Economics 302
Microeconomic Theory
Fall 2005 *EF*, Dr. Shirley Liu

Midterm Exam 1

To receive full credit for any question, you must show all necessary work

Math Review: Let a, b, and c be constants,

- $x^a x^b = x^{a+b}$
- $\frac{x^a}{x^b} = x^{a-b}$
- $x^0 = 1$
- $x^{-a} = \frac{1}{x^a}$
- $(x^a)^b = x^{ab}$
- $x^a y^a = (xy)^a$
- If $f(x) = ax^b$, then $f'(x) = abx^{b-1}$
- If $f(x) = c$, then $f'(x) = 0$
- If $f(x) = g(x) \pm h(x)$, then $f'(x) = g'(x) \pm h'(x)$
- If $f(x) = g(x)h(x)$, then $f'(x) = g'(x)h(x) + g(x)h'(x)$
- If $f(x) = \frac{g(x)}{h(x)}$, then $f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{[h(x)]^2}$
- If $f(x) = g(h(x))$, then $f'(x) = g'(h)h'(x)$

1. [17] Short answer questions:

   (a) [5] Define "Optimal Bundle".

   The best bundle the consumer can afford

   (b) [7] A consumer's preferences for two goods can be represented by: $U(x_1, x_2) = 2x_1 + x_2$. Suppose $p_1 = \$2$ and $p_2 = \$3$, and the consumer has an income of $m = \$30$. How many units of good one would this consumer choose? How many units of good two would this consumer choose? *Show all work.*

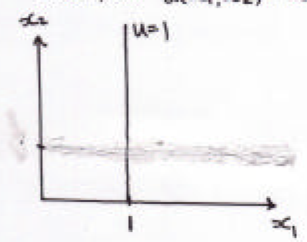
   MRS = −2 → The consumer is always willing to trade 2 units of good two for 1 more unit of good one.

   $$\left.\begin{array}{l} \cdot\ 2P_2 = 6 \\ \cdot\ P_1 = 3 \end{array}\right\} \quad 2P_2 > P_1 \Rightarrow \text{Buy good one only}$$

   $$\begin{cases} x_1^* = \frac{m}{P_1} = \frac{30}{2} = 15 \\ x_2^* = 0 \end{cases}$$

   (c) [5] A consumer consumes two goods (x_1, x_2) such that $MU_1 > 0$ and $MU_2 = 0$. This consumer's preferences are NOT well-behaved. Explain. *Clearly state which of the four properties of "well-behaved" preferences is violated. If more than one of the properties are violated, simply choose ONE of violated properties and show how it is violated. Additionally, you can construct examples and/or use graphs as aids for your answer.*

   Example: $U(x_1, x_2) = x_1$



   ① Violates Strong Monotonicity

   Let $X = (1,0)$, $Y = (1,1)$

   By SM, $Y \succ X$

   But $U(Y) = U(X) = 1$

   $Y \sim X$

   ② Violates Strict Convexity

   Let $X = (1,0)$, $Y = (1,2)$

   Z = Any weighted Avg of X and Y $= (1,1)$ for example.

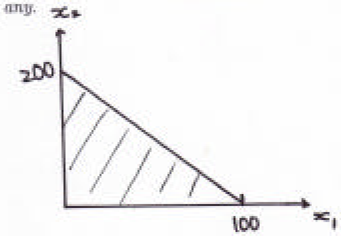
   By SC: $Z \succ X, Y$

   But $U(Z) = U(X) = U(Y) = 1$

   $Z \sim X \sim Y$.

2. [26] Sean enjoys driving and consumes: x_1 = (gallons of gasoline) and x_2 = (\$ spent on everything else). Suppose the price for each gallon of gasoline is \$2, and Sean has a weekly income of $m = \$200$.

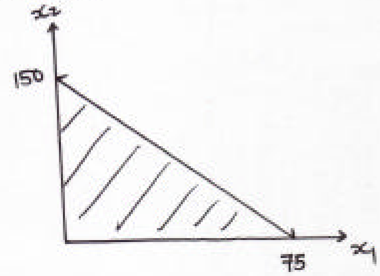
(a) [10] Graphically illustrate Sean's <u>Budget Set</u>. *Make sure you label all axis, intercepts, and kinks, if any.*



(b) [16] Suppose there is a shortage of gasoline and the government is considering the following two different policies for gasolines. Graphically illustrate Sean's budget set under each of the two policy schemes. <u>*Make sure you label all axis, intercepts, and kinks, if any.*</u>

i. [8] A lump sum tax of \$50 on each consumer.

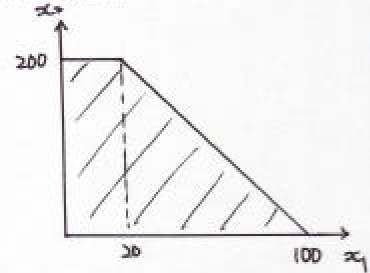
$2x_1 + x_2 \leq 150$



ii. [8] The first 20 gallons of gasoline are free. A quantity tax of \$0.50 is imposed on each gallon of gasoline after the purchase of the 20th gallon.

$$P_1 = \begin{cases} 0 & \text{if } x_1 \leq 20 \\ 2.5 & \text{if } x_1 > 20 \end{cases}$$

Total amt of gas affordable $= 20 + \frac{200}{2.5} = 100$



3. [27] A consumer's preferences for two goods: x_1 = (tea) and x_2 = (cookies) can be represented by: $U(x_1, x_2) = \text{Min}\{x_1, 3x_2\}$. Suppose the price for each cup of tea is $p_1 = \$2$, and the price for each cookie is $p_2 = \$1$, and the consumer has an income of $m = \$140$.

(a) [5] What type of preferences does this consumer have?

