Homework 6 Solutions.

(1)

(a) Given $C(y) = 5y^2 + 25$

- $\boxed{MC(y) = 10y}$

- $\boxed{AC(y) = \frac{C(y)}{y} = 5y + \frac{25}{y}}$

Find minimum of AC:

$$\frac{d\,AC(y)}{dy} = 5 - \frac{25}{y^2} = 0$$

$$\frac{25}{y^2} = 5$$

$$25 = 5y^2$$

$$5 = y^2$$

$$y = \sqrt{5}$$ ← where AC is at its minimum.

verify $MC(\sqrt{5}) = AC(\sqrt{5})$

$MC(\sqrt{5}) = 10\sqrt{5}$ ✓

$$AC(\sqrt{5}) = 5(\sqrt{5}) + \frac{25}{(\sqrt{5})} = 5\sqrt{5} + \frac{\sqrt{5}\cdot\sqrt{5}\cdot\sqrt{5}\cdot\cancel{\sqrt{5}}}{\cancel{\sqrt{5}}}$$

$$= 5\sqrt{5} + 5\sqrt{5}$$

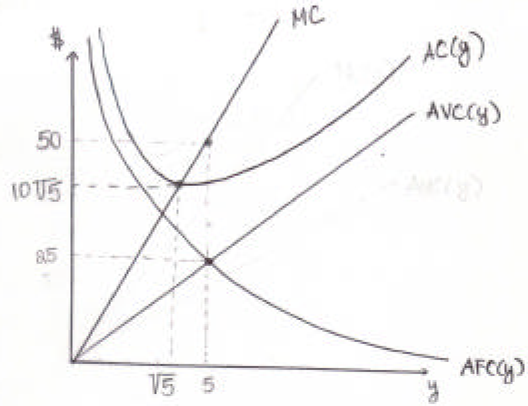
$$= 10\sqrt{5}$$ ✓

(b) $AC(y) = 5y + \frac{25}{y}$

$AVC(y) = 5y$

$AFC(y) = \frac{25}{y}$

$MC(y) = 10y.$



(c) See ▬ in part (b)

(d) No, at every level of market price (P), the firm will choose a level of output where $P = MC(y)$. At each of these output levels (y), $P > AVC(y)$.

(e) At $P = 50$

Firm's Supply: $P = MC(y)$

$P = 10y$

$50 = 10y$

$y =$ [redacted] $.5$

Firm's Profit $= Py - C(y)$

$= (50)(5) - [5(5)^2 + 25]$

$= 250 - [125 + 25]$

$= 250 - 150$

$= 100$

(f) Producer's Surplus $= Py - C(y)$

$$= (50)(5) - 5(5)^2$$

$$= 250 - 125$$

$$= 125$$

(2) (a) Given that firm's target level of output $= \bar{y}$

$$f(x_1, x_2) = \left[\text{Min}\{x_1, 2x_2\}\right]^{1/2} = \bar{y}$$

$$\text{Min}\{x_1, 2x_2\} = \bar{y}^2$$

At kink: $x_1 = 2x_2 = \bar{y}^2$

$$x_1^* = \bar{y}^2$$

$$x_2^* = \bar{y}^2/2$$

(b) $C(w_1, w_2, \bar{y}) = w_1 x_1^* + w_2 x_2^*$

$$= w_1(\bar{y}^2) + w_2(\bar{y}^2/2)$$

(c) $C(y) = (1)\, y^2 + (1)\,(y^2/2)$

$$= \frac{3y^2}{2}$$

$MC(y) = 3y$

(d) $AC(y) = \dfrac{C(y)}{y} = \dfrac{3y^2/2}{y} = \dfrac{3y}{2}$

(e) Inverse Supply: $P = MC(y) = 3y$.

Supply Function: $\underbrace{S(p)}_{= y} = P/3$

(f) If $P = 48$.

$S(48) = 48/3 = 16 \rightarrow$ Firm's Output.

Profit $= Py - C(y)$

$= (48)(16) - \left[\frac{3}{2}(16)^2\right]$

$= 768 - 384$

$= 384$.