Lagging the Dog?: The Robustness of Panel Corrected Standard Errors in the Presence of Serial Correlation and Observation Specific Effects

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Preliminary.

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Abstract

This paper examines the performance of the method of panel corrected standard errors (PCSEs) for time-series cross-section data when a lag of the dependent variable is included as a regressor. The lag specification can be problematic if observation-specific effects are not properly accounted for, leading to biased and inconsistent estimates of coefficients and standard errors. We conduct Monte Carlo studies to assess how problematic the lag specification is, and find that, although the method of PCSEs is robust when there is little to no correlation between unit effects and explanatory variables, the method's performance declines as that correlation increases. A fixed effects estimator with robust standard errors appears to do better in these situations.

1 Introduction

The method of panel corrected standard errors, developed by Beck and Katz (1995, 1996), is one of the most influential methodological innovations ever introduced in political science.[1] It is difficult to find a recent quantitative analysis of time-series cross-section (TSCS) data that does not use this method. It has proven to be extremely successful for helping researchers answer important political science questions. Yet there has been very little discussion of the robustness of this technique despite its wide application.

In this paper, we examine the robustness of panel corrected standard errors (PCSEs) in certain situations where the usefulness of the method may be compromised. We focus on potential problems introduced by using lagged dependent variables along with PCSEs. This is not a trivial issue since the method of PCSEs is widely employed with lagged specifications in the social sciences.[2] We read a random sample of 80 articles citing Beck and Katz (1995) and found that that approximately 40 percent of them report PCSEs obtained from models that include lagged dependent variables.[3] Lags are used with PCSEs for a couple of reasons. PCSEs are appropriate only if serial correlation is not present in the data. One standard method (arguably the preferred method) for removing the serial correlation that often occurs in TSCS data is to include a lagged dependent variable in the model specification. It is also quite common for researchers to include lagged dependent variables to capture temporal dynamics that theory indicates may exist. This can present a problem, however, if unobserved, observation-specific effects are also present in the data (or if a relevant, time-invariant explanatory variable has been mistakenly left out of the specification). Given the panel structure of the data, if such effects are not properly accounted for, introducing a lag can lead to biased and inconsistent coefficient estimates. This presents a problem because the attractiveness of the method of PCSEs depends crucially on the consistency of ordinary least squares point estimates. It is also possible that the consistency property of PCSEs themselves will fail to obtain.

[1]A search of the Social Science Citation Index currently produces 241 citations to Beck and Katz (1995).

[2]To make our terminology clear, by "method of PCSEs" we mean using OLS estimates of coefficients but robust estimates of standard errors of the coefficients.

[3]The sample was taken from the 203 articles that were published in the past five years and cite Beck and Katz.

In our study, we consider how problematic the lag specifications are for the performance of the method of PCSEs. We conduct Monte Carlo experiments to assess the severity of the problem and consider alternative corrections for serial correlation. We find that, although the method of PCSEs is robust when there is little to no correlation between unit effects and explanatory variables, the method's performance declines as that correlation increases. A fixed effects estimator with robust standard errors appears to do better in these situations. We reemphasize that researchers need to test for unit effects and their correlation with explanatory variables before proceeding to use PCSEs.

2 A review of the method of PCSEs

The key motivation for using PCSEs is to improve inferences made from TSCS data by taking into account the complexity of the error process, but in a way that does not ask too much of the data. The errors in TSCS models are likely to be nonspherical, exhibiting any or all of the following:

- Contemporaneous correlation: the errors across cross-sectional units are correlated due to common shocks in a given time period.
- Panel heteroskedasticity: the error variance differs across cross-sectional units due to characteristics unique to the units.
- Serial correlation: the errors within units are temporally correlated.

Ordinary least squares (OLS) is not the best linear unbiased estimator (BLUE) and can produce incorrect standard errors when the errors are nonspherical. Generalized least squares (GLS), which incorporates information about the errors and thereby makes up for the inefficiency of OLS, is BLUE and will give correct standard errors. However, GLS assumes that the variance-covariance matrix ($\mathbf{\Omega}$), which is used to weight the data, is known when in practice it is not. Instead, we can employ feasible generalized least squares (FGLS), which involves using an estimate of the variance-covariance matrix ($\hat{\mathbf{\Omega}}$).

Beck and Katz (1995) show, however, that the FGLS method advocated by Parks (1967) and Kmenta (1986) produces incorrect standard errors when applied to TSCS data. The poor statistical properties of this technique stem from the fact that it estimates an inordinate number of parameters in the variance-covariance matrix (Beck 2001, 280). Although FGLS works fine in large samples, TSCS data typically does not provide enough observations to estimate these parameters with much precision. The method gives overconfident standard errors because it does not fully take into account the variability in the estimates of the error parameters.

Beck and Katz (1995) argue that a superior way to handle complex error structures in TSCS analysis is to estimate the coefficients by OLS and then compute PCSEs. In this method, $\mathbf{\Omega}$ is an $NT \times NT$ block diagonal matrix with $\mathbf{\Sigma}$, an $N \times N$ matrix of contemporaneous correlations along the diagonal. OLS residuals, denoted $e_{i,t}$ for unit i at time t, are used to estimate the elements of $\mathbf{\Sigma}$:

$$\hat{\mathbf{\Sigma}}_{i,j} = \frac{\sum_{t=1}^{T} e_{i,t}e_{j,t}}{T}. \tag{1}$$

Then the standard errors of the coefficients are computed using the square roots of the diagonal elements of

$$(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\hat{\mathbf{\Omega}}\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}, \tag{2}$$

where $\mathbf{X}$ denotes the $NT \times NT$ matrix of stacked vectors of explanatory variables, $\mathbf{x}_{i,t}$. Although this approach estimates the same number of parameters as the FGLS method, it has better small sample properties. The intuition as to why this is the case is that PCSEs are similar to White's heteroskedasticity-consistent standard errors for cross-sectional estimators, but are better because they take advantage of the information provided by the panel structure of the data (Beck and Katz 1996, 34). Through Monte Carlo studies, Beck and Katz (1995, 1996) demonstrate that PCSEs produce more reliable standard errors than FGLS methods.

Based on these results, the method of PCSEs has been widely adopted in political science research. It is available in the most commonly used statistical software, and requires no more effort or cost to estimate than a standard regression model. The ease with which researchers can apply the method can lead us to use it without giving serious consideration to particular methodological nuances that may present problems. The method has been applied in research situations that are very different from those simulated in Beck and Katz's original analysis, raising concerns about its robustness. In the next section we discuss how it may be inappropriate to use this method for certain applications.

3 Lagged specifications for TSCS data

A crucial assumption for the method of PCSEs is that the errors are free of serial correlation. Yet it is reasonable to expect that such correlation would be common in TSCS data. Before this method is applied, the serial correlation must be removed. One popular way to do this is to include a lagged dependent variable in the model specification. Beck and Katz (1996) conduct simulations which show that the lag correction generally outperforms a FGLS estimator that uses the Prais-Winsten transformation, and therefore recommend that researchers begin with the former. Beck (2001, 279–280) states that "the modeling of dynamics via a lagged dependent variable allows researchers to estimate their specification using" OLS with PCSEs. Introducing a lag, can be problematic, however, because if serial correlation is not entirely removed by the lag, OLS will be inconsistent. Beck (2001, 279-280) advocates a Lagrange multiplier test to make sure the lag (or additional lags) takes care of the temporal dependence.