Perfect Complements

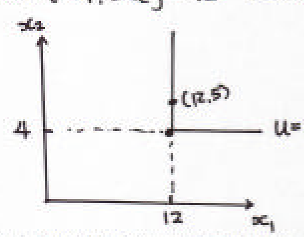
(b) [5] What is this consumer's opportunity cost of consuming an extra cookie (x_2)?

$\frac{1}{2}$ cup of tea

(c) [7] Graphically illustrate all the bundles (x_1, x_2) that the consumer prefers equally to the bundle (12, 5). *Make sure you label all axis, intercepts, and kinks, if any.*

$U(12,5) = \text{Min}\{12, 3(5)\} = 12$

$\Rightarrow$ Any (x_1, x_2) that satisfy $\text{Min}\{x_1, 3x_2\} = 12$ is on the same indiff. curve as (12,5)



(d) [10] How many cups of tea (x_1^*) and cookies (x_2^*) would this consumer choose? *Derive your answers rigourously. NO CREDIT will be given for the correct answers without showing proper work.*

At (x_1^*, x_2^*)

① $x_1 = 3x_2$

② $P_1x_1 + P_2x_2 = m \rightarrow 2x_1 + x_2 = 140$

$2(3x_2) + x_2 = 140$

$7x_2 = 140$

$$\begin{cases} x_2^* = 20 \\ x_1^* = 3(20) = 60 \end{cases}$$

4. [30] Sam consumes two goods: x_1 = (comic books) and x_2 = (video games). Sam's preferences can be represented by: $U(x_1, x_2) = 3x_1x_2$. Let (p_1, p_2, m) denote the price of each comic book, a video game, and Sam's income, respectively. [If needed: $MU_1 = 3x_2$, and $MU_2 = 3x_1$].

(a) [5] Derive the expression for Sam's marginal rate of substitution between comic books and video games (MRS). *Simplify your answer.*

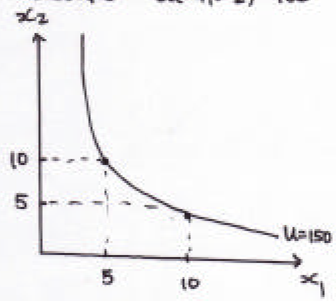
$$MRS = -\frac{MU_1}{MU_2} = -\frac{3x_2}{3x_1} = -\frac{x_2}{x_1}$$

(b) [5] Suppose Sam is currently consuming the bundle (5, 10), how many video games is Sam willing for give up for an additional comic book, to keep him equally as well-off? *Numeric Answer.*

$$MRS\Big|_{(5,10)} = -\frac{(10)}{(5)} = -2$$ * 2 video games

(c) [6] Does Sam's preferences exhibit "diminishing marginal rate of substitution? *Come up with your own examples to explain your answer. NO CREDIT will be given for the correct answer without proper explanation.*

Example: $U(x_1, x_2) = 150$



$MRS|_{(5,10)} = -2$

$MRS|_{(10,5)} = -\frac{1}{2}$

As the consumer acquires more good one (from (5,10) to (10,5)), the amt of good two Sam is willing to give up for an additional unit of good two decreases from (2) to ($\frac{1}{2}$)

yes

(d) [10] Derive Sam's demand functions for both comic books ($x_1(p_1, p_2, m)$) and video games ($x_2(p_1, p_2, m)$). *Show all proper steps. NO CREDIT will be given for the correct answers without showing proper work.*

At (x_1^*, x_2^*):

① $MRS = -P_1/P_2$

② $P_1x_1 + P_2x_2 = m$

$$MRS = -\frac{x_2}{x_1} = -\frac{P_1}{P_2} \longrightarrow x_2 = \frac{P_1x_1}{P_2}$$

$$P_1x_1 + P_2\left(\frac{P_1x_1}{P_2}\right) = m$$

$$2P_1x_1 = m$$

$$\begin{cases} x_1^* = \dfrac{m}{2P_1} \\ x_2^* = \dfrac{P_1}{P_2}\left(\dfrac{m}{2P_1}\right) = \dfrac{m}{2P_2} \end{cases}$$

(e) [4] What fraction of Sam's income would he choose to spend on comic books?

$$\frac{1}{2}$$

Extra Credit Question [5 points]:

Professor Mitnik's class has the following grading scheme. There are two exams in his class. Let x_1 = (Grade on Exam 1) and x_2 = (Grade on Exam 2). A student's final course grade is the weighted average of the two exam grades such that a weight of 25% is placed on Exam 1, and a weight of 75% is placed on Exam 2. More specifically, a student's Final Grade = $0.25x_1 + 0.75x_2$.

Susan is taking Professor Mitnik's class. She is determined to get the highest final course grade possible. Susan decided that she will spend a total of 1000 minutes studying for the two exams. If she spends m_1 minutes studying for Exam 1, her grade for Exam 1 will be $x_1 = \frac{m_1}{20}$; if she spends m_2 minutes studying for Exam 2, her grade for Exam 2 will be $x_2 = \frac{m_2}{10}$. If she spends no time studying for either exam, her grade for that exam will be 0.

Given that Susan wants to get the highest final course grade possible, and that she is spending a total of 1000 minutes studying for the two exams, what will be Susan's final course grade? ***There are no partial credits for the extra credit question.***

$$\left.\begin{array}{l} MRS = -\frac{1}{3} \\ -\frac{P_1}{P_2} = -2 \end{array}\right\} \quad MRS > -\frac{P_1}{P_2} \Rightarrow \begin{array}{l} x_1^* = 0 \\ x_2^* = \frac{m}{P_2} = \frac{1000}{10} = 100 \end{array}$$

$$\text{Final Grade} = 0.25(0) + 0.75(100) = \underline{\underline{75}}$$