1 John and Jim's parameters

1.1 The parameters

marginal abatement cost function: $18 - \theta e$

8 firms.

Half of them are *High MC* subjects ($\theta = \theta_H = 1$) and half *Low MC* subjects ($\theta = \theta_L = 2$).

To be compatible with this assumption we assume that

$$c(e) = 100 - 18e + \frac{\theta}{2}e^2$$
de tal manera que
$$c'(e) = -18 + \theta e$$
y
$$-c'(e) = 18 - \theta e$$

We took their *low aggregate standard experiment*: $L = 28$, and their "uniform" allocation ($l_H^0 = 3$ and $l_L^0 = 4$).

The marginal penalty function is $f'(e - l) = \phi + \gamma(e - l)$.

We assume $f(e_i - s_i) = \phi(e_i - s_i) + \frac{\gamma}{2}(e_i - s_i)^2$

The *High* enforcement strategy is $\pi_H = 0.7$, $\phi = 17.5$, and $\gamma = 1.43$). This enforcement strategy should induce perfect compliance by risk neutral firms.

The *Low* enforcement strategy is $\pi_H = 0.35$, $\phi = 2$, and $\gamma = 2.9$).

In our problem π is a choice variable.

1.2 Perfect information

1.2.1 The Cost-Effectiveness of Inducing Perfect Compliance

$f"(0) = \gamma = 1.43$

$f'(0) = \phi = 17.5$

We assume.

$\mu = \mu_i = 5$

$\beta = \beta_i = 0.5$

The condition to induce compliance or not is therefore

$\mu_i \frac{f''(0)}{f'(0)} \leq \beta_i f'(0) => 5 \times \frac{1.43}{17.5} \leq 0.5 \times 17.5?$

$$5 \times \frac{1.43}{17.5} = 0.408\,57 < 0.5 \times 17.5 = 8.\,75$$

It is therefore cost effective to induce perfect compliance.

1.2.2 The allocation of emissions and probabilities in a perfectly enforced program based on emission standards

The regulator's problem is

$$\min_{\substack{(s_L,s_H)\\(\pi_L,\pi_H)}} 4\times\left(100-18\times s_H+\frac{s_H^2}{2}\right)+4\times\left(100-18\times s_L+s_L^2\right)+5\times 4\times(\pi_L+\pi_H)$$

subject to:

$$4\times s_H+4\times s_L=28$$

$4\times\left(100-18\times s_H+\frac{s_H^2}{2}\right)+4\times\left(100-18\times s_L+s_L^2\right)=2s_H^2-72s_H+4s_L^2-72s_L+800$

$4\times s_H+4\times s_L=28 => s_H+s_L=7$

$$\min_{\substack{(s_L,s_H)\\(\pi_L,\pi_H)}} 2s_H^2-72s_H+4s_L^2-72s_L+800+5\times 4\times(\pi_L+\pi_H)$$

subject to:

$$s_H+s_L=7$$

Y como el regulador va a elegir $\pi_L^*=\frac{-c_i'(s_L^*)}{f'(0)}=\frac{18-2s_L^*}{17.5}$ y $\pi_H^*=\frac{-c_i'(s_H^*)}{f'(0)}=\frac{18-s_H^*}{17.5}$

Lagrange:

$L=2s_H^2-72s_H+4s_L^2-72s_L+800+5\times 4\times(\pi_L+\pi_H)+\lambda\times(7-s_H-s_L)$

$L=2s_H^2-72s_H+4s_L^2-72s_L+800+5\times 4\times\left(\frac{18-2s_L}{17.5}+\frac{18-s_H}{17.5}\right)+\lambda\times(7-s_H-s_L)$

$\frac{\partial L}{\partial s_H}=\frac{\partial\left(2s_H^2-72s_H+4s_L^2-72s_L+800+5\times 4\times\left(\frac{18-2s_L}{17.5}+\frac{18-s_H}{17.5}\right)+\lambda\times(7-s_H-s_L)\right)}{\partial s_H}=4s_H-\lambda-73.143=0$

$\frac{\partial L}{\partial s_L}=\frac{\partial\left(2s_H^2-72s_H+4s_L^2-72s_L+800+5\times 4\times\left(\frac{18-2s_L}{17.5}+\frac{18-s_H}{17.5}\right)+\lambda\times(7-s_H-s_L)\right)}{\partial s_L}=8s_L-\lambda-74.286=0$

$\frac{\partial L}{\partial \lambda}=\frac{\partial\left(2s_H^2-72s_H+4s_L^2-72s_L+800+5\times 4\times\left(\frac{18-2s_L}{17.5}+\frac{18-s_H}{17.5}\right)+\lambda\times(7-s_H-s_L)\right)}{\partial \lambda}=7-s_L-s_H=0$

El sistema queda

$4s_H-\lambda-73.143=0,\ \lambda=4s_H-73.143$

$8s_L-\lambda-74.286=0,\ \lambda=8s_L-74.28$

Por lo que

$4s_H-73.143=8s_L-74.28$

$s_L=\frac{1}{2}s_H+0.142\,13$

Sustituyendo en la tercera:

$7-\frac{1}{2}s_H-0.142\,13-s_H=0$, Solution is: $\mathbf{s}_H=\mathbf{4.571\,9}$

$\mathbf{s}_L=\frac{1}{2}\times 4.571\,9+0.142\,13=2.428\,1$

$\mathbf{s}_L=\mathbf{2.428\,1}$

$\pi_L^*=\frac{18-2\times\mathbf{2.428\,1}}{17.5}=\mathbf{0.751\,07}$ y $\pi_H^*=\frac{18-\mathbf{4.571\,9}}{17.5}=\mathbf{0.767\,32}$

Costo total del programa:

$4\times\left(100-18\times s_H+\frac{s_H^2}{2}\right)+4\times\left(100-18\times s_L+s_L^2\right)+5\times 4\times(\pi_L+\pi_H)=$

$4\times\left(100-18\times\mathbf{4.571\,9}+\frac{\mathbf{4.571\,9}^{21}}{2}\right)+4\times\left(100-18\times\mathbf{2.428\,1}+\mathbf{2.428\,1}^2\right)+5\times 4\times(\mathbf{0.751\,07}+\mathbf{0.767\,32})=\mathbf{1.455\,7\times 10}^{14}$

Total emissions:
$4 * \mathbf{4.5719} + \mathbf{4} * \mathbf{2.4281} = \mathbf{28.0}$