Inference and Estimation in Small Sample Dynamic Panel Data Models

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Abstract: We study the finite sample properties of the most important methods of estimation of dynamic panel data models in a special class of small samples: a two-sided small sample (i.e., a sample in which the time dimension is not that short but the cross-section dimension is not that large). This case is encountered increasingly in applied work. Our main results are the following: the estimator proposed by Kiviet (1995) outperforms all other estimators considered in the literature. However, standard statistical inference is not valid for any of them. Thus, to assess the true sample variability of the parameter estimates, bootstrap standard errors have to be computed. We find that standard bootstrapping techniques work well except when the autoregressive parameter is close to one. In this last case, the best available solution is to estimate standard errors by means of the Grid-t bootstrap estimator due to Hansen (1999).

Key-Words: Bias correction, Dynamic panel data model, GMM estimators, Standard and Grid-t bootstrap estimators.
JEL Classification: C12; C13; C15; C23; E24

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1 Introduction

This work is motivated by the existent concern with the finite-sample properties of the methods of estimation of the parameters of dynamic panel data models. When a panel data model includes lagged dependent explanatory variables, the within-group estimator is asymptotically valid only when the time dimension of the panel gets large. Since the time series dimension (T) of most panel data sets is a single-digit number, Instrumental Variables (IV) and Generalized Method of Moments (GMM) estimators, which are consistent for finite T when the number of cross-section observations (N) tends to infinite, have been considered in the literature (see, Anderson and Hsiao, 1982; Arellano and Bond, 1991 and Blundell and Bond, 1998). Nevertheless, for example, panel data sets where the units of analysis are the regions of a country (or cross-country panels) most likely have a time dimension larger than a single-digit number, even though, this gain normally comes at the cost of not having a very large number of cross-section observations. This leads us to address the following question: how to estimate and conduct inference in dynamic panel data models in small samples in which the time dimension of the panel is not short and the cross-section dimension is not that large? Lets denote this case as a two-sided small sample in opposition to the most standard one-sided small sample panels, in which T is very small and N is very large. The panels we consider are small in the sense that, even though $N \times T$ may be large, none of the sides gets very large itself.

Many interesting variables exhibit state dependence, that is, the current state of a variable depends on its last period's state, even after controlling for unobserved heterogeneity. Thus, very often, we use panel data to estimate dynamic relationships, namely, models containing lagged dependent variables among the regressors. A nice example is the wage curve of Blanchflower and Oswald (1994). In its simplest form, regional wages are modeled as a dynamic two-way fixed effect error component model in which regional unemployment is assumed to affect regional wages negatively (i.e., in self-explaining notation, $w_{i,t} = \rho w_{i,t-1} + \gamma u_{i,t} + \mu_i + \lambda_t + \epsilon_{i,t}$). For example, for the U.S., this model is estimated with samples in which $N = 50$ and T tends to be less than 20 (see, among others, Blanchard and Katz, 1997). Two issues are at the center of the debate in this literature: first, whether ρ is one (a Phillips curve form), zero (a static wage curve) or, as it is more likely the case, somewhere in between (a stable dynamic wage curve). Thus, there is interest in establishing the type of dynamic process followed by regional wages. Second, whether γ (the coefficient associated with the unemployment variable) is negative and statistically different from zero. To answer these questions, we need to obtain accurate estimates of both the parameters of the model and their sample variability in small samples like the ones normally available. Although IV and especially GMM estimators have attractive asymptotic properties, Monte Carlo simulations show that their finite sample approximations are poor and sensible to the actual parameter values as well as to the dimension of the data sets (see, among others, Kiviet, 1995). However, little is known about the reliability of asymptotic test procedures in this two-sided small sample panels (an exception is Bun and Kiviet (2001) who consider the case in which T and N are less than 20).

In this paper we consider two-sided small size panels where T is larger than a single-digit number but N is not very large. We study the finite-sample properties of the dominant methods proposed in the literature to estimate dynamic panel models. These methods are the least-squares dummy-variable (LSDV) approach, a LSDV bias-corrected estimator proposed by Kiviet (1995, LSDVC hereafter) and two GMM procedures, the one proposed by Arellano and Bond (1991) (AB hereafter) and the one developed by Blundell and Bond (1998) (BB hereafter). Our simulation design follows a standard specification of a dynamic panel data model, i.e., a first order autoregressive model with an additional explanatory variable. We consider two data generation processes. In the first Monte Carlo experiment, the exogenous variable and the unobservable time invariant effect are not correlated while in the second experiment they are correlated. The dynamic adjustment or autoregressive parameter varies between 0.2 (low persistence) and 0.8 (high persistence).

Our main results are the following: first, standard inference is not valid for any of the estimators and data generation processes considered in this paper. We find that for all the estimators studied, the true size of t-type tests may differ substantially from their asymptotic nominal level although the way they depart from the normal asymptotic approximation vary among them. Interestingly, this is also the case when we test the null hypothesis of $\gamma = a \in [-1, 1]$ (where γ is the parameter associated to the exogenous variable in the dynamic model studied) for all the estimators considered in this study. Surprisingly, this result also holds when the null hypothesis is $\gamma = 0$ and, but not necessarily, the dependent and exogenous explanatory variables are correlated in the data generating process (DGP), which is likely to be the case in practice. Thus, irrespective of which estimator performs better in terms of bias and root-mean square error (RMSE), most often the criteria considered to compare the small sample performance of these estimators, it is necessary to consider also the finite sample behavior of t-type tests in order to conduct valid statistical inference in small sample dynamic panel data models. These results are very important and have not been studied in the literature. Second, the LSDVC estimator proposed by Kiviet (1995) outperforms all other estimators considered both in terms of bias and RMSE. Thus, this estimator is recommended for estimating dynamic panel models on samples of the type studied in this paper (see also Judson and Owen, 1999). However, to assess its true sample variability, and hence, to conduct valid statistical inference in small samples, bootstrap standard errors have to be computed. Third, we find that standard bootstrapping techniques work well except when the autoregressive parameter in the model is close to one. In this last case, we find that the Grid-t bootstrap procedure due to Hansen (1999) outperforms any other alternative to estimate the standard errors of the estimates of the parameters of dynamic panel data models in small samples.

The rest of the paper is organized as follows. Section 2 presents the model and briefly reviews the estimators we study. Section 3 summarizes the results of our Monte Carlo experiments and Section 4 evaluates the performance of several bootstrap techniques to assess the sample variability of the estimates of the parameters of interest by means of the estimator proposed by Kiviet (1995). Section 5 presents two estimations of the wage curve. Finally, Section 6 concludes the paper.

2 Dynamic Unobserved Effects Model

Consider the following first order autoregressive model with an additional explanatory variable:

$$y_{i,t} = \rho y_{i,t-1} + \gamma x_{i,t} + \mu_i + u_{i,t} \tag{1}$$

where $i = 1, \cdots, N$ and $t = 1, \cdots, T$ indexes cross-section and time series observations, respectively. The unobserved effects (μ_i), which are modeled as fixed effects, are probably correlated with the included exogenous regressor x. The $\{x_{i,t}\}$ are strictly exogenous conditional on the unobserved effects. We also assume dynamic stability (i.e., $|\rho| < 1$). For simplicity, the $u_{i,t}$ are assumed to be independently distributed across units with zero mean and constant variance σ_u^2. Stacking the observations over time and cross-section units we obtain:

$$y = W\,\delta\, + (I_N \otimes\, \iota_T)\,\mu + u \tag{2}$$

where $\delta = (\rho,\, \gamma)'$, y is an $NT \times 1$ vector of stacked observations of the dependent variable and $W = [y_{-1} \vdots X]$ is an $NT \times 2$ matrix of stacked observations of the independent variables of the model. u is the $NT \times 1$ vector of disturbances and $\iota_T = (1, \ldots, 1)'$ is $T \times 1$. The time invariant unobserved effects vector $\mu = (\mu_1,\, \cdots,\, \mu_N)'$ is a vector of N unknown parameters corresponding to the fixed effects in model (1).

In this study we consider the case of panels where the sample size in the cross-section dimension (N) vary between 30 and 50, whereas its time series dimension (T) is between 20 and 40. This type of small sample panels has received little attention in the literature. Most of the work on the estimation of small sample dynamic panel data models considers the case where T is a single-digit number and an asymptotic analysis is conducted by treating T as a fixed number and letting N tend to infinite. For this specification a number of alternative estimators have been proposed. We now review those we study in this paper.

2.1 LSDV Estimator

Estimation of the parameters of model (1) can be performed by ordinary least squares by means of the LSDV or fixed effects (FE) estimator. Using standard regression results, the fixed effects estimator of δ can be expressed as:

$$\hat{\delta}_{LSDV} = (W'AW)^{-1}\, W'Ay \tag{3}$$

where the $NT \times NT$ matrix $A = I_N \otimes (I_T - \frac{1}{T}\iota_T\iota_T')$ is the within transformation which wipes out the individual fixed effects.