But the lag fix requires not only that no serial correlation remain, but also that no unit-specific effects be present in the data.[4] One of the reasons to employ methods beyond OLS for TSCS data is that observations in this data are not "exchangeable." That is, the labels on the observations matter, and we cannot arbitrarily change the position of an observation in the data set without changing the information on that observation. Panel heteroskedasticity, contemporaneous correlation, and serial correlation imply that observations are not exchangeable. The presence of unit effects also means the data are not exchangeable.

The discussion of unit-specific effects is largely absent from the articles that advocate PCSEs, yet the modeling of this kind of heterogeneity is at the heart of the analysis of data with repeated observations. Beck (2001, 282–287) addresses the issue of heterogeneity in TSCS data, but does not link this discussion with the issue of serial correlation or dynamics. He does recommend that researchers test for unit heterogeneity in their data, however. The dangers of unmodeled unit-specific effects in TSCS data are explicitly addressed by Green, Kim, and Yoon (2001). They remind us that when unmodeled unit effects are correlated with explanatory variables, OLS slope coefficients are biased and inconsistent.[5] The source

[4]Observation-specific effects can be a source of serial correlation.

[5]If they are not correlated with explanatory variables, then we simply get bias in the intercept.

of the problem is that, if they are not modeled explicitly, unit-specific effects are relegated to the disturbance term, which induces correlation between explanatory variables and the disturbance, violating the fundamental requirement for consistency. The situation is not terrible, however, because standard fixed effects estimators can be used to eliminate this problem.[6]

In response to Green et al. (2001), Beck and Katz (2001) argue that including fixed effects for models with continuous dependent variables to account for unobserved heterogeneity can be worse than leaving them out. The bias may not be that great in certain situations—namely, when the explanatory power of the unit effects is minimal. Fixed effects are perfectly collinear with time invariant variables and highly collinear with variables that move slowly. The former must be dropped if fixed effects are included in the model, while the latter will have imprecisely estimated coefficients. Thus, the loss in terms of inference on important substantive variables that are time invariant or move slowly can outweigh the gains of modeling heterogeneity. Beck and Katz (2001, 493) address the issue of dynamics, arguing that including lags of the dependent variable can make fixed effects less relevant (e.g., fixed effects are similar to including a lag with a coefficient of one). This is an interesting argument, although in the literature on dynamic panel models, unit heterogeneity and dynamics are treated as separate features to be modeled, which is consistent with Beck's (2001) position that these kind of features should be treated as substantive issues and not mere nuisance.[7] We will consider the degree to which the lag helps or hurts in our analysis reported below.

Data that requires separate modeling of unit heterogeneity and dynamics can be quite problematic for standard estimators. Even if independent variables are not correlated with the unit-specific effects, lagged dependent variables are correlated with such effects by construction. And if independent variables are correlated with the lagged dependent variable, their coefficients are biased and inconsistent. Standard fixed effects estimators do not necessarily take care of the attendant problems. For example, the least squares dummy variable (LSDV) or "within group" estimator is still biased and inconsistent (Baltagi 1995). Instru-

[6]We consider only fixed effects in this paper because, as Beck (2001) argues, it is more appropriate to think of unit specific effects in TSCS data as fixed as opposed to random effects (i.e., where the effects are drawn from some random distribution).

[7]For a review of the literature on dynamic panel models see Arellano and Honoré (2001).

mental variables (IV) estimators can be used to surmount these problems. But it is not clear that these estimators are appropriate for TSCS data. The asymptotic properties of many IV estimators are with respect to N, not T, but the latter typically dominates the former in studies where PCSEs are applied. IV estimators may be more appropriate for studies like those in the international relations literature which examine large numbers of country dyads. Although Beck and Katz (1995, 637) make clear that PCSEs are intended to address situations where T is larger, but not much larger, than N, this method has been used for data where N is much larger than T (e.g., see Blanton 1999; Keith 1999; Poe, Tate, and Keith 1999). IV estimators may be more appropriate for these kinds of studies than PCSEs.

But what are the implications for the method of PCSEs? The debate between Green et al. (2001) and Beck and Katz (2001) over the use of fixed effects in TSCS data does not address how PCSEs might be affected. If we use a lag to correct for serial correlation but do not adequately account for unit effects, OLS does not retain its properties that make it attractive for producing the point estimates used in the method of PCSEs. Furthermore, a key assumption required for the consistency of PCSEs is violated. If a lag is included on the right hand side in $\mathbf{X}$, the matrix of explanatory variables, and unit effects exist but are unmodeled and thereby relegated to the disturbance term ε, then $E[\mathbf{X}'\varepsilon]$ will not necessarily equal zero. This is because $\mathbf{X}$ contains a lag of the dependent variable, which is correlated with the the unit specific effects contained in ε. But this expectation is assumed to be zero in the proof that $\hat{\mathbf{\Sigma}} \overset{a.s.}{\to} \mathbf{\Sigma}$, and by extension $\hat{\mathbf{\Omega}} \overset{a.s.}{\to} \mathbf{\Omega}$ (see Beck and Katz 1996, 32–33 and White 1984, 59,165–166).[8]

It is not clear how much of a problem this is in practice, however. The theory that tells us PCSEs are reliable is grounded in their asymptotic properties. We are mainly concerned with small sample properties, but if large sample properties are not good, it does not bode well for the situation where we have small N or T. Still, it could be that including lags to eliminate serial correlation helps with the problem of unit effects. In fact, the existence of

[8] Since OLS coefficients are used to produce estimates of the residuals, it is possible that bias in OLS coefficients could lead to problems with the estimates of standard errors, which is the area where the Beck and Katz method gives the greatest gains. White's proof of consistency requires that $\hat{\beta} \overset{a.s.}{\to} \beta$ (i.e., the estimate of the slope coefficients converge almost surely to the population values), which will not necessarily happen if $E[\mathbf{X}'\varepsilon] \neq 0$.

such effects might appear as serial correlation to researchers who test for it. Since Beck and Katz (1995) make clear that such tests should be conducted, this pitfall might be avoided by taking steps to correct for serial correlation.[9] Since we are interested in how well different estimators perform in small samples, in the next section we conduct Monte Carlo studies to get a sense of how problematic these issues are.

4 Monte Carlo Analysis

We follow Beck and Katz (1995, 1996) in conducting simulations to determine how robust PCSEs are when lags are used to correct for serial correlation (or explicitly model dynamics) but unit effects are ignored. The data for the simulations are generated using error structures involving contemporaneous correlation, panel heteroskedasticity, and serial correlation. We assume that temporal dependence is the same across cross-sectional units within a simulation, although we vary the level of serial correlation as an experimental condition. To induce serial correlation, we generate the data in two ways. The first is by including dynamics in the model through a lagged dependent variable:

$$y_{i,t} = \rho y_{i,t-1} + \beta x_{i,t} + \alpha_i + u_{i,t}. \tag{3}$$

This sets up situations where we can have unmodeled dynamics in the data—an issue raised in Beck and Katz (2001)—when we omit $y_{i,t-1}$ from the estimation equation.[10] The lag correction should be most appropriate for serial correlation induced by this approach. We also generated serial correlation according to

$$y_{i,t} = \beta x_{i,t} + \alpha_i + u_{i,t} + \rho u_{i,t-1} + v_{i,t}. \tag{4}$$

Using this first-order autoregressive process (AR1) is less favorable to the lag correction than is generating the data by including a lag in the specification. This sets up a particularly

[9] For those who want to apply standard fixed effects estimators, theory tells us that the bias can go away as T becomes large. While it is not clear how large T has to get, the relatively large T in TSCS data (at least compared with standard panel data sets) may mean that the bias is not severe.

