]

- $P \cdot MP_1 = W_1 \rightarrow P \cdot x_1^{-3/4}x_2^{1/2} = W_1 \rightarrow Py = 4W_1x_1 \rightarrow x_1 = Py/4W_1$
- $P \cdot MP_2 = W_2 \rightarrow P \cdot 2x_1^{1/4}x_2^{-1/2} = W_2 \rightarrow Py = 2W_2x_2 \rightarrow x_2 = Py/2W_2$

$$y = 4\left(\frac{Py}{4W_1}\right)^{1/4}\left(\frac{Py}{2W_2}\right)^{1/2}$$

$$= 4\left(\frac{P}{4W_1}\right)^{1/4}\left(\frac{P}{2W_2}\right)^{1/2} y^{3/4}$$

$$y^{1/4} = 4\left(\frac{P}{4W_1}\right)^{1/4}\left(\frac{P}{2W_2}\right)^{1/2}$$

$$y^* = 4^4\left(\frac{P}{4W_1}\right)\left(\frac{P}{2W_2}\right)^2$$

$$= \frac{16P^3}{W_1W_2^2}$$

$$x_1^* = \frac{P}{4W_1}\left[\frac{16P^3}{W_1W_2^2}\right]$$

$$x_2^* = \frac{P}{2W_2}\left[\frac{16P^3}{W_1W_2^2}\right]$$

4. [22] Starbucks uses x_1 = (shots of expresso) and x_2 = (cups of milk) to make y = (cups of lattes). Suppose the technology Starbucks uses is represented by: $f(x_1, x_2) = \text{Min}\{\frac{1}{2}x_1, x_2\}$. Each shot of expresso costs \$1, each cup of milk costs \$2, and each cup of latte costs \$5.

(a) [12] To produce 50 lattes in the least costly way, how many shots of expresso (x_1^*) and how many cups of milk (x_2^*) should Starbucks use?

$x_1^* = 100$

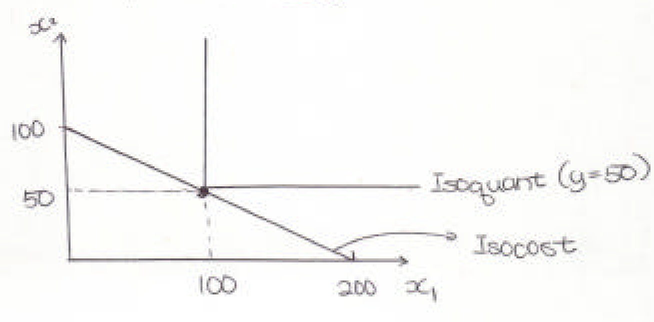
$x_2^* = 50$

(b) [10] On the same graph, graphically illustrate the associated <u>isoquant</u> and <u>isocost</u> lines that pass through Starbucks' cost-minimizing choice of inputs (x_1^*, x_2^*) in producing 50 lattes. *Make sure you label which line is the Isoquant, which line is the Isocost, all axis, intercepts, kinks, if applicable.*

Cost of $(100, 50) = (1)(100) + (2)(50) = 200$

Isoquant: $\text{Min}\{\frac{1}{2}x_1, x_2\} = 50$

Isocost: $x_1 + 2x_2 = 200$



5. [28] A perfectly competitive, profit-maximizing firm faces the following total cost function: $C(y) = y^4 - 27y^2 + 100y$.

(a) [12] Derive the firm's Average Variable Cost function ($AVC(y)$), Average Fixed Cost function ($AFC(y)$), and the Marginal Cost Function ($MC(y)$).

$AVC(y) = y^3 - 27y + 100$

$AFC(y) = 0$

$MC(y) = 4y^3 - 54y + 100$

(b) [10] Suppose the market price for each unit of output is \$140. Calculate the firm's profits and producer's surplus.

$P = 4(6)^3 - 54(6) + 100 = 640$

Profit $= Py - C(y) = 3840 - 924 = 2916$

Producer's Surplus $= Py - C_v(y) = 3840 - 924 = 2916$

(c) [3] If the firm is supplying 5 units of output. What is the current market price?

$P = 4(5)^3 - 54(5) + 100$

$= 330$

(d) [3] Over what price range should this firm choose to shut down? *Numeric Range.*

$P < \min AVC \rightarrow \frac{dAVC}{dy} = 3y^2 - 27 = 0$

$y = 3$

$AVC(3) = 46$

$\boxed{\text{Shut Down if } P < 46}$

Extra Credit Question [10 Points]:

It is not possible for a good to be observed as an "Inferior Good" over every possible income level. Clearly explain why.

For a good to be observed as an "Inferior Good" over some income level, it must have been a normal good over some lower range of income

Otherwise, suppose a good is an inferior good over all income levels:

At $m = 0$: $x = 0$

At $m > 0$: $x < 0$

$x \downarrow$ as $m \uparrow$

* This is NOT possible.