As it is well known, the within-group LSDV estimator of the parameters of model (1) is semi-inconsistent since in the transformed model, the lagged dependent variable is correlated with the error term. Nevertheless, this estimator is consistent when $T \to \infty$. Thus, the LSDV estimator is supposed to perform well for panels with a large T dimension. But how large T should be before the bias of the LSDV estimator is ignorable is left unanswered in the literature.

2.2 GMM Estimators

Several consistent instrumental variables estimators have been proposed in the literature to estimate the parameters of model (1) for panels of moderate T size. Here we restrict our analysis only to those proposed by Arellano and Bond (1991) and Blundell and Bond (1998). When there are no instruments available that are uncorrelated with the individual effects μ_i, the transformation of the model must eliminate this component from the error term. Arellano and Bond (1991) suggest differencing the regression function (1) to eliminate the individual specific effects, and estimate the parameters of the differenced model by a GMM estimator using appropriately lagged endogenous and predetermined variables as instruments in the transformed equations since, after differencing, $\Delta y_{i,t-1}$ is correlated with the differenced equation error, $\Delta u_{i,t}$. However, as long as $u_{i,t}$ is serially uncorrelated, all lags on y and x beyond $t-1$ are valid instruments for the differenced equation at period t. Because the number of instruments increases with the time series dimension T, the model generates many overidentifying restrictions even for moderate values of T, although the quality of these instruments is often poor. When there are instruments available that are uncorrelated with the individual effects μ_i, these variables can be used as instruments for the equations in levels. Blundell and Bond (1998) propose an estimator, which combines a set of moment conditions relating to the equations in first differences and a set of moment conditions relating to the equations in levels to obtain an efficient GMM estimator. They show that this system estimator has superior properties in terms of small sample bias and RMSE than the estimator proposed by Arellano and Bond (1991), specially when the DGP presents a high level of persistence.

These GMM estimators are of the form:

$$\hat{\delta}_{GMM} = [(W^{*'}Z)\, A_N\, (Z'W^*)]^{-1}(W^{*'}Z)\, A_N\, (Z'y^*) \tag{4}$$

where

$$A_N = \left(\frac{1}{N} \sum_i Z_i' H_i Z_i \right)^{-1}$$

and W^* and y^* denote some transformation of W and y (e.g. levels, first differences, etc.), Z_i is a matrix of instrumental variables, and H_i is an individual specific weighting matrix.

The estimator proposed in Arellano and Bond (1991) uses the first difference transformation and

$$H_i^{AB} = \begin{pmatrix} 2 & \text{-1} & \cdots & & 0 \\ \text{-1} & 2 & \cdots & & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ \cdots & \cdots & \cdots & & \text{-1} \\ 0 & 0 & \cdots & \text{-1} & 2 \end{pmatrix}$$

The corresponding instrumental variable matrix is:

$$Z_i^{AB} = \begin{pmatrix} y_{i1} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & \Delta x_{i3} \\ 0 & y_{i1} & y_{i2} & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & \Delta x_{i4} \\ 0 & 0 & 0 & y_{i1} & y_{i2} & y_{i3} & & & & & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & y_{i1} & \cdots & y_{i,(T-2)} & \Delta x_{i,T} \end{pmatrix} \tag{5}$$

The estimator proposed by Blundell and Bond (1998) adds to the first-difference equations the levels equations. In this case $y_i^* = (\Delta\, y_{i3}, \ldots, \Delta y_{iT}, y_{i3}, \ldots, y_{iT})'$,

$$W_i^* = \begin{pmatrix} \Delta\, y_{i2} & \ldots & \Delta y_{i,(T-1)} & y_{i2} & \cdots & y_{i,(T-1)} \\ \Delta x_{i3} & \ldots & \Delta x_{i,T} & x_{i3} & \ldots & x_{i,T} \end{pmatrix}'$$

and

$$Z_i^{BB} = \begin{pmatrix} Z_i^{AB} & 0 & \cdots & 0 & 0 \\ 0 & \Delta\, y_{i2} & \cdots & 0 & 1 \\ . & . & & . & . \\ 0 & 0 & \cdots & \Delta\, y_{i,(T-1)} & 1 \end{pmatrix}$$

The specific weighting matrix used is:

$$H_i^{BB} = \begin{pmatrix} H_i^{AB} & 0 \\ 0 & \frac{1}{2} I_{T-2} \end{pmatrix}$$

where I_{T-2} is the identity matrix with dimension equal to the observed number of levels equations. Unlike $\hat{\delta}_{LSDV}$, both GMM estimators, AB and BB, are consistent for finite T when $N \longrightarrow \infty$.

2.3 Corrected LSDV Estimation

Finally, we also consider a bias corrected version of the LSDV estimator due to Kiviet (1995). This estimator is computed by subtracting an approximation, of order $O(N^{-1}T^{-3/2})$, of the asymptotic bias of the LSDV estimator. Kiviet (1995) demosntrates that:

$$\begin{aligned}
E(\hat{\delta}_{LSDV} - \delta) = -\sigma_u^2(\bar{D})^{-1}\Big(\frac{N}{T}(\iota_T' C \iota_T)[2q - \bar{W}' A \bar{W}(\bar{D})^{-1}q] \\
+ \operatorname{tr}\{\bar{W}'(I_N \otimes A_T C A_T)\bar{W}(\bar{D})^{-1}\}q \\
+ \quad \bar{W}'(I_N \otimes A_T C A_T)\bar{W}(\bar{D})^{-1}q + \sigma_u^2 N q'(\bar{D})^{-1}q \\
\times[-\frac{N}{T}(\iota_T' C \iota_T)\operatorname{tr}\{C' A_T C\} + 2\operatorname{tr}\{C' A_T C A_T C\}]q\Big) \\
+\mathrm{O}(N^{-1}T^{-3/2}) \qquad (6)
\end{aligned}$$

where tr denotes the trace operator, $\bar{D} = \bar{W}' A \bar{W} + \sigma_u^2 N \operatorname{tr}\{C' A_T C\} q q'$, $A_T = I_T - \frac{1}{T}\iota_T \iota_T'$, $q = (1, 0, \cdots, 0)$, $A\bar{W} = E(AW)$ and

$$C = \begin{bmatrix} 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 1 & 0 & & & & & \cdot \\ \rho & 1 & 0 & & & & \cdot \\ \rho^2 & \rho & 1 & \cdot & & & \cdot \\ \vdots & \vdots & \vdots & \vdots & & & \vdots \\ \rho^{T-2} & \cdot & \cdot & \cdot & \rho & 1 & 0 \end{bmatrix}$$

Therefore, the asymptotic bias of the LSDV estimator is a function of the true parameters of the model. Thus, to compute the LSDVC estimator, an estimation of this asymptotic bias is subtracted from the LSDV estimate. And to obtain an estimation of this asymptotic bias, we estimate the paremeters of the model by means of the simple IV estimator proposed by Anderson and Hsiao (1981).

3 Monte Carlo Simulations

In this section we study the finite-sample properties of the estimators presented in the previous section. Our simulation follows closely the experimental design adopted in Arellano and Bond

(1991). The dependent variable is generated by model (1), where $u_{i,t} \sim IN(0,\ 1)$, $\mu_i \sim IN(0,\ 1)$, $i = 1, \cdots, N$; $t = 1, \cdots, T + 10$ and $y_{i,0} = 0$. The first ten cross-sections are discarded so that the actual sample size is NT. The exogenous regressor $x_{i,t}$ is generated by the following DGP:

$$x_{i,t} = 0.8x_{i,t} + \lambda\mu_i + v_{i,t}$$

where $v_{i,t} \sim N(0,\ 0.9)$, $x_{i,0} = 0$ and λ takes the values zero or one.

When $\lambda = 1$, the exogenous regressor in model (1) is correlated with the unobserved fixed effect in that model, while they are uncorrelated when $\lambda = 0$. This latter case is the one studied in Arellano and Bond (1991). The results of the Monte Carlo experiments are very similar for both DGPs. Thus, we only report those corresponding to the case in which $\lambda = 0$.

The choice of the parameters is as follows: $\rho = 0.2, 0.5$ and 0.8, $\gamma = -1, 0$ and 1, $N = 30, 50$ and $T = 20, 30, 40$. Table 1 summarizes the resultant combination of parameter values used in the Monte Carlo experiments. Tables 2, 3 and 4 and Figure 1 summarize the most important results of these experiments when $\lambda = 0$.[1]

Table 1 about here

Table 2 presents the bias and RMSE for both ρ and γ, for each estimator. It is clear that the estimator proposed by Kiviet (1995) (K in the table) outperforms the other estimators in all cases both in terms of bias and RMSE, not only for the estimator of the autoregressive parameter ρ but also for the estimator of the coefficient of the exogenous regressor γ when the true parameter value is different from zero.

Table 2 about here

The LSDV estimator of ρ is largely biased in most specifications. As expected, the bias decreases as ρ and T increase. The bias in the estimate of γ is small. It is less than one percent when $\gamma = 0$ and ranges between 1.2 and 2.7 percent when γ is different from zero.

Figure 1 about here

[1]The whole set of results, including the case in which $\lambda = 1$, is available from the authors upon request.