[10] We generated 50 values for $y_{i,t}$ starting from $y_{i,0} = 0$, which we discarded before generating the data used for the analysis.

difficult test of the lag correction. If we find that the lag correction performs well in this instance, then we can be very confident about using it in practice.[11]

We also vary the degree of correlation between the unit-specific effect α_i and the explanatory variable.[12] We did this by drawing the α_i from a uniform distribution and then scaling random normal deviates by α_i to produce the $x_{i,t}$. We are unable to precisely manipulate this correlation, because it is affected by the degree of panel heteroskedasticity in the data, which is tied to the value of $x_{i,t}$, as in Beck and Katz's original analyses. The level of correlation and heteroskedasticity is also likely to affect the amount of serial correlation induced in the data. If the value of the disturbance term was big last period, it will be big this period, especially if the disturbance is tied to size of explanatory variable, which is in turn tied to the size of the unobserved unit effect. It is quite possible that these different factors will be intertwined in real data as well. Thus, even crude manipulation of the correlation between α_i and $x_{i,t}$ is informative for the performance of different estimators.

As far as the explanatory power of $x_{i,t}$ and α_i goes, the data were generated so that the Schwartz Criterion only slightly favored a model that took fixed effects into account over plain OLS.[13] Our intention here is to avoid the situation where the results are driven by selecting unreasonably influential unit effects. The coefficient on $x_{i,t}$ was on average four times its standard error.[14]

We are primarily interested in the performance of the different approaches for estimating

[11] Beck and Katz (1996) use a more complicated procedure for generating serial correlation, using a combination of the autoregressive and lag structure approaches.

[12] Obviously, typical model specifications used with TSCS data have more than one explanatory variable. We include only one for ease of exposition and to keep our analysis as transparent and comparable as possible to Beck and Katz's simulations.

[13] Beck and Katz (2001) argue that the Schwartz Criterion (SC) is superior to the standard F test for the presence of fixed effects, because the SC imposes a higher penalty for including more explanatory variables. The SC provides a particularly difficult test for the LSDV model where separate dummies for each cross-sectional unit are specified. FE is identical to LSDV, except that FE performs a transformation of the model that obviates the need for including cross-section dummies. Thus it is not clear to us that the SC, when used with the FE estimator instead of LSDV, has the same properties that lead Beck and Katz to favor it.

[14] The coefficients in Beck and Katz (1996) were between two and three times their standard errors. We generated data to produce larger t statistics so that we would produce fewer models where $\hat{\beta}$ was not bounded away from zero.

β and its standard error. The main concern is that the method of PCSEs may perform poorly both in terms of point estimates and standard errors under certain conditions. To gage the accuracy of the point estimates for β, we computed the mean and root mean squared error (RMSE) of the estimated βs across replications. To assess the performance of the standard errors, we computed the Beck and Katz measure of optimism:

$$100\frac{\sum_{l=1}^{1000}\left(\beta_{OLS}^{(l)}-\bar{\beta}_{OLS}\right)^2}{\sum_{l=1}^{1000}\left[\mathrm{SE}\left(\beta_{OLS}^{(l)}\right)\right]^2}, \tag{5}$$

where l denotes replications and β_{OLS} denotes OLS estimates. Values above 100 indicate that true sampling variability is greater than the reported estimate of that variability, while values less than 100 indicate that the estimate understates true variability.[15] In each replication, we also performed a Lagrange Multiplier (LM) test to determine the existence of serial correlation, even after the inclusion of the lagged dependent variable to remove it. Even though the lag correction has been favored, it is valuable to know how it will perform when we know unit effects are present in the data.

In addition to examining the method of PCSEs, we also consider other approaches that are appropriate for data with repeated observations on cross-sectional units. We employed the standard within-group fixed effects estimator (FE). Theory tells us that this estimator is also biased and inconsistent when a lagged dependent variable is included, although the bias decreases at T gets large. The bias is of order $1/T$ (Nickell 1981), and therefore may not be much of a problem in TSCS data where T is larger than in typical panel data sets.

Given the error structure in the data, we also tried a technique for computing robust standard errors for the fixed-effects estimator proposed by Arellano (1987). Arellano's technique

[15] We also measured bias in the standard errors by computing

$$\frac{\sigma_{\beta^{(l)}}-\frac{1}{1000}\sum_{l=1}^{1000}\hat{\sigma}_{\hat{\beta}^{(l)}}}{\sigma_{\beta^{(l)}}} \tag{6}$$

where $\sigma_{\beta^{(l)}}$ is the standard error of the $\beta^{(l)}$ over the replications and $\hat{\sigma}_{\hat{\beta}^{(l)}}$ is the estimated standard error of β in replication l. Additionally, we considered the coverage of the confidence intervals, determining how often the true value of β falls within the estimated 95% confidence interval, while taking into account the length of the interval. We report only the measure of optimism because it makes our analysis comparable with Beck and Katz's original experiments. The bias other measures produced results consistent with the optimism statistic. Details of the other measures are available upon request from the authors.

has the same flavor as PCSEs, in that it employs a White estimator of the variance-covariance matrix.[16] The robust estimates of FE standard errors are computed from

$$\left(\tilde{\mathbf{X}}'\tilde{\mathbf{X}}\right)^{-1}\left(\sum_{i=1}^{N}\tilde{\mathbf{X}}_i{}'\hat{\tilde{u}}_i\hat{\tilde{u}}_i'\tilde{\mathbf{X}}_i\right)\left(\tilde{\mathbf{X}}'\tilde{\mathbf{X}}\right)^{-1}, \tag{7}$$

where $\tilde{\mathbf{X}}$ denotes the matrix of explanatory variables from the within group transformation and $\hat{\tilde{u}}_i$ are the estimated residuals obtained from running OLS on the transformed equation.[17]

We begin by reporting experiments where there is no serial correlation in the data (i.e., $\rho = 0$). This establishes a baseline for assessing the degree to which the lag specification is problematic. It also gives us a clean test of whether the lag specification helps to address the problem of unmodeled unit heterogeneity. For this first round of experiments, we set $\beta = 10$, $N = 15$, and $T = 20$.

Table 1 reports the results from these experiments. As the column labeled "% reject $\rho = 0$" in Table 1 indicates, the presence of unit effects almost always leads us to reject the null of serial correlation when no lag is included in the estimation model. The only cases where there is a nontrivial chance of not rejecting the null of no serial correlation is with very high contemporaneous correlation and heteroskedasticity. Even in those cases, the OLS point estimates and PCSEs do fine.

The fixed effects model does somewhat better in estimating β when there is minimal correlation between α_i and $x_{i,t}$, and a lot better when there is low correlation. The bias in β_{OLS} relative to the fixed effects estimator β_{FE}, should come as no surprise, and appears only to be a serious problem when there is correlation between α_i and $x_{i,t,}$ (displayed in

[16]This technique assumes that N is large and T is small, which is the opposite of what we usually think of with TSCS data. But we will consider experiments where N is large relative to T since researchers have applied PCSEs to data that takes this form. Other robust estimators for the standard errors were considered, but we do not report results on them because they performed so poorly.

