10.1 An illustration of a game with no Nash equilibrium in pure strategies. There is an equilibrium in mixed strategies, however.

10.2 Another example of a game with a mixed-strategy equilibrium.

10.1

 **B's Strategies**

 **H T**

 **A's Strategies** **H** +1 1

 **T** 1 +1

Payoffs are recorded from *A*'s perspective. Clearly, there is no Nash Equilibrium here in pure strategies. If *B* knows what *A* will do, it can achieve +1, but *A* can see this by putting herself into *B*'s position. If each flips the coin, the expected outcome is zero. This mixed strategy is a Nash Equilibrium. If *B* knows *A* is following such a strategy, she could play *H* with probability *P*, *T* with probability 1  *P*. Expected payoff is

2(*P* )(+1) + 2(*P* )(1) + 2(1  *P* )(+1) + 2(1  *P* )(1) = 0

but any choice of *P* other than 2 gives *A* an incentive to depart from 2. Let *q* be *A*'s probability of heads. Then her expected payoff is

*Pq*(+1) + *P*(1  *q*)(1) + (1  *P*) *q*(1) + (1  *P*)(1  *q*) (+ 1)

= *Pq  P + Pq  q + Pq* + 1  *P  q + Pq*

= 4*Pq*  2*P*  2*q* + 1.

If *P* > 2 *A* should choose *q* = 1 so expected payoff is 2*P*  1 > 0.

If *P* < 2 *A* should choose *q* = 0 and expected payoff is 1  2*P* > 0.

10.2 a. Payoffs from Smith's perspective are +3, 1.

**Jones' Strategies**

 **1 2 3**

**1** +3 2 1

**Smith's**

**Strategy** **2** 1 +3 1

**3** 1 1 +3

This game is similar to the one in Problem 10.1 in that it has no equilibrium pair.

b. Expected payoff to mixed strategy is for Smith

 a(3) + a(1) + a(1) = a .

For Jones it is

a(3) + a(1) + a(1) = a .

This mixed strategy is an equilibrium since neither player has an incentive to depart from it even if he or she knows what the other is doing (assuming the strategy choices are truly random).