In most of the specifications, both AB and BB estimators perform better than the LSDV estimator, both in terms of bias and RMSE. Finally, as expected, when T increases, there are not differences among these three estimators. Nevertheless, even for T as high as 40, the LSDVC estimator dominates the other estimators both in terms of bias and RMSE. Thus, the estimator proposed by Kiviet (1995) is preferred for estimating the parameters of model (1) in the class of small samples that we study in this paper.

Tables 3 and 4 present the quantile tabulation of the 1st, 5th, 10th, 90th, 95th and 99th percentiles of the distribution of the t-statistic for the following null hypotheses: $H_0 : \rho = 0.2$, $\rho = 0.5$, and $\rho = 0.8$ (Table 3) and $H_0 : \gamma = 0$, $\gamma = 1$ and $\gamma = -1$ (Table 4).

Table 3 about here

The quantiles of the distribution of the t-test do not coincide with those of the asymptotic standard normal approximation, not only for the LSDV and GMM estimators but also, and more relevant, for the LSDVC estimator. This result is extremely important because it casts doubts about the appropriateness of conducting standard asymptotic statistical inference in small sample dynamic panel data models, irrespective of the method of estimation adopted.

The distribution of the t-test when ρ is estimated by means of the LSDV estimator is clearly skewed to the left. The same result holds for the two GMM estimators, although the skewness of the distribution of the t-test seems to be less severe in these cases. More importantly, even though the distribution of the t-test when ρ is estimated by means of the LSDVC estimator is not skewed, it is neither a standard normal distribution.

Table 4 shows the critical values of the t-statistics of the postulated null hypothesis for γ. Irrespective of the method of estimation adopted, the distribution of these tests do not seem to be skewed, but again, they are not a standard normal distribution even when $\gamma = 0$ under the null hypothesis.

Table 4 about here

Thus, the evidence presented suggest that the LSDVC estimator must be preferred for estimating the parameters of model (1) in the class of small samples that we study in this paper. However, the results reported in Tables 3 and 4 also suggest that, even in this case, standard statistical inference is misleading and hence, bootstrap standard errors have to be computed to conduct valid statistical inference. Though, which bootstrap estimator performs better is not known. In the next section we address this issue.

4 Small Sample Statistical Inference

In this section we consider the problem of constructing bootstrap confidence intervals of 90% coverage for the estimates of the parameters of model (1) in two-sided small samples. A correctly constructed confidence interval has the property that in 10% of the samples, the true value of the parameter lies outside the limits of the interval.

All the experiments reported in this section are based on 1000 replications of samples generated by model (1), where $T = 20$, $N = 30$, $\rho = 0.2$, 0.5 and 0.8, $\gamma = 1$, $\lambda = 0$ and the errors are independent and Gaussian as in section 3.

We compare several methods to assess the sample variability of the estimates of the parameters of model (1). Conventional asymptotic confidence intervals are computed as $\hat{\alpha} \pm 1.645\, s(\hat{\alpha})$, where α is either ρ or γ and $s(\hat{\alpha})$ is the estimated standard deviation of the coefficient. Standard bootstrapping confidence intervals are constructed by means of the Percentile-t bootstrap technique (see Hall, 1992). We generate $B = 1999$ simulated samples to construct bootstrapped confidence intervals for the estimates of the parameters of model (1).

Each bootstrapping sample is generated as follows:

1. Obtain LSDVC estimates of ρ, γ and $\mu = (\mu_1, \cdots, \mu_N)'$. Denote these estimates as: $\hat{\rho}$, $\hat{\gamma}$, and $\hat{\mu} = (\hat{\mu}_1, \cdots, \hat{\mu}_N)'$ respectively. Using these coefficients, generate the series of predicted residuals $\hat{u}_{i,t}$.

2. Generate a simulated sample of $y^b_{i,t}$, $t = 1, \cdots, T$, for each $i = 1, \cdots, N$, by drawing errors independently from the set of estimated residuals $(\hat{u}_{i,1}, \cdots, \hat{u}_{i,T})$, and then, by computing

$$y^b_{i,t} = \hat{\rho} y^b_{i,t-1} + \hat{\gamma} x_{i,t} + \hat{\mu}_i + \hat{u}^b_{i,t}, \quad t = 1, \cdots, T$$

   where $x_{i,t}$ is taken as fixed and $y^b_{i,0} = 0$.

For each resampled data set $\{y^b_{i,t},\, x_{i,t}\}$, $b = 1, \ldots, B$, estimate model (1) and obtain bootstrap LSDVC estimates of $\hat{\rho}^b$, $\hat{\gamma}^b$ and of their respective standard deviations $s^b(\hat{\rho}^b)$ and $s^b(\hat{\gamma}^b)$.

Then, the bootstrap confidence intervals are constructed in a standard way. First, compute the 5% and 95% quantiles of the t-statistic distribution $(t_1, t_2, \ldots, t_B)$, where $t_b = \frac{\hat{\alpha}^b - \hat{\alpha}}{s^b(\hat{\alpha}^b)}$ and $\alpha = \rho, \gamma$. Denote these quantiles q_5^b and q_{95}^b. Second, for each coefficient, its confidence interval is given by: $[\hat{\alpha} - q_5^b s(\hat{\alpha}),\ \ \hat{\alpha} + q_{95}^b s(\hat{\alpha})]$.

Since the standard bootstrap confidence interval fails to provide an asymptotic correct coverage when the autoregressive coefficient is close to one (see Basawa et al. (1991)), we also consider three other bootstrap methods when $\rho = 0.8$. The biased-corrected percentile bootstrap due to Kilian (1998), and the Grid-α and Grid-t bootstrap due to Hansen (1999).

The bootstrap method proposed by Killian (1998) is as follows: first, compute the bootstrap bias of the estimate of the autoregressive parameter of the model as: $bias = \bar{\rho}^b - \hat{\rho}$ where $\bar{\rho}^b$ is the mean of the bootstrap LSDVC estimate $\hat{\rho}^b$. Second, compute a bias corrected estimate of ρ by means of: $\hat{\rho}^* = 2\hat{\rho} - \bar{\rho}^b$. Finally, generate $B = 1999$ simulated samples, $\{y_{i,t}^b,\ x_{i,t}\},\ b = 1, \ldots, B$, following the procedure described above to construct standard bootstrap confidence intervals with the only difference that in step 2, instead of using the LSDVC estimate of ρ to generate $y_{i,t}^b$, the bias corrected estimate $\hat{\rho}^*$ is used.

Finally, we consider the estimation of the grid bootstrap confidence intervals. First, we need to estimate bootstrap quantiles as a function of ρ, $q_c^g(\rho)$, where c is the relevant quantile (i.e. 5 and 95%). In order to estimate these functions we first select a fine grid of values of the autoregressive parameter, $A_G = [\rho_1,\ \rho_2,\ \ldots, \rho_G]$. Second, we compute $q_c^g(\rho)$ for each $\rho \in A_G$. Third, the grid-α (grid-t) bootstrap confidence interval is computed as the intersection between the $\hat{\rho} - \rho$ (t-statistic) function and the $q_5^g(\rho)$ and $q_{95}^g(\rho)$ quantile functions. In practice, to implement any of these grid bootstrap methods we construct a grid of $G = 50$ evenly spaced points ($\hat{\rho}^g\ \ g = 1, \ldots, G$) spread over the interval $[\hat{\rho} \pm 6s(\hat{\rho})]$, where $\hat{\rho}$ and $s(\hat{\rho})$ are the LSDVC estimates of the autoregressive parameter and its standard deviation respectively. Then, generate $B = 1999$ simulated samples **at each grid point** following the procedure described above to construct standard bootstrap confidence intervals with the only difference that in step 2, for each $g = 1, \ldots, G$, instead of using the LSDVC estimate of ρ to generate $y_{i,t}^b$, $\hat{\rho}^g$ is used (see Hansen, 1999).

Table 5 summarizes the results. Each cell of the table reports the percentage of samples in which the true value of the parameter lies outside the estimated confidence interval. Ideally, each of these percentages should be 0.1.

As expected, in the case of ρ, inference based on the asymptotic normal approximation rejects too often the null hypothesis under consideration. In addition, the coverage of the confidence interval based on the asymptotic normal approximation deteriorates substantially as the true value of ρ increases. The standard bootstrap confidence interval provides a very accurate coverage for low values of ρ, instead. However, it provides a very conservative coverage for $\rho = 0.8$ (high values of ρ). The three alternative bootstrap methods perform better than the standard bootstrap technique when $\rho = 0.8$. The bootstrap-after-bootstrap confidence interval (Killian, 1998) rejects the null hypothesis 4.6% of the times, and even though this coverage is still conservative, it performs better than the standard bootstrap procedure. The percentage of samples in which the true value of the parameter lies outside of the estimated Grid-α and Grid-t confidence intervals are 5.2 and 6.3% respectively.

Table 5 about here

For γ, both the confidence interval based on the normal approximation and the standard bootstrap estimator provide very good coverage in the cases where $\rho = 0.2$, 0.5. When $\rho = 0.8$, however, standard asymptotic inference is misleading while the standard bootstrap technique provides a slightly conservative coverage.