[17]Another option is to estimate the least squares dummy variable (LSDV) model for the point estimates of β and then compute PCSEs. LSDV is equivalent to the FE estimator we employ, but requires the inclusion of $N - 1$ additional dummy variables in the model. Even with moderately sized Ns, including these additional dummies can be cumbersome and violate limits on matrix size in commonly employed software. We ran some experiments and found that the results for LSDV with PCSEs were nearly identical to the results for FE with the robust standard error estimate. We are note aware of any studies that have used PCSEs with LSDV, but this is certainly an option for researchers.

the column marked $\rho_{\alpha,x}$). OLS with PCSEs tends to overstate both the size of β and the true sampling variability (although not by much for the latter). Overstating variance implies that we might not reject the null of a zero coefficient when that null is false, leading us to conclude that variables do not have effects when in fact they do. However, unless there is no heteroskedasticity in the data, researchers are advised to use the robust estimator of the standard errors rather than the normal fixed effects standard errors, which do quite poorly.[18]

Including a lag helps to improve the quality of the OLS estimates and the PCSEs, although the former are still on average quite far from the truth even with low correlation between α and x. We have about a 50–50 chance of rejecting the null of no serial correlation and thus proceeding with the method of PCSEs. While the PCSEs are in general the most accurate of any of the standard errors we considered, the bias in β_{OLS} suggests we would be better off using a fixed effects estimator, possibly even without a lag. Ancillary analysis indicated that with high correlations between α and x and no serial correlation, PCSEs are "pessimistic" by only 5 to 10 percent, implying that omitted variable bias is not completely driving the results on PCSEs.

What we are mainly concerned with is using the lag to correct for serial correlation, so the remaining analyses focuses on models that actually include some temporal dependence apart from cross-sectional effects. Table 2 reports results with minimal correlation between α_i and $x_{i,t}$. The average of β_{OLS} is always closer to truth than is the average of β_{FE}, and their RMSEs are comparable. The FE point estimates get worse as ρ increases, while the OLS estimates get slightly better, most likely due to the increasing collinearity between α_i and the lagged dependent variable. PCSEs produce virtually exact estimates of sampling variability, and are a few percentage points more accurate than the robust FE standard errors. The method of PCSEs is recommended in this situation.

[18]We do not report results for the estimated coefficient on the lagged dependent variable because researchers are not generally interested in making substantive inferences about the lag. We note however, that we did see substantial bias in the estimated lag coefficient, just as theory would have indicated. The OLS estimates of the lag coefficient were generally biased upward, as we would expect because of the unit effect. The bias was so large in some cases that it would have falsely raised concerns about unit roots. If researchers are genuinely interested in making inferences about dynamics, they need to be much more careful about the estimator they choose.

However, the performance of the method of PCSEs begins to deteriorate as the (positive) correlation between α_i and $x_{i,t}$ increases. Table 3 shows that when this correlation is moderate (i.e., between .45 and .55) and serial correlation is low, the OLS point estimates are very far from truth. FE point estimates do much better both in terms of the average point estimate and RMSE (until ρ reaches .9). PCSEs do fairly well according to the measure of optimism, as long as heteroskedasticity is below .3. When heteroskedasticity is .5, PCSEs overstate true sampling variability between 10 and 20 percent. But according to Beck and Katz (1996, 20), this degree of heteroskedasticity is rare in real data sets, so we may not have to worry about the reliability of PCSEs in practice. The FE robust standard errors are overconfident by only a few percentage points, and remain solid across the ranges of contemporaneous correlation, heteroskedasticity, and serial correlation examined. When the performance of the point estimates and standard errors is considered jointly, researchers can do a fair amount better by using the FE estimator with robust standard errors instead of OLS with PCSEs. It should be noted that the standard errors given by the FE estimator do very badly. Even though accounting for unit effects appears to be important, doing so with the FE estimator without correcting for the standard errors is not likely to lead researchers to correct inferences

Table 4 shows the same pattern with very high correlation between α_i and $x_{i,t}$. OLS tends to overestimate β by a great deal when $\rho \leq .5$, especially when there is no heteroskedasticity in the data, although PCSEs are most accurate with homoskedasticity (they are dead on with high serial correlation). At their worst PCSEs are overly "pessimistic" by about 25%. Still it is important to note that PCSEs generally do not do that badly, especially when they only need to correct for contemporaneous correlation. The FE estimator outperforms OLS both in terms of the average of the estimates of β and RMSE. The robust FE standard errors are slightly overoptimistic, but generally do well across the experimental conditions.

The results are essentially the same when we generate the data using an AR1 process. Table 5 reports the results for some of those experiments. For the OLS point estimates, including a lag to correct for serial correlation is clearly superior to not including it. But even with the lag, β_{OLS} is still often quite far from the true β. For the FE estimator, we do slightly better by estimating the model without a lag and then using the robust estimator for the standard errors to take care of serial correlation. Although we do not report those results here, the difference in the performance of the FE estimator without the lag was increasing in ρ (i.e., it did worse as ρ increases, not surprisingly).

To summarize the results to this point, the method of PCSEs is fairly robust to situations where both serial correlation and observation specific effects are present in the data. The lag correction for serial correlation does not seem to be terribly problematic for inferences about exogenous variables when those variables are uncorrelated with α_i, although it does not help much with the problem of unobserved unit effects. When there is medium to high correlation between unobserved unit effects and explanatory variables, the bias in the OLS estimates of the coefficients on those variables can be quite high and the performance of PCSEs deteriorates somewhat.[19] In these cases, the within group estimator with robust standard errors generally works better. Thus, although theory tells us that introducing a lag when there are unobserved individual effects produces bias, in practice it appears better to include the lag to eliminate serial correlation in TSCS data if we do not explicitly model individual effects. But we can do much better in some cases by accounting for this heterogeneity, and possibly leaving out the lag in a FE model.

So far we have kept N and T constant. Do we see the same patterns when we vary sample size? Table 6 reports results when $T = 40$ and correlation between α_i and $x_{i,t}$ is moderate. Comparing these results with Table 3, which has smaller T but the same level of correlation, we find that the increase in the number of time periods slightly improves the performance of both OLS with PCSEs and FE with robust standard errors. The gain for the latter is a bit better, which is consistent with what theory tells us. Unless ρ is very high, the FE point estimates are to be preferred to OLS point estimates.

If we drop N to 5 but keep the same level for $\rho_{\alpha,x}$, as we did for the experiments reported in Table 7, we find that β_{FE} generally does better than β_{OLS} in terms of the point estimates of β, although the performance of the former is still not very good. PCSEs generally do better than the robust fixed effects standard errors, so it is a toss-up between which estimator is better in this situation.

[19] With low levels of serial and contemporaneous correlation and homoskedasticity, OLS does very badly for the point estimates, producing values that are on average almost twice the size of true β. However, these estimates are farther off on average without the lag correction.

Perhaps a more interesting case is when we increase N to 100. Although the original Monte Carlo experiments that Beck and Katz performed did not consider values of N that were this large, the method of PCSEs has been applied in research with this number of cross-sectional units, so it is valuable to assess performance with simulated data of these dimensions. Table 2 reports results when the average value for $\rho_{\alpha,x}$ is 0.[20] The performance of OLS and FE in terms of RMSE are nearly identical, as is the performance of PCSEs and FE robust standard errors. We note that there is a greater tendency to reject the null of no serial correlation even though a lag has been included, but PCSEs perform fine even with the serial correlation induced by the unobserved unit effects.

