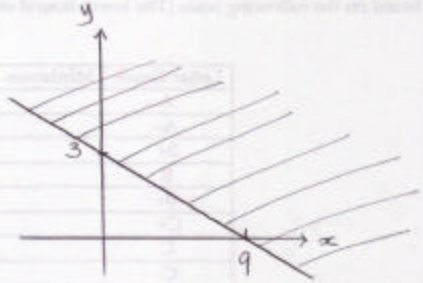
Answer key for Math Review Questions

(Incl. additional questions for partial derivatives

(1)

(a) $x - 3y \geq 9$

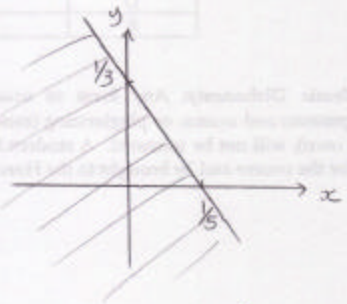
$\Rightarrow \frac{x}{9} - \frac{y}{3} \geq 1$



(b) $10x + 6y \leq 2$

$\Rightarrow 5x + 3y \leq 1$

$\Rightarrow \frac{x}{1/5} + \frac{y}{1/3} \leq 1$



(2) Given $\begin{cases} 2x + y = 24 \\ 5y - x = 10 \end{cases}$

$x = 5y - 10$

$2(5y - 10) + y = 24$

$10y - 20 + y = 24$

$11y = 44$

$\boxed{y = 4}$

$x = 5(4) - 10$

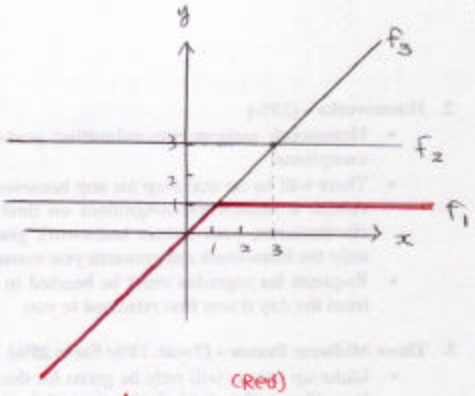
$\boxed{x = 10}$

(3) $f(x) = \text{Min}\{1, 3, x\}$

Let $f_1 = 1$

$f_2 = 3$

$f_3 = x$



Answer : (Red) — line

$$f(x) = \text{Min}\{1, 3, x\} = \begin{cases} x & \text{if } x \le 1 \\ 1 & \text{if } x > 1 \end{cases}$$

(4) Given $\dfrac{(4y)^2 (x+1)^4 (x-1)}{4y^2 (x+1)(x^2-1)}$

$$= \frac{16y^2 (x+1)^4 (x-1)}{4y^2 (x+1)(x+1)(x-1)}$$

$$= \frac{4(x+1)^3}{(x+1)}$$

$$= 4(x+1)^2$$

$$= 2^2 (x+1)^2$$

$$= (2x+2)^2$$ yes

(5)

(a) $f(x) = (5x^3 - 3x - 13)^5$

$$\frac{df(x)}{dx} = 5(5x^3 - 3x - 13)^4 (15x^2 - 3)$$

(b) $f(x) = \frac{1}{x^2}$

$$\frac{df(x)}{dx} = -\frac{2}{x^3}$$

(6) ~~$f(x_1, x_2) =$~~

$$f(x,y) = 4x^3y^2 + xy + 2x + y + 10$$

$$\frac{\partial f(x,y)}{\partial x} = 12x^2y^2 + y + 2$$

$$\frac{\partial f(x,y)}{\partial y} = 8x^3y + x + 1$$

Additional Review Questions for Partial Derivatives (Answer Key)

(1) $f(x_1, x_2) = 5x_1^2 x_2^2 + 3x_1 x_2 + 2x_1^2 + 3x_2 + x_1 - 12$

$$\frac{\partial f(x_1, x_2)}{\partial x_1} = 10x_1 x_2^2 + 3x_2 + 4x_1 + 1$$

$$\frac{\partial f(x_1, x_2)}{\partial x_2} = 10x_1^2 x_2 + 3x_1 + 3$$

(2) $f(x_1, x_2) = (x_1^2 + x_2^2)^{1/2}$

$$\frac{\partial f(x_1, x_2)}{\partial x_1} = \frac{1}{2}(x_1^2 + x_2^2)^{-1/2}(2x_1)$$

$$\frac{\partial f(x_1, x_2)}{\partial x_2} = \frac{1}{2}(x_1^2 + x_2^2)^{-1/2}(2x_2)$$

(3) $f(x_1, x_2) = \frac{(x_1+1)^2}{(x_2-1)}$

$$\frac{\partial f(x_1, x_2)}{\partial x_1} = \frac{2(x_1+1)}{(x_2-1)}$$

$$\frac{\partial f(x_1, x_2)}{\partial x_2} = [illegible] - \frac{(x_1+1)^2}{(x_2-1)^2}$$

(4) $f(x_1, x_2) = 2x_1^3$

$$\frac{\partial f(x_1, x_2)}{\partial x_1} = 6x_1^2$$

$$\frac{\partial f(x_1, x_2)}{\partial x_2} = 0$$