In light of the evidence presented in this section, the best alternative to assess the true sample variability of the estimates of the parameter of model (1) in two-sided small samples is to rely on standard bootstrap procedures when the true value of ρ, appraised by the point estimate obtained by means of the LSDVC estimator, is not large; and to rely on the Grid-t bootstrap method when the true value of ρ approaches one.

5 Empirical Application: The Wage Curve

The responsiveness of real wages to unemployment is a fundamental parameter in macroeconomic analysis. A higher degree of wage flexibility implies, *ceteris paribus*, a lower equilibrium unemployment rate. Early empirical work on the relationship between wages and unemployment is based on time-series data. More recently, in a very important contribution, Blanchflower and Oswald (1994) shifted the emphasis to the use of micro data sets. They use repeated cross-sectional data at the individual level to study the wage-unemployment relationship for several countries. They find that in any given region, if local unemployment rises, wages fall *ceteris paribus*. They have labeled this negative relationship between local wages and local unemployment, the wage curve. Moreover, they claim that the relationship between local wages and local unemployment is static and that the unemployment elasticity of pay is approximately -0.1 for most countries. However, these two last results have been questioned. Card and Hyslop (1997) and Blanchard and Katz (1997) present evidence which supports that regional wages are highly persistent and also cast doubts about the degree of responsiveness of wages to local unemployment.

Generally, the wage curve refers to the following dynamic two-way fixed effect error component model:

$$w_{i,t} = \rho w_{i,t-1} - \gamma u_{i,t} + \lambda_t + \mu_i + \epsilon_{i,t} \tag{7}$$

where $w_{i,t}$ is a measure of regional wages and $u_{i,t}$ is a measure of regional unemployment.

Model (7) is estimated in two-steps. In the first-step, individual earnings are modeled as a log-linear function of a set of regional dummy variables and a set of individual characteristics including education, gender, industry affiliation and age or potential experience. In the second-step, equation (7) is estimated using the regional dummy variables estimated in the first stage of the analysis as the measure of regional wages (i.e., the regional expected wages).

There are two important questions associated to the parameters of model (7). First, the fact that aggregate wages seem to be non-stationary does not imply that ρ equals one since the time effects themselves may be a unit-root non-stationary process. Contrary, market equilibrium may impose ρ to be strictly less than one since wages across regions must be cointegrated (see Galiani, 1999). Hence, it is important to establish whether the true value of ρ is less than one, and, in that case, whether it is different from zero. Thus, there is interest in establishing the type of dynamic process followed by regional wages. Second, does regional wages fall if local unemployment increases? And, more specifically, is the unemployment elasticity of pay -0.1? To answer these questions, it is necessary to obtain accurate estimates of both the parameters of the model and their sample variability. In the previous section we show that the best approach to estimate model (7) is to estimate their coefficients by means of the LSDVC estimator and to assess their sample variability by means of standard bootstrap techniques or the Grid-t bootstrap estimator. We now illustrate this method by estimating a wage curve for both Argentina and the U.S.

Table 6 reports the estimate of the wage curve for the U.S. for the 1980-1991 period. We report the estimates of the parameters of the model by means of the LSDV, GMM AB and LSDVC estimators. In addition, in the latter case we also report 95% bootstrap confidence intervals. The only difference in the parameter estimates is between the AB estimator and both the LSDV and LSDVC estimators of γ. This is consistent with our finding that, for high values of ρ and large N (as is the case in this example), all the estimators of ρ converges among them. Additionally, standard inference on the AB estimate does not reject the null hypothesis of $\rho = 1$ at the 5% confidence level while this is rejected in the other cases reported in Table 6.

Figure 2 illustrates the results reported in Table 6. The dashed lines plot the 5% and 95% quantile functions of the standard bootstrap distribution of the t-statistic for the LSDVC estimator of ρ, while the doted and dashed lines, constant at -1.96 and 1.96, represent the quantile functions of the asymptotic normal approximation for the same estimator. It is clear from the figure that these two pairs of quantile functions do not coincide, invalidating statistical inference that relies on conventional asymptotic approximations.

Figure 2 also allow us to read confidence intervals. The solid line plots the t-statistic function of the autoregressive coefficient for several values of ρ. The two open arrows projected from the intersection between the solid line and the doted and dashed lines (marked by a star in the figure) onto the ρ-axis give the asymptotic normal confidence interval of the parameter estimate. The parametric percentile-t bootstrap confidence interval for the estimate of ρ is constructed by evaluating the sampling t-statistic distribution at the estimate of ρ (in this case by means of the LSDVC estimator). This interval is obtained as follows: first, the point estimate of ρ ($\hat{\rho} = 0.9113$), marked by a filled black circle, is projected vertically onto the 5% and 95% bootstrap quantile functions, with intersections marked by open diamonds. Second, these two points are horizontally projected onto to the t-statistic function, where the intersection points are marked by open squares. Finally, projecting these points onto the ρ-axis (white arrow heads) gives the 95% percentile-t bootstrap interval [0.903, 0.974].

As Hansen (1999) points out, the percentile-t bootstrap confidence interval assumes implicitly that the bootstrap quantile functions are constant for any parameter value. Figure 2 shows that this is not the case, and, in that way, it explains why the conventional bootstrap fails to provide a correct coverage. As we show in the previous section, this confidence interval was too conservative for large values of ρ. Finally, the projection of the intersections between the bootstrap quantile functions and the t-statistic function onto the ρ-axis gives the 95% Grid-t confidence interval of the estimated autoregressive coefficient.

Figure 2 about here

Table 7 reports the estimation of the wage curve for Argentina for the 1991-1997 period using six-monthly data (i.e., $T = 14$). Again, we report the estimates of the parameters of the model by means of the LSDV, GMM AB and LSDVC estimators. In addition, in the latter case we also report 95% bootstrap confidence intervals. Now, there is a large difference between the LSDVC and both the LSDV and AB estimates of ρ. This is consistent with our finding that the estimator proposed by Kiviet (1995) performs substantially better than the other estimators when ρ is not that large and N is small (as is the case in this example). The null hypothesis of $\rho = 1$ as well as the hypothesis of $\rho = 0$ are rejected. In addition, there are also important differences among the estimates of γ. Furthermore, standard inference on the AB estimate does not reject the null hypothesis of $\gamma = 0$ at the 5% confidence level while this is clearly rejected in the case of the LSDVC estimate. Evidently, we do reject that the short-run unemployment elasticity of pay is -0.1 in all cases. However we do not reject that the long-run unemployment elasticity of pay is -0.1 when the coefficients are estimated by means of the LSDVC estimator. Clearly, it is invalid to test this latter hypothesis by means of standard asymptotic statistical inference. Thus, we conduct a bootstrap test by computing the statistic of contrast of the test for each of the 1999 bootstrap samples and by obtaining the 2.5 and 97.5 percentiles of the distribution of this statistic. The interval delimited by these two percentiles determines the zone of nonrejection of the null hypothesis of the test.

Finally, Figure 3 illustrates the results reported in Table 7. In this case, the 95% Grid-t confidence interval is included in the 95% percentile-t bootstrap interval, illustrating why, in most cases, the former confidence interval gives a better coverage than the latter one.

Figure 3 about here

6 Conclusions

In this paper we study the inference and estimation of dynamic panel data models in a special and increasingly important class of small samples that we denoted two-sided small samples (i.e., panels where the time dimension (T) is larger than a single-digit number but where the cross-section dimension (N) is not that large neither). We study the finite-sample properties of the most important methods of estimation proposed in the literature. Our main results are the following:

Even though one may have expected the LSDV estimator to perform well in samples where T is large, the bias of the fixed effect estimator was sizeable, even for $T = 30$ when $N = 50$. This result demonstrates the poor performance of this estimator in two-sided small samples. Thus, it is invalid to use it in most of the panel data sets available.

The LSDVC estimator proposed by Kiviet (1995) performs much better than all other estimators considered in the literature both in terms of bias reduction and by the RMSE criteria. This estimator is quite accurate and, hence, must be the one adopted to estimate dynamic panel data models in small samples.

More importantly, we find that standard inference is not valid for any of the estimators and data generating process considered in this paper. We find that for all the estimators studied, the true size of t-type tests may differ substantially from their asymptotic nominal level although the way they depart from this asymptotic approximation vary among them. Interestingly, this result holds for ρ as well as for the true value of the coefficient associated to the exogenous variable (γ). Surprisingly, this result also holds for the null hypothesis $\gamma = 0$ and, but not necessarily, when the dependent and exogenous explanatory variables are correlated in the DGP. Indeed, in our application to the U.S. data, where N is reasonable large ($N = 51$), we find that the main bias in the GMM estimates occur in the case of γ, where the coefficient estimated by LSDVC is 60% higher than the one estimated by means of the AB estimator. In the application to the Argentine data, where N is not large ($N = 17$), we find that the GMM estimates of both coefficients are substantially biased downward. In this case, based on the GMM estimate and standard statistical inference we do not reject the null hypothesis of no impact of local unemployment on local wages, contradicting a standard finding of the literature and what is know about wages and unemployment in Argentina during the period studied. Consequently, irrespective of which estimator performs better in terms of bias reduction and

RMSE in the class of small samples we study, it is necessary to consider also the finite sample behavior of t-type tests in order to conduct valid statistical inference.