As with the other experiments, things are more problematic for the methods of PCSEs when there is correlation between α_i and $x_{i,t}$. As Table 9 reports, with moderate to high values for $\rho_{\alpha,x}$, OLS does about the same in terms of the average estimate of β and RMSE as it did with smaller N. The performance of PCSEs is somewhat worse, overstating variance by as much as 30 percent in some cases. One saving grace is that we are more likely to reject the null of no autocorrelation with larger N, although there is still a nontrivial chance that we would not reject and proceed with the method of PCSEs in certain cases. The increase in N enhances the performance of the fixed effects estimator, with the FE robust estimates of the standard errors matching true variability in the point estimates. Although the FE estimator does worse as ρ increases, these results suggest that with low to moderate serial correlation, researchers with large Ns in their data will probably do fine with this estimator, and do not need to employ more complicated dynamic panel data estimators.

To summarize, researchers who want to use OLS with PCSEs appear to be better off by including a lag in their specifications when there is serial correlation (whether generated by "true" dynamics or an autoregressive error process) and observation-specific effects in the data. The bias that can result from including the lag is less of a problem than the bias that results from doing nothing about the heterogeneity in the data. The method of PCSEs will work fine as long as there is no correlation between the unit effects and explanatory variables. When such correlation exists, the method of PCSEs can lead to very inaccurate point estimates and Type II errors. In these cases, researchers are probably better off using

[20] We conducted experiments for a smaller range for ρ because of the excessive amount of time it takes for these experiments to run.

a fixed effects estimator with Arellano's robust standard errors, which may even obviate the the inclusion of the lag to correct for serial correlation. If the number of cross-sectional units is large compared with the number of time periods, the gains of using FE with robust standard errors instead of OLS with PCSEs can be substantial.

It should be reemphasized then that analysts of TSCS data should include a test for unit effects in their battery of tests to determine the appropriate estimation approach. Tests for serial correlation that should be *de rigeuer* in TSCS analysis can reveal the presence of unit effects. A test of correlation between such effects and explanatory variables is also important. One problem with this recommendation is that the standard Hausman test which compares the within-group and generalized least squares estimators for panel data is invalid when the errors are heteroskedastic and/or serially correlated, which compromises its usefulness for TSCS data. The only test that we are aware of that is possibly appropriate for the kinds of nonspherical errors that we typically see in TSCS analysis is that proposed by Arellano (1993), which relies on the forward orthogonal deviations transformation developed by Arellano and Bover (1995). But this test is designed for large N data and models without lagged dependent variables, and so therefore may not work very well in the situations were it is most needed.

5 Discussion

Researchers who analyze TSCS data owe a large debt to Beck and Katz for not only raising our level of consciousness about potential problems for standard estimators, but also for providing a robust method that corrects for some of those problems. This paper follows in the spirit of Beck and Katz's seminal articles on TSCS data—that TSCS data presents unique challenges for standard methods, and that there may be room for improvement over these methods. PCSEs are very robust and will often serve researchers well. We have sought to shed more light on how the method of PCSEs performs when lags of the dependent variable are included as regressors and there are unobserved, observation-specific effects in the data. Methods that account for observation-specific effects can in some cases do better than PCSEs.[21]

[21]We note that the improvements over PCSEs that we found are not nearly as substantial as the improvements that Beck and Katz found over the method of panel weighted least squares.

Our intention is not to discourage the use the method of PCSEs. The method is very robust and deserves to be part of our statistical toolkit. We do, however, want to encourage researchers to consider that their data might present challenges that require methods beyond that of PCSEs. It is important to check for unit effects and to assess their correlation with explanatory variables. A priority should be coming up with a good method for determining the degree of this correlation.

here are several issues which we have not addressed that will require more work. First, we did not consider what happens when more than one lag of the dependent variable is included as a regressor. Adding more lags can help remove serial correlation that remains after one lag is included and may help eliminate some of the problems that we found with the performance of the methods we examined. Additional experiments can be done to assess how more lags may help. Second, even though the FE estimator with robust standard errors outperformed the method of PCSEs in certain cases, its performance was still not as good as we might like. There is undoubtedly room for improvement over this approach. For large N, IV estimators developed in the literature on dynamic panel data have some promise, but a more thorough investigation of them is required before we can recommend using them for data sets that have a relatively large number of time periods. These methods are certainly not appropriate when the time dimension dominates the cross-sectional dimension. Third, although our reading of the literature leads us to conclude that the Prais-Winsten transformation for eliminating serial correlation is inferior to the lag correction, it may be worth revisiting to see how it performs in the presence of observation specific effects. Finally, we have sidestepped the issue of what to do if a model contains substantively important variables that vary little, if at all, over time. Time invariant variables have to be dropped when the FE estimator is employed, and inferences may change on slow moving variables due to collinearity with the unit effects. It may be difficult to tell whether the changes in the estimated effects of slow moving variables are due to bias or simply collinearity (although the two issues are related). More simulations of the kind reported in this paper can help guide researchers who confront this problem.

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Table 1: Monte Carlo Analysis With No Serial Correlation in the Data (N=15; T=20)

Cor.	Het.	ρ	% reject $\rho = 0$	$\rho_{\alpha,x}$	$\bar{\beta}_{OLS}$	RMSE$_{OLS}$	$\bar{\beta}_{FE}$	RMSE$_{FE}$	Optimism PCSE	Optimism FE	Optimism FE Robust
No lag in estimation model											
0.25	0.0	0.00	1.00	0.003	10.248	4.623	10.060	3.028	100	102	106
0.25	0.3	0.00	1.00	0.002	10.146	3.304	10.059	2.457	99	84	104
0.25	0.5	0.00	0.85	0.001	10.088	2.248	10.045	1.731	99	59	102
0.50	0.0	0.00	1.00	0.003	10.245	4.625	10.058	3.038	100	102	106
0.50	0.3	0.00	0.99	0.002	10.147	3.313	10.059	2.464	100	84	104
0.50	0.5	0.00	0.80	0.001	10.092	2.259	10.045	1.736	100	59	102
0.25	0.0	0.00	1.00	0.299	28.049	18.501	9.996	3.075	74	103	108
0.25	0.3	0.00	1.00	0.287	21.210	11.582	9.980	2.501	80	85	105
0.25	0.5	0.00	0.76	0.265	16.610	6.925	9.980	1.763	86	60	103
0.50	0.0	0.00	1.00	0.299	28.034	18.484	9.971	3.066	74	103	107
0.50	0.3	0.00	0.98	0.287	21.196	11.566	9.959	2.491	80	85	105
0.50	0.5	0.00	0.71	0.265	16.599	6.930	9.965	1.755	87	60	103
Lag included in estimation model											
0.25	0.0	0.00	0.55	0.003	10.272	3.769	10.012	3.103	100	104	106
0.25	0.3	0.00	0.65	0.002	10.185	2.969	10.015	2.520	99	86	104
0.25	0.5	0.00	0.55	0.001	10.109	2.127	10.007	1.777	99	61	103
0.50	0.0	0.00	0.53	0.003	10.296	3.780	10.011	3.110	101	105	106
0.50	0.3	0.00	0.60	0.002	10.202	2.973	10.016	2.524	100	87	104
0.50	0.5	0.00	0.52	0.001	10.121	2.131	10.009	1.780	99	61	103
0.25	0.0	0.00	0.48	0.299	17.157	8.135	9.921	3.148	104	105	107
0.25	0.3	0.00	0.55	0.288	16.401	7.025	9.920	2.565	99	87	105
0.25	0.5	0.00	0.42	0.265	14.827	5.246	9.933	1.812	94	62	104
0.50	0.0	0.00	0.50	0.299	17.135	8.127	9.895	3.122	104	105	107
0.50	0.3	0.00	0.54	0.288	16.394	7.036	9.899	2.546	99	87	105
0.50	0.5	0.00	0.47	0.265	14.832	5.283	9.918	1.800	94	62	104

