

### Homework 5

Due: Monday, November 28, within the first 10 minutes at the beginning of lecture

1. [50] A firm uses two inputs  $x_1 =$  (labor) and  $x_2 =$  (capital) to produce its output  $y =$  (bicycles), via the following technology:  $f(x_1, x_2) = x_1^{\frac{1}{3}}x_2^{\frac{1}{3}}$ . Let  $(p, w_1, w_2)$  denote the per unit price of output, input 1, and input 2.
  - (a) [6] Suppose in the short run the level of capital is fixed at  $\bar{x}_2 = 8$ . State the firm's short-run production function.
  - (b) [6] Solve for the firm's short-run profit-maximizing choice of labor.
  - (c) [8] State the difference between a "Factor demand function" and a "Conditional factor demand function".
  - (d) [12] Solve for the firm's factor demand functions:  $x_1(p, w_1, w_2)$  and  $x_2(p, w_1, w_2)$ .
  - (e) [12] Assuming that the firm wishes to produce exactly  $(\bar{y})$  bicycles, solve for the firm's conditional factor demand functions:  $x_1(w_1, w_2, \bar{y})$  and  $x_2(w_1, w_2, \bar{y})$ .
  - (f) [6] Solve for the firm's minimum cost function for producing  $(\bar{y})$  bicycles:  $C(w_1, w_2, \bar{y})$ .
  
2. [25] A firm uses two inputs  $(x_1, x_2)$  to produce its output  $(y)$ . This firm employs perfect substitutes technology as specified by:  $f(x_1, x_2) = 2x_1 + x_2$ . Suppose the firm wishes to produce exactly (60) units of output in the least costly way, and  $w_1 = w_2 = \$6$ .
  - (a) [12] To produce (60) units of output in the least costly way, how many units of input one and how many units of input two would this firm choose to use? *Numeric answers.*
  - (b) [5] What is the firm's minimum cost in producing (60) units of output? *Numeric answer.*
  - (c) [8] On the same graph, graphically illustrate the associated isoquant and isocost lines that passes through the firm's cost-minimizing choice of inputs for producing exactly (60) units of output. *Numerically label all intercepts, and the cost-minimizing choice of inputs  $(x_1^*, x_2^*)$ .*
  
3. [25] Joe's Handicrafts produces plastic deer for lawn ornaments. His production function is given by  $f(x_1, x_2) = \text{Min}\{x_1, 2x_2\}$ , where  $x_1$  is the amount of plastic used and  $x_2$  is the amount of labor used.
  - (a) [5] Graphically illustrate all the input combinations that will produce (4) deers. *Label all axis and the kink.*
  - (b) [10] If the input prices are  $(w_1, w_2) = (1, 1)$ , and Joe wants to produce (4) deers in the cheapest way possible. How many units of each input would Joe use? How much would it cost?
  - (c) [10] If the input prices are  $(w_1, w_2) = (2, 3)$ , and Joe wants to produce (4) deers in the cheapest way possible. How many units of each input would Joe use? How much would it cost?