Thus, the evidence presented in this paper shows that the LSDVC estimator must be preferred for estimating the parameters of a dynamic panel data model in two-sided small panels. However, it also shows that standard statistical inference is misleading and, hence, bootstrap standard errors have to be computed to conduct valid statistical inference on the parameters of this model.

Finally, we find that standard bootstrap techniques work well except when the autoregressive parameter in the model is close to one. In this case we find that the Grid-t bootstrap estimator due to Hansen (1999) outperforms any other alternative to estimate the standard errors of the estimates of the parameters of dynamic panel data models in two-sided small samples. Thus, we recommend to estimate the parameters of the model by means of the estimator proposed by Kiviet (1995) and to assess their sample variability by means of standard bootstrap procedures when the true value of ρ, appraised by the point estimate of it, is not large; and to rely on the Grid-t bootstrap method due to Hansen (1999) when the true value of ρ approaches one.

Table 1: Monte Carlo Design. 45 **different parameter combinations.**

Case	T	N	ρ	γ	Case	T	N	ρ	γ	Case	T	N	ρ	γ
I	20	30	0.2	0	XVI	20	30	0.2	1	XXXI	20	30	0.2	-1
II	20	30	0.5	0	XVII	20	30	0.5	1	XXXII	20	30	0.5	-1
III	20	30	0.8	0	XVIII	20	30	0.8	1	XXXIII	20	30	0.8	-1
IV	30	30	0.2	0	XIX	30	30	0.2	1	XXXIV	30	30	0.2	-1
V	30	30	0.5	0	XX	30	30	0.5	1	XXXV	30	30	0.5	-1
VI	30	30	0.8	0	XXI	30	30	0.8	1	XXXXVI	30	30	0.8	-1
VII	20	50	0.2	0	XXII	20	50	0.2	1	XXXVII	20	50	0.2	-1
VIII	20	50	0.5	0	XXIII	20	50	0.5	1	XXXVIII	20	50	0.5	-1
IX	20	50	0.8	0	XXIV	20	50	0.8	1	XXXIX	20	50	0.8	-1
X	30	50	0.2	0	XXV	30	50	0.2	1	XL	30	50	0.2	-1
XI	30	50	0.5	0	XXVI	30	50	0.5	1	XLI	30	50	0.5	-1
XII	30	50	0.8	0	XXVII	30	50	0.8	1	XLII	30	50	0.8	-1
XIII	40	50	0.2	0	XXVIII	40	50	0.2	1	XLIII	40	50	0.2	-1
XIV	40	50	0.5	0	XXIX	40	50	0.5	1	XLIV	40	50	0.5	-1
XV	40	50	0.8	0	XXX	40	50	0.8	1	XLV	40	50	0.8	-1

Table 2: Monte Carlo Results.

		% Bias		RMSE				% Bias		RMSE	
		ρ	γ	ρ	γ			ρ	γ	ρ	γ
I	FE	−31.866	0.067	0.076	0.036	XIII	FE	−16.277	0.042	0.039	0.018
	K	0.880	0.066	0.044	0.034		K	−0.549	0.042	0.022	0.017
	AB	−22.258	−0.216	0.067	0.051		AB	−11.459	0.018	0.034	0.023
	BB	−14.777	−0.221	0.059	0.052		BB	−7.579	−0.068	0.031	0.023
II	FE	−16.841	0.076	0.093	0.037	XIV	FE	−8.254	0.044	0.046	0.018
	K	0.100	0.075	0.043	0.035		K	−0.236	0.044	0.020	0.017
	AB	−13.774	−0.252	0.087	0.051		AB	−6.565	−0.028	0.041	0.023
	BB	−10.174	−0.295	0.073	0.052		BB	−4.937	−0.090	0.036	0.024
III	FE	−14.023	0.095	0.117	0.039	XV	FE	−6.527	0.043	0.054	0.019
	K	−2.024	0.088	0.046	0.037		K	−0.152	0.045	0.017	0.018
	AB	−16.291	−0.348	0.144	0.052		AB	−6.824	−0.060	0.059	0.024
	BB	−12.674	−0.406	0.116	0.054		BB	−5.702	−0.093	0.051	0.025
XVI	FE	−14.998	2.436	0.042	0.047	XXVIII	FE	−7.567	1.368	0.021	0.026
	K	0.713	−0.038	0.031	0.041		K	−0.157	0.071	0.015	0.022
	AB	−13.634	1.981	0.047	0.056		AB	−7.965	1.515	0.024	0.030
	BB	−11.032	1.946	0.045	0.056		BB	−6.795	1.415	0.024	0.030
XVII	FE	−5.353	2.715	0.036	0.049	XXIX	FE	−2.556	1.533	0.017	0.026
	K	0.293	−0.055	0.024	0.041		K	−0.02	0.056	0.011	0.022
	AB	−5.682	2.338	0.044	0.057		AB	−2.957	1.797	0.021	0.031
	BB	−5.362	2.531	0.043	0.059		BB	−2.902	1.879	0.021	0.032
XVIII	FE	−2.303	2.067	0.023	0.042	XXX	FE	−0.951	1.252	0.010	0.023
	K	0.658	0.229	0.019	0.041		K	0.035	0.016	0.006	0.020
	AB	−2.777	1.351	0.031	0.049		AB	−1.216	1.359	0.013	0.026
	BB	−2.570	1.315	0.030	0.052		BB	−1.198	1.301	0.013	0.028
XXXI	FE	−15.412	−2.366	0.042	0.047	XLIII	FE	−7.689	−1.307	0.022	0.026
	K	0.320	0.115	0.030	0.041		K	−0.280	−0.011	0.016	0.022
	AB	−12.658	−2.164	0.044	0.055		AB	−6.039	−1.167	0.022	0.028
	BB	−10.185	−2.156	0.042	0.056		BB	−4.822	−1.138	0.021	0.028
XXXII	FE	−5.629	−2.710	0.036	0.049	XLIV	FE	−2.601	−1.476	0.017	0.027
	K	0.016	0.065	0.024	0.041		K	−0.065	0.000	0.012	0.022
	AB	−5.130	−2.440	0.040	0.055		AB	−2.356	−1.419	0.018	0.029
	BB	−4.821	−2.683	0.039	0.057		BB	−2.258	−1.515	0.018	0.030
XXXIII	FE	−2.431	−2.029	0.024	0.043	XLV	FE	−0.967	−1.190	0.010	0.023
	K	0.497	−0.213	0.018	0.040		K	0.020	0.045	0.006	0.020
	AB	−2.526	−1.516	0.029	0.048		AB	−0.940	−1.037	0.011	0.024
	BB	−2.267	−1.569	0.027	0.05		BB	−0.932	−1.025	0.011	0.025

Note: 1000 replications.

Table 3: Monte Carlo Results. t-statistic for $\hat{\rho}$

		FE	K	AB	BB			FE	K	AB	BB
I	1%	−3.956	−1.794	−3.449	−5.439	XIII	1%	−3.759	−1.727	−4.020	−5.768
	5%	−3.055	−1.165	−2.669	−4.273		5%	−2.999	−1.179	−2.998	−4.670
	10%	−2.765	−0.922	−2.181	−3.378		10%	−2.640	−0.899	−2.296	−3.541
	90%	−0.241	0.967	0.443	1.355		90%	−0.247	0.848	0.309	1.349
	95%	0.041	1.199	0.817	1.854		95%	0.102	1.093	0.650	1.767
	99%	0.584	1.610	1.373	2.852		99%	0.623	1.455	1.410	3.081
II	1%	−4.599	−1.833	−3.936	−6.088	XIV	1%	−4.353	−1.822	−4.808	−7.114
	5%	−3.744	−1.247	−3.217	−4.952		5%	−3.550	−1.189	−3.475	−5.183
	10%	−3.417	−0.967	−2.758	−4.051		10%	−3.226	−0.938	−2.723	−4.456
	90%	−0.882	1.040	0.000	0.553		90%	−0.838	0.841	−0.167	0.47
	95%	−0.614	1.232	0.472	1.237		95%	−0.445	1.123	0.089	1.145
	99%	−0.071	1.674	0.921	2.254		99%	0.094	1.597	1.269	2.375
III	1%	−5.772	−2.718	−5.311	−7.482	XV	1%	−5.629	−1.921	−5.040	−7.907
	5%	−5.155	−2.052	−4.544	−6.273		5%	−4.953	−1.281	−4.798	−6.266
	10%	−4.815	−1.683	−4.109	−5.621		10%	−4.552	−0.993	−4.106	−5.974
	90%	−2.409	0.905	−1.166	−1.124		90%	−2.258	0.985	−1.422	−1.524
	95%	−2.087	1.185	−0.853	−0.601		95%	−1.900	1.281	−0.935	−0.530
	99%	−1.466	1.701	−0.303	0.376		99%	−1.185	1.888	−0.509	0.523
XVI	1%	−3.496	−1.761	−3.243	−5.295	XXVIII	1%	−3.129	−1.570	−3.503	−5.756
	5%	−2.648	−1.171	−2.523	−3.832		5%	−2.538	−1.132	−2.909	−4.498
	10%	−2.271	−0.885	−2.195	−3.227		10%	−2.225	−0.904	−2.043	−3.140
	90%	0.264	0.981	0.557	1.204		90%	0.203	0.828	0.373	0.744
	95%	0.584	1.236	0.952	1.871		95%	0.632	1.135	0.630	1.317
	99%	1.274	1.722	1.663	2.833		99%	1.199	1.555	1.427	1.924
XVII	1%	−3.682	−1.743	−3.407	−5.074	XXIX	1%	−3.320	−1.559	−3.926	−5.893
	5%	−2.827	−1.153	−2.708	−3.936		5%	−2.665	−1.137	−2.782	−4.375
	10%	−2.465	−0.856	−2.328	−3.486		10%	−2.319	−0.871	−2.201	−3.288
	90%	0.137	1.024	0.427	0.796		90%	0.115	0.896	0.205	0.274
	95%	0.475	1.266	0.694	1.133		95%	0.504	1.148	0.517	0.693
	99%	1.105	1.796	1.489	2.622		99%	1.159	1.637	0.991	1.604
XVIII	1%	−4.108	−1.872	−3.451	−5.188	XXX	1%	−3.555	−1.587	−4.210	−5.905
	5%	−3.120	−1.136	−2.811	−4.012		5%	−2.961	−1.193	−3.164	−4.733
	10%	−2.686	−0.857	−2.370	−3.477		10%	−2.648	−0.947	−2.651	−3.640
	90%	−0.068	1.434	0.205	0.405		90%	−0.055	0.915	0.119	0.202
	95%	0.223	1.989	0.644	1.052		95%	0.405	1.279	0.449	0.626
	99%	1.062	3.064	1.310	2.422		99%	0.967	1.706	0.959	1.547