Table 2: Monte Carlo Analysis With Dynamics Included and Minimal Correlation Between α_i and $x_{i,t}$ ($N = 15; T = 20$)

Cor.	Het.	ρ	% reject $\rho = 0$	$\rho_{\alpha,x}$	$\bar{\beta}_{OLS}$	RMSE$_{OLS}$	$\bar{\beta}_{FE}$	RMSE$_{FE}$	Optimism		
									PCSE	FE	FE Robust
0.25	0.0	0.25	0.17	−0.003	9.913	3.576	9.847	3.087	100	103	105
0.25	0.3	0.25	0.24	−0.002	9.920	2.881	9.870	2.513	100	86	103
0.25	0.5	0.25	0.29	−0.002	9.933	2.098	9.897	1.773	100	61	102
0.50	0.0	0.25	0.33	−0.003	9.875	3.553	9.851	3.067	100	103	104
0.50	0.3	0.25	0.37	−0.002	9.896	2.871	9.873	2.507	100	86	103
0.50	0.5	0.25	0.38	−0.002	9.921	2.099	9.898	1.772	100	61	102
0.25	0.0	0.50	0.32	−0.003	9.937	3.387	9.811	3.082	101	103	105
0.25	0.3	0.50	0.26	−0.002	9.941	2.752	9.834	2.510	100	86	103
0.25	0.5	0.50	0.23	−0.002	9.947	2.007	9.859	1.773	100	61	102
0.50	0.0	0.50	0.44	−0.003	9.912	3.366	9.814	3.062	100	103	104
0.50	0.3	0.50	0.40	−0.002	9.924	2.745	9.835	2.504	100	86	103
0.50	0.5	0.50	0.36	−0.002	9.938	2.007	9.859	1.773	101	61	102
0.25	0.0	0.75	0.33	−0.003	9.957	3.197	9.707	3.067	101	103	104
0.25	0.3	0.75	0.31	−0.002	9.961	2.608	9.729	2.502	101	86	103
0.25	0.5	0.75	0.29	−0.002	9.967	1.881	9.753	1.774	100	61	102
0.50	0.0	0.75	0.43	−0.003	9.949	3.181	9.706	3.054	101	103	104
0.50	0.3	0.75	0.42	−0.002	9.955	2.605	9.726	2.503	101	86	103
0.50	0.5	0.75	0.41	−0.002	9.964	1.884	9.747	1.781	101	61	102
0.25	0.0	0.90	0.21	−0.003	9.961	3.070	9.501	3.069	101	103	105
0.25	0.3	0.90	0.23	−0.002	9.969	2.508	9.524	2.512	101	86	103
0.25	0.5	0.90	0.25	−0.002	9.982	1.791	9.546	1.804	100	61	102
0.50	0.0	0.90	0.37	−0.003	9.964	3.053	9.493	3.062	101	103	105
0.50	0.3	0.90	0.37	−0.002	9.971	2.507	9.511	2.522	101	86	104
0.50	0.5	0.90	0.38	−0.002	9.982	1.795	9.528	1.822	101	61	103

Table 3: Monte Carlo Analysis With Dynamics Included and Medium Correlation between α_i and $x_{i,t}$ ($N = 15; T = 20$)

Cor.	Het.	ρ	% reject $\rho = 0$	$\rho_{\alpha,x}$	$\bar{\beta}_{OLS}$	RMSE$_{OLS}$	$\bar{\beta}_{FE}$	RMSE$_{FE}$	Confidence		
									PCSE	FE	FE Robust
0.25	0.0	0.25	0.16	0.537	18.984	9.613	9.959	3.143	101	106	107
0.25	0.3	0.25	0.17	0.496	17.295	7.706	9.956	2.549	96	88	106
0.25	0.5	0.25	0.22	0.439	15.176	5.509	9.953	1.796	90	62	104
0.50	0.0	0.25	0.34	0.537	19.032	9.722	9.982	3.133	102	106	107
0.50	0.3	0.25	0.33	0.496	17.353	7.818	9.973	2.547	96	88	106
0.50	0.5	0.25	0.35	0.439	15.244	5.651	9.964	1.797	90	62	104
0.25	0.0	0.50	0.34	0.537	15.807	6.696	9.921	3.135	103	106	107
0.25	0.3	0.50	0.25	0.496	14.805	5.387	9.916	2.545	96	88	106
0.25	0.5	0.50	0.23	0.439	13.613	4.063	9.912	1.794	87	62	104
0.50	0.0	0.50	0.43	0.537	15.850	6.786	9.939	3.122	103	106	107
0.50	0.3	0.50	0.37	0.496	14.861	5.491	9.928	2.538	94	88	106
0.50	0.5	0.50	0.35	0.439	13.688	4.216	9.918	1.793	86	62	104
0.25	0.0	0.75	0.31	0.537	12.823	4.292	9.826	3.117	105	106	107
0.25	0.3	0.75	0.28	0.496	11.984	3.091	9.816	2.535	96	88	106
0.25	0.5	0.75	0.26	0.439	11.341	2.234	9.807	1.795	84	62	104
0.50	0.0	0.75	0.41	0.537	12.855	4.347	9.839	3.096	105	105	107
0.50	0.3	0.75	0.39	0.496	12.022	3.158	9.822	2.523	95	87	105
0.50	0.5	0.75	0.37	0.439	11.399	2.356	9.807	1.792	81	62	104
0.25	0.0	0.90	0.20	0.537	11.121	3.342	9.642	3.098	105	105	107
0.25	0.3	0.90	0.20	0.496	10.112	2.346	9.622	2.529	98	87	105
0.25	0.5	0.90	0.23	0.439	9.419	1.879	9.604	1.813	87	62	104
0.50	0.0	0.90	0.35	0.537	11.146	3.364	9.649	3.076	106	105	106
0.50	0.3	0.90	0.33	0.496	10.134	2.373	9.622	2.519	99	87	105
0.50	0.5	0.90	0.34	0.439	9.448	1.946	9.597	1.815	86	62	104

Table 4: Monte Carlo Analysis With Dynamics Included and High Correlation between α_i and $x_{i,t}$ ($N = 15; T = 20$)

