(1)

(a) $f(x_1, 8) = x_1^{1/3}(8)^{1/3} = 2x_1^{1/3}$

(b) $MP_1 = \frac{2}{3}x_1^{-2/3}$

$MP_1 = \frac{w_1}{p} \rightarrow \frac{2}{3}x_1^{-2/3} = \frac{w_1}{p}$

$x_1^{-2/3} = \frac{3w_1}{2p}$

$x_1^* = \left(\frac{3w_1}{2p}\right)^{-3/2} = \left(\frac{2p}{3w_1}\right)^{3/2}$

(c) Factor Demand Function: $x(p, w_1, w_2)$ relates the firm's profit-maximizing choice of an input as a fcn of all prices.

Conditional Factor Demand Function: $x(w_1, w_2, \bar{y})$ relates the firm's cost-minimizing choice of an input as a function of input prices, conditional on the firm producing $(\bar{y})$ units of output.

(d) At (x_1^*, x_2^*):

① $P \cdot MP_1 = w_1$

② $P \cdot MP_2 = w_2$

Step 1: ① $P \cdot \left(\frac{1}{3}x_1^{-2/3}x_2^{1/3}\right) = w_1 \rightarrow P\left(x_1^{1/3}x_2^{1/3}\right) = 3w_1x_1$

$Py = 3w_1x_1$

$x_1 = \frac{py}{3w_1}$ ···· ⊛

② $P \cdot \left(\frac{1}{3}x_1^{1/3}x_2^{-2/3}\right) = w_2 \rightarrow P\left(x_1^{1/3}x_2^{1/3}\right) = 3w_2x_2$

$Py = 3w_2x_2$

$x_2 = \frac{py}{3w_2}$ ···· ⊛⊛

Step 2 :

$$y = x_1^{1/3} x_2^{1/3}$$

$$y = \left(\frac{py}{3w_1}\right)^{1/3} \left(\frac{py}{3w_2}\right)^{1/3}$$

$$y = \left(\frac{p}{3w_1}\right)^{1/3} \left(\frac{p}{3w_2}\right)^{1/3} y^{2/3}$$

$$y^{1/3} = \left(\frac{p}{3w_1}\right)^{1/3} \left(\frac{p}{3w_2}\right)^{1/3}$$

$$y = \left(\frac{p}{3w_1}\right)\left(\frac{p}{3w_2}\right) = \frac{p^2}{9w_1w_2}$$

Step 3 :

$$x_1^* = \frac{py}{3w_1} = \frac{p}{3w_1}\left(\frac{p^2}{9w_1w_2}\right) = \frac{p^3}{27w_1^2w_2}$$

$$x_2^* = \frac{py}{3w_2} = \frac{p}{3w_2}\left(\frac{p^2}{9w_1w_2}\right) = \frac{p^3}{27w_1w_2^2}$$

(e) At (x_1^*, x_2^*):

① $TRS = -\frac{w_1}{w_2}$

② $f(x_1, x_2) = \bar{y}$

Step 1 : $TRS = -\frac{MP_1}{MP_2} = -\frac{\frac{1}{3}x_1^{-2/3}x_2^{1/3}}{\frac{1}{3}x_1^{1/3}x_2^{-2/3}} = -\frac{x_2}{x_1}$

Step 2 : $-\frac{x_2}{x_1} = -\frac{w_1}{w_2} \longrightarrow x_2 = \frac{w_1x_1}{w_2}$

Step 3 : $f(x_1, x_2) = \bar{y}$

$$x_1^{1/3} x_2^{1/3} = \bar{y}$$

$$x_1^{1/3}\left(\frac{w_1x_1}{w_2}\right)^{1/3} = \bar{y}$$

$$x_1^{2/3}\left(\frac{w_1}{w_2}\right)^{1/3} = \bar{y}$$

$$x_1^{2/3} = \bar{y}\left(\frac{w_1}{w_2}\right)^{-1/3} = \bar{y}\left(\frac{w_2}{w_1}\right)^{1/3}$$

$$\begin{cases} x_1^* = \left(\bar{y}\left(\frac{w_2}{w_1}\right)^{1/3}\right)^{3/2} = \bar{y}^{3/2}\left(\frac{w_2}{w_1}\right)^{1/2} \\ x_2^* = \frac{w_1}{w_2}x_1^* = \frac{w_1}{w_2}\left(\bar{y}^{3/2}\left(\frac{w_2}{w_1}\right)^{1/2}\right) \end{cases}$$

$$= \bar{y}^{3/2}\left(\frac{w_1}{w_2}\right)\left(\frac{w_1}{w_2}\right)^{-1/2}$$

$$= \bar{y}^{3/2}\left(\frac{w_1}{w_2}\right)^{1/2}$$

(f) $C(w_1, w_2, \bar{y}) = w_1 x_1(w_1, w_2, \bar{y}) + w_2 x_2(w_1, w_2, \bar{y})$

$$= w_1 \bar{y}^{3/2}\left(\frac{w_2}{w_1}\right)^{1/2} + w_2 \bar{y}^{3/2}\left(\frac{w_1}{w_2}\right)^{1/2}$$

$$= \bar{y}^{3/2}(w_1 w_2)^{1/2} + \bar{y}^{3/2}(w_1 w_2)^{1/2}$$

$$= 2\bar{y}^{3/2}(w_1 w_2)^{1/2}$$

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(2) (a) $f(x_1, x_2) = 2x_1 + x_2 = 60$

TRS $= -2 \rightarrow$ 2 units of input two can be substituted by 1 unit of input one.

* Compare the cost of $\underbrace{2x_2}_{(2w_2)}$ vs $\underbrace{x_1}_{(w_1)}$

$2w_2 = 12$
$w_1 = 6$
$\Big\}$ $2w_2 > w_1 \Rightarrow$ use x_1 only.

* $x_1^* = 30$ ← $2x_1 + (0) = 60$
$x_2^* = 0$ $x_1 = 30$

(b) $C(w_1, w_2, 60) = w_1 x_1^* + w_2 x_2^*$

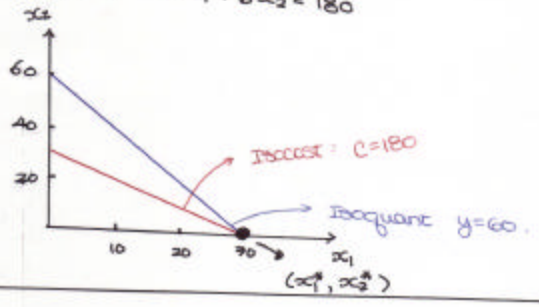
$= 6(30) + 6(0)$

$= 180$

(c)

Isoquant : $2x_1 + x_2 = 60$

Isocost : $6x_1 + 6x_2 = 180$



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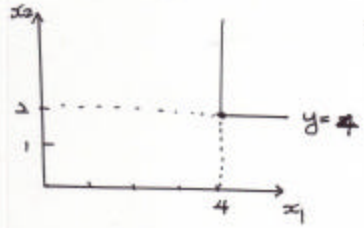
(3)

(a) $\min\{x_1, 2x_2\} = 4$



kink: $x_1 = 4$

$2x_2 = 4 \rightarrow x_2 = 2$

(b) The cheapest way to produce 4 deers is (4, 2)

$C(w_1, w_2, 4) = C(1, 1, 4) = (1)(4) + (1)(2) = 6$

(c) The cheapest way to produce 4 deers is (4, 2)

$C(w_1, w_2, 4) = C(2, 3, 4) = (2)(4) + (3)(2) = 8 + 6 = 14$