Note: 1000 replications. The 1th, 5th, 10th, 90th, 95th and 99th quantiles for the standard normal distribution are −2.32, −1.64, −1.28, 1.28, 1.64 and 2.32, respectively.

Table 3: Monte Carlo Results. t-statistic for $\hat{\rho}$ (Cont.)

		FE	K	AB	BB			FE	K	AB	BB
XXXI	1%	−3.404	−1.695	−2.899	−4.383	XLIII	1%	−3.424	−1.789	−3.267	−4.886
	5%	−2.722	−1.186	−2.264	−3.514		5%	−2.635	−1.198	−2.235	−3.577
	10%	−2.319	−0.915	−1.976	−2.905		10%	−2.230	−0.916	−1.892	−2.881
	90%	0.240	0.972	0.661	1.258		90%	0.209	0.830	0.609	1.337
	95%	0.498	1.134	1.028	1.724		95%	0.636	1.145	0.788	1.487
	99%	1.156	1.619	1.457	2.530		99%	1.258	1.602	1.167	2.462
XXXII	1%	−3.598	−1.722	−3.285	−4.971	XLIV	1%	−3.497	−1.689	−3.847	−5.128
	5%	−2.881	−1.231	−2.354	−3.562		5%	−2.749	−1.167	−2.336	−3.49
	10%	−2.543	−0.949	−2.142	−3.078		10%	−2.437	−0.959	−1.982	−2.997
	90%	0.081	0.976	0.513	0.794		90%	0.140	0.874	0.410	0.715
	95%	0.397	1.213	0.871	1.482		95%	0.487	1.164	0.887	1.187
	99%	1.062	1.661	1.448	2.292		99%	1.025	1.551	1.134	1.728
XXXIII	1%	−4.031	−1.847	−3.126	−4.793	XLV	1%	−3.603	−1.619	−3.469	−5.155
	5%	−3.256	−1.220	−2.589	−3.675		5%	−3.021	−1.173	−2.506	−3.751
	10%	−2.887	−0.913	−2.266	−3.268		10%	−2.645	−0.934	−2.245	−3.208
	90%	−0.139	1.428	0.165	0.522		90%	−0.071	0.961	0.162	0.193
	95%	0.259	1.940	0.542	1.128		95%	0.287	1.157	0.647	1.125
	99%	0.977	3.053	1.135	2.181		99%	0.850	1.583	1.193	1.543

Note: 1000 replications. The 1th, 5th, 10th, 90th, 95th and 99th quantiles for the standard normal distribution are −2.32, −1.64, −1.28, 1.28, 1.64 and 2.32, respectively.

Table 4: Monte Carlo Results. t-statistic for $\hat{\gamma}$

		FE	K	AB	BB			FE	K	AB	BB
I	1%	−2.557	−1.724	−2.736	−3.695	XIII	1%	−2.566	−1.770	−2.634	−3.755
	5%	−1.627	−1.101	−1.750	−2.593		5%	−1.662	−1.122	−1.568	−2.250
	10%	−1.252	−0.849	−1.369	−1.886		10%	−1.253	−0.883	−1.228	−1.899
	90%	1.249	0.865	1.232	1.693		90%	1.396	0.961	1.280	1.748
	95%	1.744	1.176	1.594	2.257		95%	1.761	1.224	1.487	2.269
	99%	2.413	1.644	2.429	3.319		99%	2.520	1.767	2.159	3.133
II	1%	−2.597	−1.779	−2.724	−3.651	XIV	1%	−2.628	−1.811	−2.588	−3.608
	5%	−1.629	−1.111	−1.765	−2.650		5%	−1.666	−1.129	−1.641	−2.324
	10%	−1.295	−0.853	−1.402	−1.927		10%	−1.310	−0.896	−1.240	−1.809
	90%	1.321	0.885	1.265	1.742		90%	1.409	0.960	1.294	1.683
	95%	1.749	1.188	1.577	2.318		95%	1.790	1.237	1.468	2.302
	99%	2.465	1.641	2.506	3.452		99%	2.532	1.759	2.197	3.009
III	1%	−2.724	−1.901	−2.769	−3.718	XV	1%	−2.690	−1.867	−2.610	−3.520
	5%	−1.741	−1.244	−1.859	−2.764		5%	−1.694	−1.152	−1.739	−2.248
	10%	−1.350	−0.928	−1.449	−2.047		10%	−1.370	−0.911	−1.228	−1.943
	90%	1.405	0.957	1.261	1.809		90%	1.476	0.957	1.250	1.688
	95%	1.906	1.299	1.693	2.363		95%	1.888	1.274	1.631	2.394
	99%	2.648	1.928	2.576	3.467		99%	2.714	1.778	2.247	3.079
XVI	1%	−1.633	−1.526	−1.866	−2.839	XXVIII	1%	−1.623	−1.565	−1.563	−2.269
	5%	−1.091	−1.194	−1.351	−1.792		5%	−1.048	−1.165	−1.058	−1.501
	10%	−0.675	−0.914	−0.934	−1.439		10%	−0.662	−0.890	−0.620	−1.083
	90%	1.781	0.860	1.637	2.302		90%	1.907	0.927	1.732	2.228
	95%	2.130	1.099	1.870	2.760		95%	2.230	1.170	2.152	3.008
	99%	2.830	1.598	2.662	3.796		99%	2.944	1.675	2.623	3.544
XVII	1%	−1.609	−1.619	−1.997	−3.049	XXIX	1%	−1.609	−1.620	−1.745	−2.752
	5%	−1.072	−1.213	−1.288	−1.827		5%	−0.944	−1.158	−0.710	−1.176
	10%	−0.602	−0.923	−0.836	−1.235		10%	−0.617	−0.923	−0.409	−0.647
	90%	1.868	0.858	1.789	2.654		90%	1.982	0.922	1.757	2.629
	95%	2.234	1.126	2.024	2.926		95%	2.404	1.210	2.245	3.225
	99%	2.818	1.578	2.614	3.822		99%	2.956	1.628	2.831	4.014
XVIII	1%	−1.975	−1.775	−1.933	−3.715	XXX	1%	−1.684	−1.672	−1.760	−2.958
	5%	−1.097	−1.172	−1.469	−2.485		5%	−0.948	−1.147	−0.882	−2.029
	10%	−0.716	−0.878	−1.023	−1.789		10%	−0.614	−0.895	−0.674	−1.333
	90%	1.784	1.007	1.541	2.494		90%	2.030	1.000	1.664	3.047
	95%	2.137	1.242	1.893	3.069		95%	2.381	1.252	2.144	3.455
	99%	2.765	1.907	2.532	4.251		99%	3.094	1.778	2.518	4.82

Note: 1000 replications. The 1th, 5th, 10th, 90th, 95th and 99th quantiles for the standard normal distribution are −2.32, −1.64, −1.28, 1.28, 1.64 and 2.32, respectively.

Table 4: Monte Carlo Results. t-statistic for $\hat{\gamma}$ (Cont.)