Cor.	Het.	ρ	% reject $\rho = 0$	$\rho_{\alpha,x}$	$\bar{\beta}_{OLS}$	RMSE$_{OLS}$	$\bar{\beta}_{FE}$	RMSE$_{FE}$	Optimism		
									PCSE	FE	FE Robust
0.25	0.0	0.25	0.21	0.930	23.159	13.366	9.760	3.209	89	107	108
0.25	0.3	0.25	0.22	0.780	14.367	4.675	9.769	2.620	85	89	107
0.25	0.5	0.25	0.24	0.638	11.745	2.290	9.817	1.853	88	63	105
0.50	0.0	0.25	0.35	0.930	23.379	13.696	9.744	3.188	90	107	108
0.50	0.3	0.25	0.35	0.780	14.490	4.912	9.759	2.601	87	89	106
0.50	0.5	0.25	0.36	0.638	11.862	2.566	9.810	1.838	89	63	105
0.25	0.0	0.50	0.32	0.930	20.065	10.374	9.727	3.208	89	107	108
0.25	0.3	0.50	0.20	0.780	12.184	2.768	9.733	2.621	80	89	107
0.25	0.5	0.50	0.23	0.638	10.379	1.627	9.777	1.856	83	63	105
0.50	0.0	0.50	0.41	0.930	20.328	10.743	9.710	3.185	87	107	108
0.50	0.3	0.50	0.35	0.780	12.329	3.047	9.723	2.601	80	89	106
0.50	0.5	0.50	0.35	0.638	10.531	1.925	9.770	1.842	83	63	105
0.25	0.0	0.75	0.31	0.930	15.874	6.487	9.626	3.218	94	108	109
0.25	0.3	0.75	0.25	0.780	9.161	1.858	9.626	2.634	76	90	107
0.25	0.5	0.75	0.24	0.638	8.134	2.444	9.665	1.874	77	63	106
0.50	0.0	0.75	0.40	0.930	16.086	6.773	9.607	3.193	90	107	108
0.50	0.3	0.75	0.37	0.780	9.268	1.939	9.614	2.614	75	89	107
0.50	0.5	0.75	0.37	0.638	8.267	2.473	9.654	1.863	76	63	105
0.25	0.0	0.90	0.20	0.930	12.561	3.926	9.433	3.233	101	108	109
0.25	0.3	0.90	0.21	0.780	6.712	3.696	9.428	2.658	81	90	108
0.25	0.5	0.90	0.24	0.638	5.928	4.359	9.459	1.917	78	64	106
0.50	0.0	0.90	0.36	0.930	12.667	4.046	9.400	3.213	100	108	109
0.50	0.3	0.90	0.35	0.780	6.754	3.671	9.403	2.645	81	90	107
0.50	0.5	0.90	0.36	0.638	5.989	4.334	9.437	1.918	77	64	106

Table 5: Monte Carlo Analysis With AR1 Process and Correlation between α_i and $x_{i,t}$ ($N = 15; T = 20$)

Cor.	Het.	ρ	% reject $\rho = 0$	$\rho_{\alpha,x}$	$\bar{\beta}_{OLS}$	RMSE$_{OLS}$	$\bar{\beta}_{FE}$	RMSE$_{FE}$	Optimism PCSE	Optimism FE	Optimism FE Robust
No lag in estimation model											
0.00	0.0	0.50	1.00	0.536	38.646	28.913	9.943	3.131	68	97	100
0.00	0.3	0.50	1.00	0.495	26.524	16.811	9.964	2.548	89	80	99
0.00	0.5	0.50	0.98	0.438	19.011	9.349	9.979	1.798	117	57	97
0.00	0.0	0.50	1.00	0.536	38.646	28.913	9.943	3.131	68	97	100
0.25	0.3	0.50	1.00	0.495	26.505	16.768	9.992	2.489	87	78	96
0.25	0.5	0.50	0.96	0.438	18.956	9.368	10.000	1.759	125	55	95
0.50	0.0	0.50	1.00	0.536	38.694	28.900	10.003	3.040	61	94	97
0.50	0.3	0.50	0.99	0.495	26.499	16.739	10.020	2.490	84	78	96
0.50	0.5	0.50	0.92	0.438	18.925	9.405	10.020	1.761	130	55	95
Lag included in estimation model											
0.00	0.0	0.50	0.14	0.536	16.641	7.220	10.148	2.969	95	100	101
0.00	0.3	0.50	0.12	0.496	14.133	4.628	10.165	2.423	90	83	100
0.00	0.5	0.50	0.14	0.439	11.669	2.312	10.180	1.728	91	59	98
0.00	0.0	0.50	0.14	0.536	16.641	7.220	10.148	2.969	95	100	101
0.25	0.0	0.50	0.22	0.536	16.761	7.347	10.168	2.907	95	98	99
0.25	0.3	0.50	0.20	0.496	14.247	4.769	10.189	2.384	93	81	98
0.25	0.5	0.50	0.22	0.439	11.747	2.524	10.199	1.705	96	58	97
0.50	0.0	0.50	0.36	0.536	16.917	7.557	10.203	2.904	96	98	99
0.50	0.3	0.50	0.34	0.496	14.402	4.997	10.220	2.388	96	82	98
0.50	0.5	0.50	0.35	0.439	11.878	2.826	10.221	1.711	101	58	97

Table 6: Monte Carlo Analysis With Dynamics Included and Medium Correlation between α_i and $x_{i,t}$ ($N = 15; T = 40$)

Cor.	Het.	ρ	% reject $\rho = 0$	$\rho_{\alpha,x}$	$\bar{\beta}_{OLS}$	RMSE$_{OLS}$	$\bar{\beta}_{FE}$	RMSE$_{FE}$	Optimism		
									PCSE	FE	FE Robust
0.25	0.0	0.25	0.20	0.535	18.929	9.236	9.939	2.130	100	103	106
0.25	0.3	0.25	0.22	0.495	17.212	7.407	9.950	1.725	93	85	105
0.25	0.5	0.25	0.31	0.439	15.077	5.230	9.962	1.214	87	60	103
0.50	0.0	0.25	0.36	0.535	18.975	9.314	9.963	2.128	100	103	106
0.50	0.3	0.25	0.36	0.495	17.249	7.474	9.970	1.725	93	85	104
0.50	0.5	0.25	0.39	0.439	15.107	5.302	9.976	1.215	87	60	103
0.25	0.0	0.50	0.50	0.535	15.758	6.191	9.930	2.130	100	103	106
0.25	0.3	0.50	0.35	0.495	14.732	5.005	9.942	1.725	92	85	105
0.25	0.5	0.50	0.23	0.439	13.507	3.712	9.954	1.214	83	60	103
0.50	0.0	0.50	0.54	0.535	15.803	6.255	9.953	2.128	99	103	106
0.50	0.3	0.50	0.44	0.495	14.773	5.071	9.961	1.725	90	85	104
0.50	0.5	0.50	0.39	0.439	13.550	3.797	9.967	1.215	80	60	103
0.25	0.0	0.75	0.47	0.535	12.783	3.544	9.900	2.129	101	103	106
0.25	0.3	0.75	0.41	0.495	11.939	2.499	9.913	1.725	92	85	105
0.25	0.5	0.75	0.32	0.439	11.266	1.706	9.925	1.214	77	60	103
0.50	0.0	0.75	0.50	0.535	12.821	3.586	9.921	2.125	100	103	106
0.50	0.3	0.75	0.46	0.495	11.976	2.546	9.930	1.724	89	85	104
0.50	0.5	0.75	0.41	0.439	11.309	1.786	9.936	1.216	74	60	103
0.25	0.0	0.90	0.25	0.535	11.080	2.406	9.832	2.126	102	103	106
0.25	0.3	0.90	0.25	0.495	10.088	1.566	9.841	1.725	93	85	105
0.25	0.5	0.90	0.25	0.439	9.391	1.279	9.850	1.218	78	60	103
0.50	0.0	0.90	0.38	0.535	11.112	2.429	9.846	2.119	102	103	106
0.50	0.3	0.90	0.37	0.495	10.116	1.582	9.850	1.722	93	85	104
0.50	0.5	0.90	0.36	0.439	9.420	1.307	9.854	1.219	76	60	103

Table 7: Monte Carlo Analysis With Dynamics Included and Medium Correlation between α_i and $x_{i,t}$ ($N = 5; T = 20$)

