Economics 302
Microeconomic Theory
Fall 2005, Dr. Shirley Liu

Homework 4

Due: Monday, November 14, in lecture

Note: You can always hand it in early, but no homeworks passed the due time will be accepted

1. [20] For each of the technologies given below, verify whether the technology exhibits increasing, decreasing, or constant returns to scale.

   (a) $f(x_1, x_2) = 2x_1 + x_2$

   (b) $f(x_1, x_2) = \text{Min}\{x_1, 2x_2\}$

   (c) $f(x_1, x_2) = x_1^2 x_2$

   (d) $f(x_1, x_2) = (x_1^2 + x_2^2)^{\frac{1}{2}}$

2. [80] A firm uses two inputs, $x_1 =$ (Labor) and $x_2 =$ (capital), to produce one input (Y). The technology facing the firm is represented by: $f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{4}}$.

   (a) [5] Given the firm's technology, is the production plan $(x_1, x_2, y) = (4, 16, 20)$ technologically feasible?

   (b) [10] Derive the firm's marginal product of each input: MP_1 and MP_2.

   (c) [6] Derive the expression for the firm's technical rate of substitution: TRS.

   (d) [6] Suppose in the short run the level of input one is fixed: $x_2 = 16$. State the firm's short-run production function.

   (e) [9] Graphically illustrate the firm's short-run production set. *Label all axis and label two points on the production function.*

   (f) [14] Let $(p, w_1, w_2) = (4, 1, 0.5)$. Solve for the firm's short-run profit maximizing choices of input one (x_1^*) and output level (y^*).

   (g) [7] Let $(p, w_1, w_2) = (4, 1, 0.5)$. Calculate the firm's short-run maximum profits: π_{SR}.

   (h) [9] Let $(p, w_1, w_2) = (4, 1, 0.5)$. On the same graph as in part (e), graphically illustrate the "isoprofit line" corresponding to the firm's short-run maximum profit level. *Label the intercept and the slope of the isoprofit line.*

   (i) [14] Now suppose the firm is operating in the LONG RUN. Derive the firm's factor demand functions for each input (i.e. Firm's long-run profit maximizing choice of each input): $x_1(p_1, w_1, w_2)$ and $x_1(p_1, w_1, w_2)$. *Do not plug in numbers, keep your answers general.*