		FE	K	AB	BB			FE	K	AB	BB
XXXI	1%	−2.758	−1.594	−2.980	−4.207	XLIII	1%	−3.030	−1.709	−2.280	−3.182
	5%	−2.149	−1.095	−2.096	−2.967		5%	−2.316	−1.239	−1.746	−2.579
	10%	−1.758	−0.820	−1.640	−2.333		10%	−1.867	−0.913	−1.617	−2.412
	90%	0.700	0.915	0.821	1.206		90%	0.722	0.933	0.830	1.175
	95%	1.060	1.182	1.040	1.637		95%	1.156	1.257	1.109	1.681
	99%	1.693	1.661	1.751	2.483		99%	1.696	1.635	1.650	2.243
XXXII	1%	−2.891	−1.603	−3.026	−4.403	XLIV	1%	−3.060	−1.711	−2.604	−4.310
	5%	−2.243	−1.156	−2.148	−3.104		5%	−2.387	−1.218	−1.995	−2.906
	10%	−1.878	−0.859	−1.737	−2.630		10%	−2.033	−0.967	−1.748	−2.536
	90%	0.610	0.931	0.725	1.073		90%	0.612	0.932	0.654	0.876
	95%	0.911	1.155	1.036	1.490		95%	1.014	1.230	0.891	1.452
	99%	1.537	1.606	1.629	2.426		99%	1.590	1.645	1.729	2.011
XXXIII	1%	−3.017	−1.961	−2.787	−4.298	XLV	1%	−3.106	−1.737	−3.198	−5.465
	5%	−2.179	−1.321	−2.010	−3.236		5%	−2.243	−1.134	−1.861	−3.308
	10%	−1.763	−0.968	−1.642	−2.576		10%	−1.917	−0.900	−1.674	−2.608
	90%	0.739	0.888	0.897	1.560		90%	0.764	0.979	0.637	1.285
	95%	1.068	1.145	1.216	2.323		95%	1.076	1.213	1.082	1.572
	99%	1.676	1.560	1.754	2.940		99%	1.833	1.755	1.677	2.653

Note: 1000 replications. The 1th, 5th, 10th, 90th, 95th and 99th quantiles for the standard normal distribution are −2.32, −1.64, −1.28, 1.28, 1.64 and 2.32, respectively.

Table 5: Monte Carlo Design. 90% **Confidence Level Intervals.**

	ρ	0.2	0.5	0.8
Asymptotic		0.138	0.197	0.408
Percentile-t		0.104	0.115	0.036
Kilian				0.046
Grid-α				0.052
Grid-t				0.063
	γ	1	1	1
Asymptotic		0.098	0.115	0.168
Percentile-t		0.116	0.113	0.071

Table 6. The Wage Curve: U.S. States, 1980-1991

	Dependent variable = Log State Wage (w_{it})		
	LSDV	GMM (AB)	LSDVC
Lagged log wage (w_{it-1})	0.9054	0.9095	0.9113
Standard Inference	(0.871, 0.939)	(0.814, 1.004)	(0.875, 0.948)
Standard Bootstrap			(0.903, 0.974)
Kilian Bias-Corrected			(0.901, 0.951)
Grid-α			(0.903, 0.965)
Grid-t			(0.899, 0.998)
Log unemployment rate (u_{it})	−0.0417	−0.0296	−0.0477
Standard Inference	(−0.049, −0.035)	(−0.041, −0.018)	(−0.055, −0.040)
Standard Bootstrap			(−0.048, −0.032)
State Fixed Effects	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes

Notes: 1. All regressions contain 612 observations (51 states over 12 years) for the period 1980 to 1991. Wages and individual controls are from the Merged Outgoing Rotation Group Files of CPS. Wages are earnings per hour. We restrict the sample only to employee workers. Unemployment is the state unemployment rate.

2. Figures in parentheses are 95% confidence intervals.

Table 7. The Wage Curve: Argentine Regions, 1991-1997

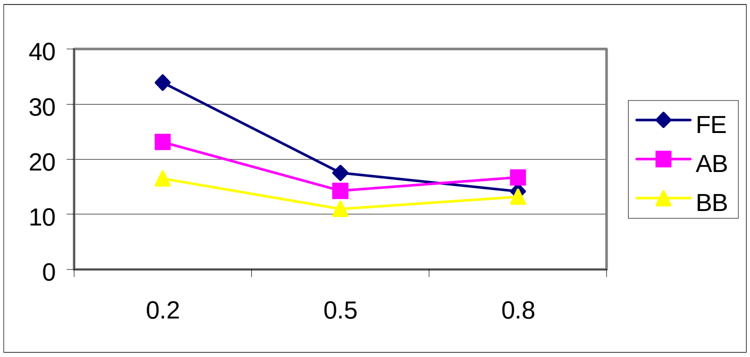
	Dependent variable = Log Region Wage (w_{it})		
	LSDV	GMM (AB)	LSDVC
Lagged log wage (w_{it-1})	0.5327	0.5698	0.6877
Standard Inference	(0.424, 0.641)	(0.406, 0.734)	(0.582, 0.794)
Standard Bootstrap			(0.481, 0.731)
Kilian Bias-Corrected			(0.537, 0.712)
Grid-α			(0.558, 0.709)
Grid-t			(0.538, 0.704)
Log unemployment rate (u_{it})	-0.0314	-0.0270	-0.0485
Standard Inference	(-0.059, -0.004)	(-0.059, 0.005)	(-0.076, -0.021)
Standard Bootstrap			(-0.076, -0.023)
Region Fixed Effects	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes

Notes: 1. All regressions contain 238 observations (17 regions over 14 semester) for the period 1991 to 1997. Wages and individual controls are from the Permanent Household Survey conducted by INDEC. Wages are earnings per hour. We restrict the sample only to employee workers. Unemployment is the regional unemployment rate for males.

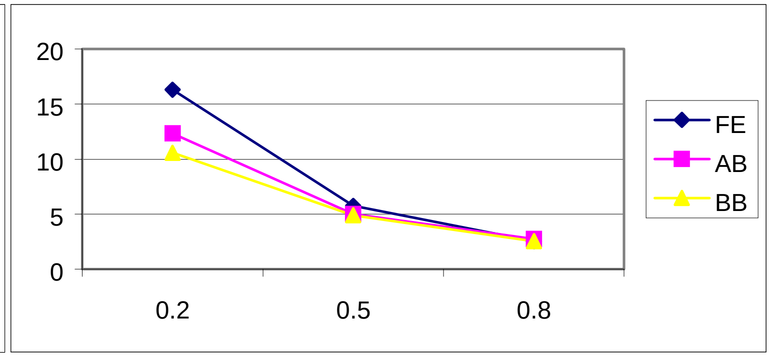
2. Figures in parentheses are 95% confidence intervals.