Cor.	Het.	ρ	% reject $\rho = 0$	$\rho_{\alpha,x}$	$\bar{\beta}_{OLS}$	RMSE$_{OLS}$	$\bar{\beta}_{FE}$	RMSE$_{FE}$	Optimism PCSE	Optimism FE	Optimism FE Robust
0.25	0.0	0.25	0.10	0.538	19.199	10.902	9.836	5.396	101	103	115
0.25	0.3	0.25	0.14	0.402	15.871	7.315	9.838	4.551	100	88	112
0.25	0.5	0.25	0.18	0.262	12.745	3.980	9.874	3.028	98	58	106
0.50	0.0	0.25	0.17	0.538	19.362	11.090	9.926	5.438	102	104	116
0.50	0.3	0.25	0.18	0.402	15.997	7.447	9.900	4.579	101	88	113
0.50	0.5	0.25	0.21	0.262	12.829	4.062	9.909	3.043	99	59	106
0.25	0.0	0.50	0.17	0.538	15.992	8.265	9.790	5.389	102	103	115
0.25	0.3	0.50	0.16	0.402	13.711	5.651	9.793	4.546	98	88	112
0.25	0.5	0.50	0.20	0.262	11.696	3.338	9.827	3.026	93	58	106
0.50	0.0	0.50	0.24	0.538	16.150	8.421	9.875	5.427	102	104	116
0.50	0.3	0.50	0.21	0.402	13.832	5.755	9.850	4.574	99	88	113
0.50	0.5	0.50	0.22	0.262	11.792	3.414	9.858	3.043	93	59	106
0.25	0.0	0.75	0.17	0.538	12.938	6.234	9.677	5.373	102	103	115
0.25	0.3	0.75	0.17	0.402	11.294	4.327	9.684	4.536	97	88	112
0.25	0.5	0.75	0.20	0.262	10.051	2.810	9.709	3.024	87	58	106
0.50	0.0	0.75	0.22	0.538	13.076	6.339	9.752	5.398	102	103	116
0.50	0.3	0.75	0.22	0.402	11.394	4.370	9.733	4.556	98	88	113
0.50	0.5	0.75	0.23	0.262	10.134	2.829	9.731	3.040	87	59	106
0.25	0.0	0.90	0.10	0.538	11.140	5.489	9.457	5.358	102	103	116
0.25	0.3	0.90	0.13	0.402	9.679	4.110	9.468	4.529	98	88	112
0.25	0.5	0.90	0.17	0.262	8.496	3.265	9.482	3.038	88	59	106
0.50	0.0	0.90	0.15	0.538	11.257	5.543	9.514	5.360	103	104	117
0.50	0.3	0.90	0.18	0.402	9.759	4.087	9.506	4.529	99	88	113
0.50	0.5	0.90	0.22	0.262	8.554	3.231	9.497	3.043	88	59	106

Table 8: Monte Carlo Analysis With Dynamics Included and Minimal Correlation Between α_i and $x_{i,t}$ ($N = 100; T = 20$)

Cor.	Het.	ρ	% reject $\rho = 0$	$\rho_{\alpha,x}$	$\bar{\beta}_{OLS}$	RMSE$_{OLS}$	$\bar{\beta}_{FE}$	RMSE$_{FE}$	Optimism PCSE	Optimism FE	Optimism FE Robust
0.25	0.0	0.25	0.47	−0.001	10.010	1.449	9.957	1.153	102	101	98
0.25	0.3	0.25	0.48	−0.001	10.000	1.190	9.955	0.963	102	84	98
0.25	0.5	0.25	0.62	−0.001	9.994	0.872	9.955	0.693	102	61	98
0.50	0.0	0.25	0.73	−0.001	10.006	1.446	9.955	1.140	103	100	98
0.50	0.3	0.25	0.66	−0.001	9.997	1.184	9.953	0.953	103	84	97
0.50	0.5	0.25	0.67	−0.001	9.993	0.866	9.952	0.687	101	60	97
0.25	0.0	0.50	0.77	−0.001	10.014	1.341	9.921	1.152	102	101	98
0.25	0.3	0.50	0.64	−0.001	10.007	1.114	9.919	0.962	102	84	98
0.25	0.5	0.50	0.49	−0.001	10.000	0.818	9.919	0.695	101	60	98
0.50	0.0	0.50	0.77	−0.001	10.009	1.338	9.913	1.142	102	100	98
0.50	0.3	0.50	0.74	−0.001	10.003	1.109	9.911	0.955	102	84	97
0.50	0.5	0.50	0.71	−0.001	9.997	0.813	9.911	0.692	101	60	98
0.25	0.0	0.75	0.76	−0.001	10.013	1.236	9.817	1.160	101	101	98
0.25	0.3	0.75	0.72	−0.001	10.010	1.033	9.816	0.973	101	84	98
0.25	0.5	0.75	0.64	−0.001	10.005	0.753	9.815	0.713	100	61	98
0.50	0.0	0.75	0.75	−0.001	10.008	1.232	9.797	1.158	101	101	98
0.50	0.3	0.75	0.74	−0.001	10.005	1.028	9.796	0.975	101	84	98
0.50	0.5	0.75	0.73	−0.001	10.002	0.748	9.795	0.721	100	61	99

Table 9: Monte Carlo Analysis With Dynamics Included and Medium to High Correlation between α_i and $x_{i,t}$ ($N = 100; T = 20$)

Cor.	Het.	ρ	% reject $\rho = 0$	$\rho_{\alpha,x}$	$\bar{\beta}_{OLS}$	RMSE$_{OLS}$	$\bar{\beta}_{FE}$	RMSE$_{FE}$	Optimism PCSE	Optimism FE	Optimism FE Robust
0.25	0.0	0.25	0.54	0.590	19.519	9.634	10.003	1.169	94	102	100
0.25	0.3	0.25	0.47	0.508	17.694	7.791	9.996	0.976	89	85	100
0.25	0.5	0.25	0.53	0.425	15.407	5.513	9.985	0.703	82	61	100
0.50	0.0	0.25	0.73	0.590	19.545	9.748	10.004	1.157	96	101	100
0.50	0.3	0.25	0.72	0.508	17.744	7.931	9.997	0.967	93	85	99
0.50	0.5	0.25	0.71	0.425	15.482	5.706	9.985	0.697	87	61	99
0.25	0.0	0.50	0.79	0.590	16.045	6.190	9.968	1.171	94	102	100
0.25	0.3	0.50	0.67	0.508	14.923	5.044	9.961	0.978	85	85	100
0.25	0.5	0.50	0.53	0.425	13.556	3.692	9.949	0.706	74	62	100
0.50	0.0	0.50	0.77	0.590	16.083	6.289	9.964	1.161	90	102	100
0.50	0.3	0.50	0.76	0.508	14.986	5.186	9.956	0.972	83	85	100
0.50	0.5	0.50	0.73	0.425	13.653	3.919	9.944	0.704	77	62	100
0.25	0.0	0.50	0.77	0.825	19.008	9.109	9.913	1.201	83	105	103
0.25	0.3	0.50	0.57	0.705	14.820	4.932	9.912	1.007	67	88	102
0.25	0.5	0.50	0.48	0.570	12.253	2.479	9.913	0.728	65	63	102
0.50	0.0	0.50	0.78	0.825	19.126	9.316	9.898	1.187	79	104	102
0.50	0.3	0.50	0.72	0.705	14.940	5.173	9.898	0.996	71	87	102
0.50	0.5	0.50	0.70	0.570	12.400	2.863	9.902	0.722	71	63	102