Homework 2 Solutions

(1) Perfect substitutes Pref: consumer is always willing to substitute $\left(\frac{a}{b}\right)$ units of good two for (1) more unit of good one.

$\boxed{\text{General Form: } U(x_1,x_2) = ax_1 + bx_2}$

Here: The consumer is always willing to substitute ($\frac{1}{3}$) units of good two for (1) more unit of good one.

$$\left(\frac{a}{b}\right) = \frac{1}{3} \rightarrow \begin{matrix} \text{Let } a=1 \\ b=3 \end{matrix}$$

$$\boxed{\text{Utility Function: } U(x_1,x_2) = x_1 + 3x_2}$$

(2) Perfect Complements Pref: consumer always wants $\left(\frac{b}{a}\right)$ times as much good one as good two.

General Form

$U(x_1,x_2) = \text{Min}\{ax_1, bx_2\}$

Here: The consumer always wants (3) times as much good one as good two

$$\left(\frac{b}{a}\right) = 3 \rightarrow \begin{matrix} \text{Let } b=3 \\ a=1 \end{matrix}$$

$$\boxed{\text{Utility Function: } U(x_1,x_2) = \text{Min}\{x_1, 3x_2\}}$$

(3) Given $MU_1 > 0 \rightarrow x_1$ is a regular good

$MU_2 = 0 \rightarrow x_2$ is a neutral good (levels of x_2 has NO Effect on utility)

※ $\boxed{\rightarrow x_2 \text{ should not enter the utility function}}$

Utility Function: $U(x_1,x_2) = x_1$

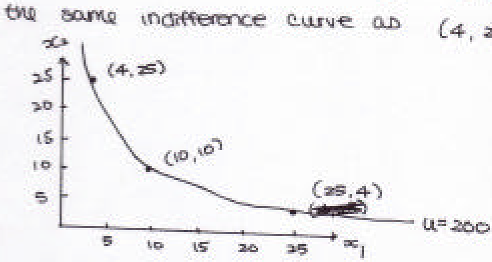
Examples

or

$U(x_1,x_2) = 5x_1$

(2) ⓐ $U(4,25) = 2(4)^{1/2}(25)^{1/2} = 200$

$\Rightarrow$ Any bundle (x_1, x_2) that satisfy $2x_1^{1/2}x_2^{1/2} = 200$ should be on the same indifference curve as $(4, 25)$



ⓑ $MU_1 = \dfrac{\partial U(x_1, x_2)}{\partial x_1} = x_1^{-1/2} x_2^{1/2}$

$MU_2 = \dfrac{\partial U(x_1, x_2)}{\partial x_2} = x_1^{1/2} x_2^{-1/2}$

ⓒ $MRS = -\dfrac{MU_1}{MU_2} = -\dfrac{x_1^{-1/2} x_2^{1/2}}{x_1^{1/2} x_2^{-1/2}} = -\dfrac{x_2}{x_1}$

ⓓ $MRS\big|_{(20,5)} = -\dfrac{(5)}{(20)} = -\frac{1}{4}$

$MRS\big|_{(5,20)} = -\dfrac{(20)}{(5)} = -4$

Frank's preferences does exhibit "Diminishing MRS". At (5, 20) he is willing to give up (4) units of good 2 to get (1) more unit of good one ~~the~~ to stay just as well-off. As he obtains more and more good one, such as at (20, 5), his willingness to substitute good two for good one diminished (Now he is only willing to give up ($\frac{1}{4}$) units of good two for one more unit of good one.)

(e) Given Frank has Well-behaved Pref.

At the optimal / demand bundle (x_1^*, x_2^*)

1. $MRS = -P_1/P_2$

2. $P_1 x_1 + P_2 x_2 = m$

Here $MRS = -\frac{x_2}{x_1}$

1. set $-\frac{x_2}{x_1} = -\frac{P_1}{P_2} \Rightarrow x_2 = \frac{P_1 x_1}{P_2}$

2. given $P_1 x_1 + P_2 x_2 = m \Rightarrow P_1 x_1 + P_2 \left(\frac{P_1 x_1}{P_2}\right) = m$

$$2P_1 x_1 = m$$

Demand Functions for x_1, x_2
$$\begin{cases} x_1^* = \dfrac{m}{2P_1} \\ x_2^* = \dfrac{P_1}{P_2}\left(\dfrac{m}{2P_1}\right) = \dfrac{m}{2P_2} \end{cases}$$

(3)

(a) MRS = − 3

(b) No, for perfect substitutes preferences, MRS are always the same (Does not change)

(c) $\frac{1}{2}$ of a sandwich.