Figure 1

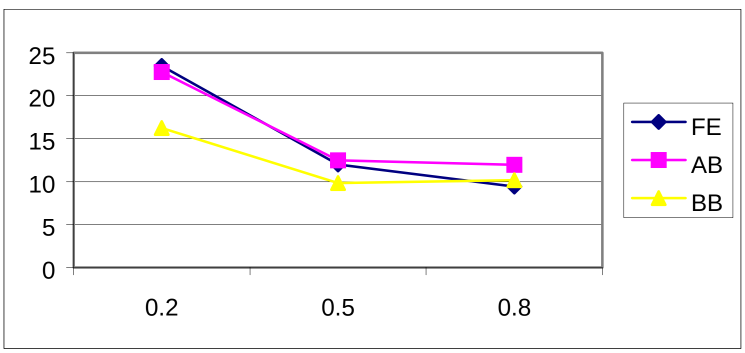
(a) T=20, N=30, γ=0



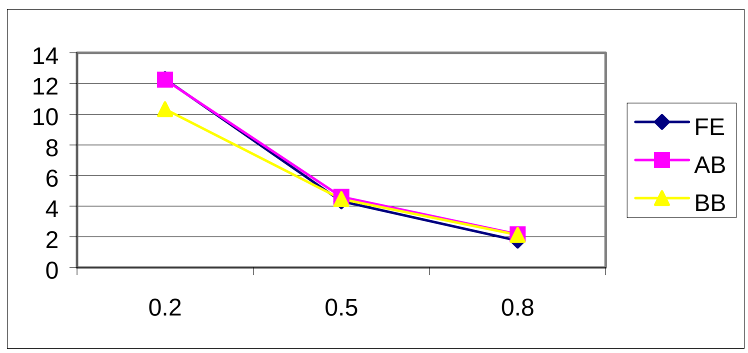
(b) T=20, N=30, γ=1



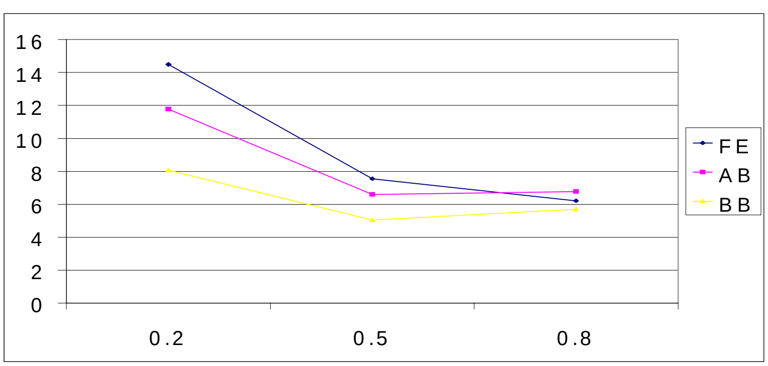
(c) T=30, N=30, γ=0



(d) T=30, N=30, γ=1



(e) T=40, N=50, γ=0



(f) T=40, N=50, γ=1

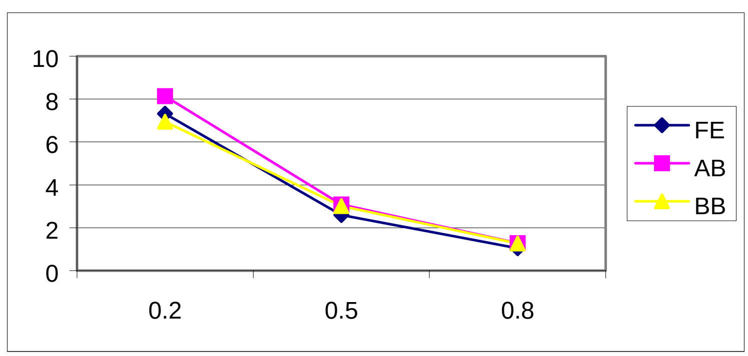


Figure 2

The Wage Curve: U.S. States 1980-1991

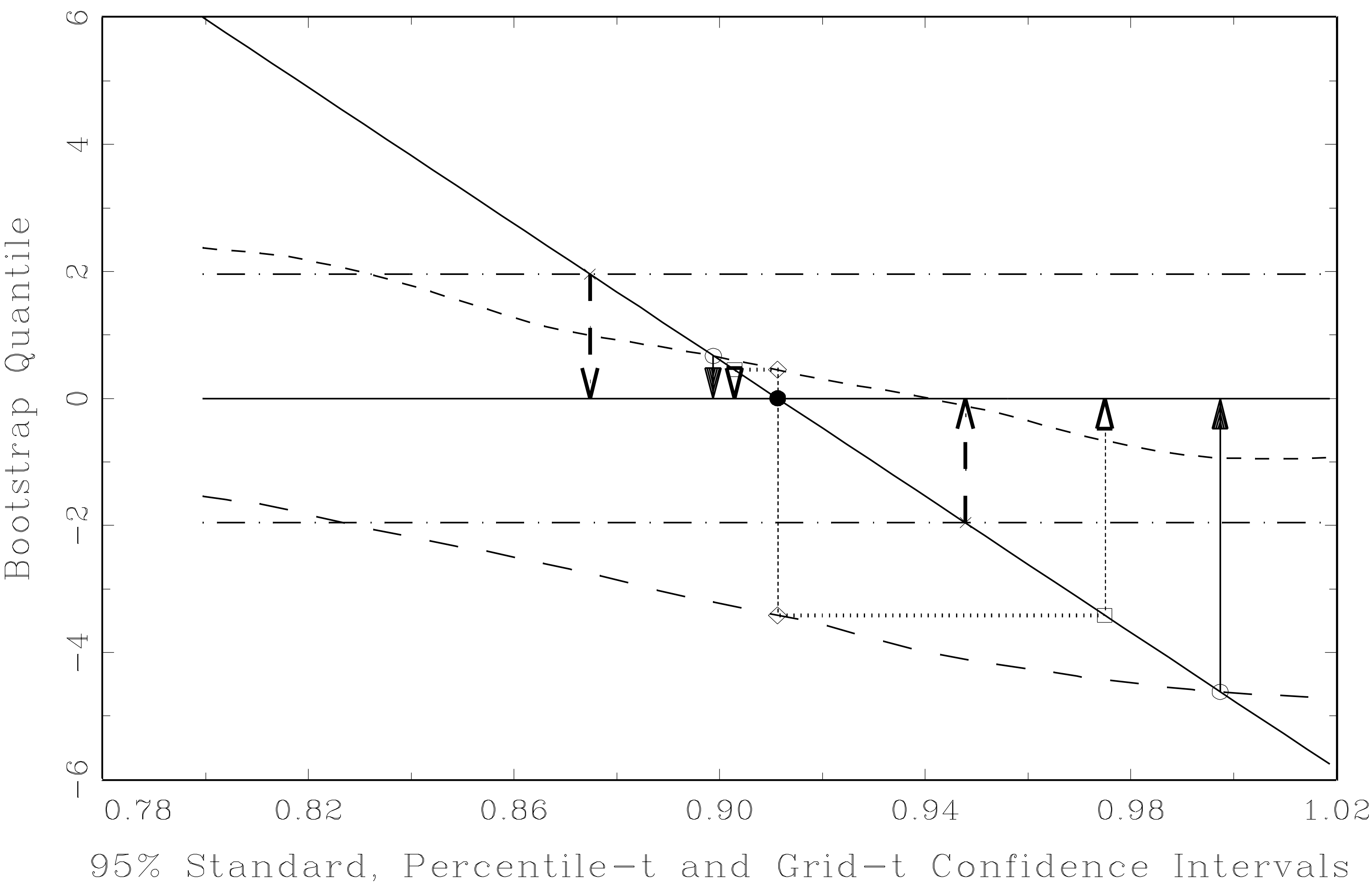
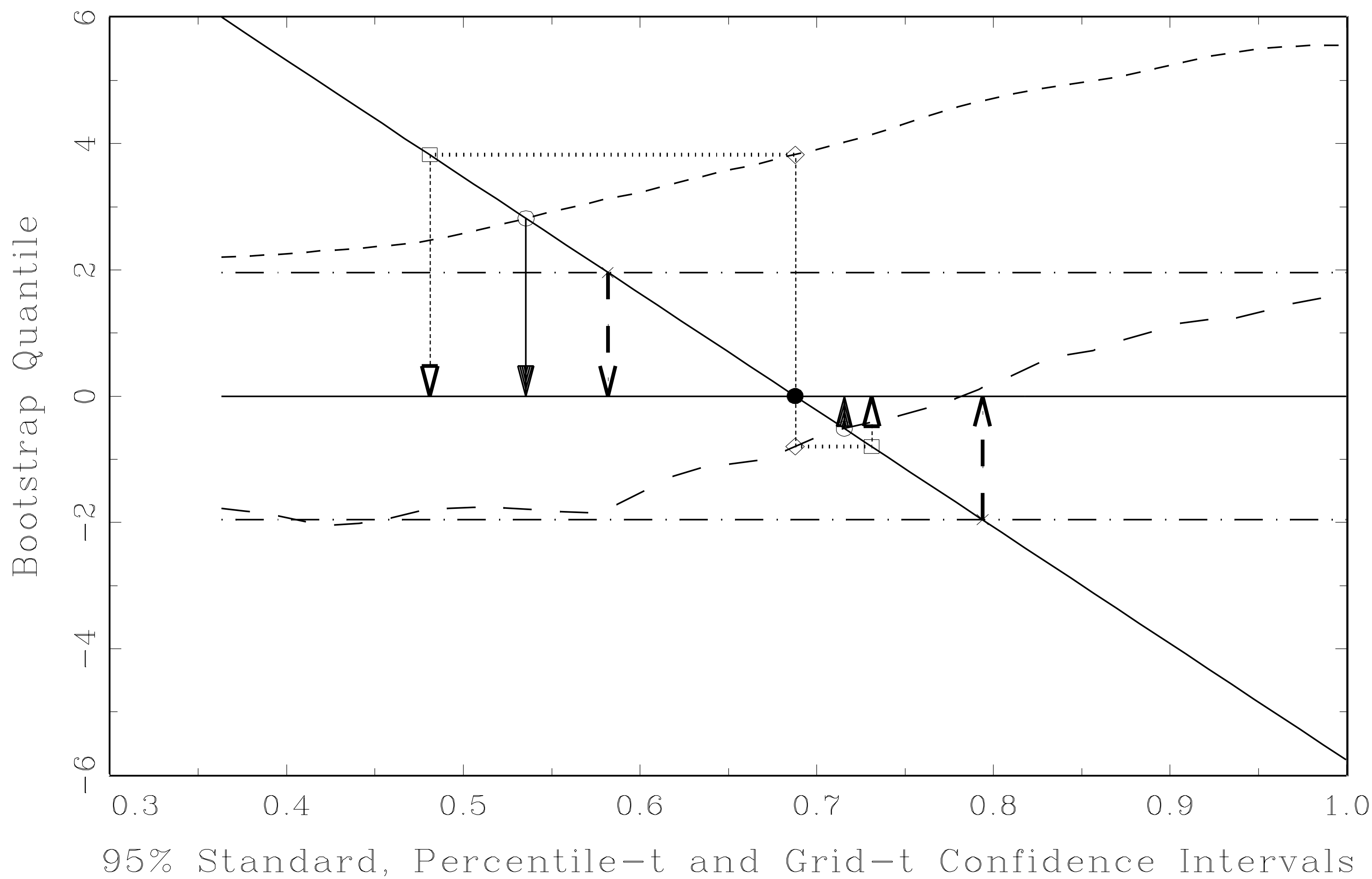


Figure 3

The Wage Curve: Argentine Regions 1991–1997



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