

Math Quiz

To get full credit on any question, you must show ALL work

1. [22] Find the value of x for which the function $f(x) = \frac{1}{3}x^3 - x$ reaches its MINIMUM value.
Show all steps.

$$f'(x) = x^2 - 1 = 0$$

$$(x+1)(x-1) = 0$$

$$x = \pm 1 \text{ (min or max)}$$

$$f''(x) = 2x$$

$$f''(1) = 2(1) \geq 0 \rightarrow \text{convex}$$

$$f''(-1) = -2 \leq 0 \rightarrow \text{concave}$$

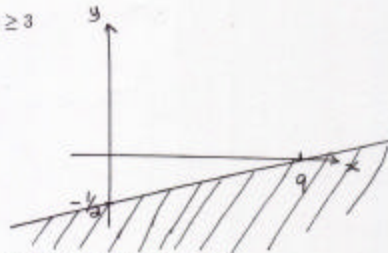
* $x = 1 \Rightarrow f(x)$ reaches
its minimum
value

2. [22] Graphically illustrate all the points (x, y) that satisfy the following inequality:

$$\frac{x}{3} - 6y \geq 3$$

$$\rightarrow \frac{x}{9} - 2y \geq 1$$

$$\rightarrow \frac{x}{9} + \frac{y}{-1/2} \geq 1$$



3. [24] Solve the following system of simultaneous equations for both x and y :

$$10x - 3y = 19$$

$$x + y = 11$$

$$y = 11 - x$$

$$10x - 3(11 - x) = 19$$

$$10x - 33 + 3x = 19$$

$$13x = 52$$

$$\boxed{x = 4}$$

$$\boxed{y = 11 - 4 = 7}$$

4. [12] Differentiate the following function, $f(x) = \frac{1}{x}$, with respect to x .

$$f(x) = x^{-1}$$

$$f'(x) = -x^{-2} = -\frac{1}{x^2}$$

5. [20] Given $f(x_1, x_2) = x_1^3 x_2^2 + x_1 x_2 + 4x_1 + x_2$, derive the partial derivative of $f(x_1, x_2)$ with respect to x_1 (i.e. $\frac{\partial f(x_1, x_2)}{\partial x_1}$); and derive the partial derivative of $f(x_1, x_2)$ with respect to x_2 (i.e. $\frac{\partial f(x_1, x_2)}{\partial x_2}$).

$$\frac{\partial f(x_1, x_2)}{\partial x_1} = 3x_1^2 x_2^2 + x_2 + 4$$

$$\frac{\partial f(x_1, x_2)}{\partial x_2} = 2x_1^3 x_2 + x_1 + 1$$