(For each hamburger, which costs \$2, he could have used the \$ 2 to buy $\frac{1}{2}$ of a sandwich)

(d) given

$$\left.\begin{array}{l} MRS = -3 \\ -\frac{P_1}{P_2} = -2 \end{array}\right\} \quad \begin{array}{c} MRS < -\frac{P_1}{P_2} \\ \shortparallel \\ -3 < -\frac{P_1}{P_2} \\ 3 > \frac{P_1}{P_2} \\ \boxed{3P_2 > P_1} \end{array}$$

To Oscar, each ~~hamburger~~ sandwich is just as good as 3 hamburgers (They are perfect substitutes in this way). If 3 hamburgers will cost him more than 1 sandwich ($3P_2 > P_1$), Oscar will choose to spend all his income on sandwiches, and none on hamburgers.

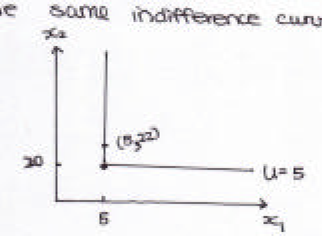
$$\begin{cases} x_1^* = \frac{m}{P_1} = \frac{60}{4} = 15 \\ x_2^* = 0 \end{cases}$$

(4)

(a) Perfect Complements

(b) $U_S(5,22) = \text{Min}\{5, \tfrac{1}{4}(22)\} = 5$

$\Rightarrow$ Any bundle (x_1, x_2) that satisfy $\text{Min}\{x_1, \tfrac{1}{4}x_2\} = 5$ is on the same indifference curve as $(5, 22)$.



kink: $x_1 = 5$

$\tfrac{1}{4}x_2 = 5 \rightarrow x_2 = 20$

(c) No, violates both strong monotonicity & strict convexity.

Strong Monotonicity

Ex: consider $X = (5,20)$

$Y = (5,22)$

By strong monotonicity: $X \succ Y$

But here: $U(5,20) = U(5,22)$

$X \sim Y$

Strict Convexity.

Ex: consider $X = (5,20)$

$Y = (5,22)$

Let Z = Any weighted avg of X, Y

Suppose $Z = (5, 21)$

By strict convexity: $Z \succ X, Y$.

But here $U(Z) = U(X) = U(Y)$

$Z \sim X \sim Y$.

(d) Neutral

(e) $U_S(3,3) = \text{Min}\{(3), \tfrac{1}{4}(3)\} = 3/4$

$U_S(1,4) = \text{Min}\{(1), \tfrac{1}{4}(4)\} = 1$

Shirley: $(1,4) \succ (3,3)$

$U_D(3,3) = 5(3) = 15$

$U_D(1,4) = 5(4) = 20$

Denise: $(1,4) \succ (3,3)$

(f) No, utility is an individual-specific ordinal ranking of bundles. Utility across individuals cannot be compared

(g) Shiney has Perfect Complements preferences

The optimal bundle (x_1^*, x_2^*) is where ① $x_1 = \frac{1}{4}x_2$ (At the "kink")

② $P_1 x_1 + P_2 x_2 = m$

$$x_1 = \frac{1}{4}x_2 \quad \text{or} \quad 4x_1 = x_2$$

$$P_1 x_1 + P_2 x_2 = m$$

$$P_1 x_1 + P_2 (4x_1) = m$$

$$x_1 (P_1 + 4P_2) = m$$

$$x_1^* = \frac{m}{P_1 + 4P_2} = \frac{100}{(1) + 4(1)} = 20$$

$$x_2^* = 4x_1^* = \frac{4m}{P_1 + 4P_2} = 80$$

(h) Since x_1 = neutral good for Denise, she'll not consume any x_1 and will consume only x_2.

$$x_1^* = 0$$

$$x_2^* = \frac{m}{P_2} = \frac{100}{1} = 100$$