Homework 4 Solutions

(1) (a) $f(2x_1, 2x_2) = 2(2x_1) + (2x_2)$

$= 2(2x_1 + x_2)$

$= 2y$ * Constant Returns to Scale

(b) $f(2x_1, 2x_2) = \text{Min}\{(2x_1), 2(2x_2)\}$

$= 2 \cdot \text{Min}\{x_1, 2x_2\}$

$= 2y$ * Constant Returns to Scale

(c) $f(2x_1, 2x_2) = (2x_1)^2 (2x_2)$

$= (4x_1^2)(2x_2)$

$= 8x_1^2 x_2$

$= 8y > 2y$ * Increasing Returns to Scale

(d) $f(2x_1, 2x_2) = ((2x_1)^2 + (2x_2)^2)^{1/2}$

$= (4x_1^2 + 4x_2^2)^{1/2}$

$= [4(x_1^2 + x_2)^2]^{1/2}$

$= 2(x_1^2 + x_2^2)^{1/2} = 2y$ * Constant Returns to Scale

(2) For $(x_1, x_2) = (4, 16)$

(a)

$$f(x_1, x_2) = f(4, 16) = (4)^{1/2}(16)^{1/4} = (2)(2) = 4 < 20.$$

maximum output level using (4, 16) units of inputs.

$\boxed{\text{No}}$

(b) $$MP_1 = \frac{df(x_1, x_2)}{dx_1} = \frac{1}{2} x_1^{-1/2} x_2^{1/4}$$

$$MP_2 = \frac{df(x_1, x_2)}{dx_2} = \frac{1}{4} x_1^{1/2} x_2^{-3/4}$$

(c) $$TRS = -\frac{MP_1}{MP_2} = -\frac{\frac{1}{2} x_1^{-1/2} x_2^{1/4}}{\frac{1}{4} x_1^{1/2} x_2^{-3/4}} = -\frac{2x_2}{x_1}$$

(d) $$f(x_1, 16) = x_1^{1/2}(16)^{1/4} = 2x_1^{1/2}$$

(e) See part (h)

(f) At (x_1^*, y^*): $MP_1 = \frac{w_1}{p}$

$$MP_1 = \frac{df(x_1, 16)}{dx_1} = x_1^{-1/2}$$

$$x_1^{-1/2} = \frac{w_1}{p} = \frac{1}{4}$$

$$x_1^{1/2} = 4$$

$$\boxed{x_1^* = 16.}$$

$$\boxed{y^* = 2(x_1^*)^{1/2} = 2(4) = 8.}$$

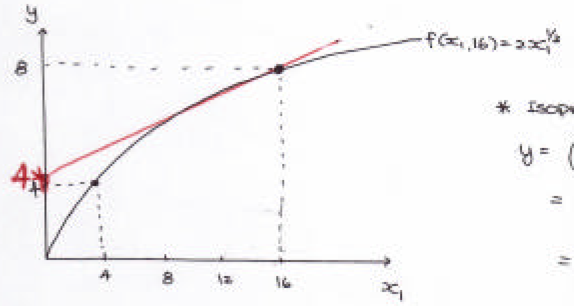
(g) $\pi_{SR} = p y^* - \omega_1 x_1^* - \omega_2 \bar{x}_2$

$= (4)(8) - (1)(16) - (0.5)(16)$

$= 32 - 16 - 8$

$= 8$

(h)



* Isoprofit line for $\pi = 8$.

$$y = \left(\frac{\pi + \omega_2 \bar{x}_2}{p}\right) + \left(\frac{\omega_1}{p}\right) x_1$$

$$= \left(\frac{8+8}{4}\right) + \left(\frac{1}{4}\right) x_1$$

$$= \underset{\text{intercept}}{4} + \underset{\text{slope}}{\left(\tfrac{1}{4}\right)} x_1$$

(i) In the LR: At (x_1^*, x_2^*)

① $P \cdot MP_1 = \omega_1 \rightarrow p \cdot \frac{1}{2} x_1^{-1/2} x_2^{1/4} = \omega_1 \rightarrow p(x_1^{1/2} x_2^{1/4}) = 2\omega_1 x_1 \rightarrow py = 2\omega_1 x_1 \rightarrow x_1 = \frac{py}{2\omega_1}$

② $P \cdot MP_2 = \omega_2 \rightarrow p \cdot \frac{1}{4} x_1^{1/2} x_2^{-3/4} = \omega_2 \rightarrow p \cdot (x_1^{1/2} x_2^{1/4}) = 4\omega_2 x_2 \rightarrow py = 4\omega_2 x_2 \rightarrow x_2 = \frac{py}{4\omega_2}$

$$y = x_1^{1/2} x_2^{1/4}$$

$$= \left(\frac{py}{2\omega_1}\right)^{1/2} \left(\frac{py}{4\omega_2}\right)^{1/4}$$

$$= \left(\frac{p}{2\omega_1}\right)^{1/2} \left(\frac{p}{4\omega_2}\right)^{1/4} y^{3/4}$$

$$y^{1/4} = \left(\frac{p}{2\omega_1}\right)^{1/2} \left(\frac{p}{4\omega_2}\right)^{1/4}$$

$$y = \left(\frac{p}{2\omega_1}\right)^{2} \left(\frac{p}{4\omega_2}\right)$$

$$= \frac{p^3}{16\omega_1^2 \omega_2}$$

$$\begin{cases} x_1^* = \dfrac{p}{2\omega_1}\left(\dfrac{p^3}{16\omega_1^2\omega_2}\right) = \dfrac{p^4}{32\omega_1^3 \omega_2} \\[2ex] x_2^* = \dfrac{p}{4\omega_2}\left(\dfrac{p^3}{16\omega_1^2\omega_2}\right) = \dfrac{p^4}{64\omega_1^2 \omega_2^2} \end{cases}$$