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Regulating a Polluting Firm Under Asymmetric Information

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Abstract. This paper reinterprets the Laffont-Tirole model of regulation under asymmetric information to cover the case of pollution control. The asymmetry of information concerns the firm's cost of lowering its pollution. The regulator has three objectives: Ensuring an efficient abatement level, generating 'green taxes' and securing the survival of the firm. We show that when optimal abatement is important relative to tax generation, the regulator cannot use the policy of offering the firm a set of linear tax schemes from which to choose. By contrast, this policy is optimal in the Laffont-Tirole model under certain not very restrictive assumptions. We proceed to establish a simple rule for when to shut-down inefficient types. In an example with specific functional forms, we derive the optimal tax function both analytically and graphically. We show the effect on the optimal tax system of a change in a technological parameter.

Key words: Laffont-Tirole model, tax generation, tax schemes, pollution, regulator

1. Introduction

Attempts at regulating pollution of a firm are sometimes met by the claim that the proposed regulation will lead the firm to either shut down or relocate. In this paper we derive the optimal system for taxing pollution, when the regulator does not know whether or not this claim is valid since he does not know the true costs to the firm of diminishing pollution. We assume that the regulator wants the firm to abate as near to the efficient level as possible, but that he is constrained by the need to secure the survival of the firm and by the fact that he cannot transfer too large sums to the firm (or we can assume that 'green taxes' are valuable through allowing the regulator to decrease taxes which distort other parts of the economy).[1] Survival of the firm may be important because the existence of the firm confers positive externalities (such as employment) on the community or, if survival is thought of as the decision not to relocate, because relocation to another jurisdiction with less strict taxation of pollution will not lower total pollution.

There is no simple solution to the regulator's problem. A simple Pigouvian marginal tax rate which is set equal to society's marginal disutility of pollution does not work when the firm's true cost-curve is unknown to the regulator, since the cost-curve is a determinant of the optimal tax rate, when society's marginal disutility is decreasing. Neither does a tax function which at each level of pollution sets the total tax payment equal to society's total disutility from pollution. This tax function yields optimal incentives but may be costly in terms of transfers to the firm when all types must survive.[2]

We derive the optimal tax function in a modified version of the adverse-selection/moral-hazard model of Laffont-Tirole (1993). We reinterpret the model to cover the case of a polluting firm, which inflicts disutility on society. The firm can lower its level of pollution through the exercise of unobservable, and hence noncontractible, effort. The problem is then to find the optimal tax function which links taxes to the level of pollution.

In the similar model in Laffont-Tirole (1993) it is shown that under certain not very restrictive assumptions, the tax function is decreasing and convex. This means that for high levels of pollution, marginal incentives for abatement are relatively small. The reason is that the regulator can extract more rents from highly efficient types if he does not induce the least efficient types to abate very much. In our model the same effect applies but another factor enters: allocational efficiency calls for *lower* marginal incentives at lower levels of pollution because society's disutility of pollution is increasing in the level of pollution.

More exactly, the reason that rent extraction (the generation of tax-revenue) calls for higher marginal incentives at lower levels of pollution (higher levels of abatement) is the following: The regulator must take into account that types which abate at low cost have the option of choosing the points on the tax function designed for types which abate at high cost. This option implies that since the high-cost types must survive the low-cost types will earn a rent. The rents earned by low-cost types depend on the extent to which the regulator induces the high-cost polluters to abate. Inducing the high-cost types to abate at a high level implies rewarding the high-cost types in the form of lower taxes. However, offering such an inducement to the high-cost types implies that the low-cost types can be taxed less at any given level of abatement. Thus, to decrease rents the regulator must accept that the high-cost types do not abate up to their socially optimal level.

The conflicting forces of efficiency and rent extraction lead to the result that *the optimal transfer function is convex or concave depending on whether rent extraction or optimal abatement has the highest priority*. This is important since only when it is convex can it be approximated by a menu of simple, linear tax schemes, consisting of a lump-sum transfer and a marginal tax rate. This implies that it will often be the correct policy to announce the function itself or, as an approximation, to announce a menu of pairs among which the firm must chose, each pair consisting of a *quantity* of pollution allowed and a corresponding tax payment to be made by the firm. We stress this result since regulators may be inclined to look fora menu of linear schemes, because 'price-regulation' has certain well-known advantages over 'quantity-regulation' and perhaps also from the impression created by Laffont and Tirole's influential book.

In the second part of the paper, we show how the optimal tax function can be derived in a concrete example with a given functional form of the abatement technology. We analyze both algebraically and graphically how the optimal tax function changes when the perceived abatement technology changes. The change in technology affects the difference in efficiency of different types. Due to an assumption of linear disutility of pollution, the example can be interpreted to cover the regulation of an entire industry and not just one firm. (The first part of this paper deals, on the other hand, exclusively with the regulation of one firm, the type of which is unknown.) Interpreting the model as dealing with the regulation of an entire industry, we compare the regulatory method of this paper to a system of tradeable permits and show the superiority of the former.

2. A model of regulation

2.1. Notation and Concepts

A regulator regulates one polluting firm (monopolist). The monopolist can, through the exercise of unverifiable effort, diminish pollution. The regulator observes the level of pollution of the firm, but its effort and technology are private information. The technology determines the firm's intrinsic cost of abatement, which we index by the firm's *type* β. We assume that the type belongs to a closed set $[\underline{\beta};\overline{\beta}]$. The type is unknown to the regulator, who only knows the cumulative probability distribution F and the associated density function f on $[\underline{\beta};\overline{\beta}]$, which we assume is strictly positive. The total level of pollution is determined by the firm's type β and its effort in pollution control e, that is $P = P(\beta, e)$. We assume that more effort results in a lower level of pollution, but at a decreasing rate. Hence, when subscripts denote partial derivatives: $P_e < 0, P_{ee} \geq 0$. Furthermore, we define the firm's type in such a way that small β's are 'clean' types and high β's are 'dirty' types: $P_\beta > 0$.

We define a function $E(P,\beta)$ as the minimum amount of effort which a firm of type β has to exercise in order to pollute (only) at the level P. The E-function can be derived from the P-function. It follows that $Ep(P,\beta) < 0, E_\beta(P,\beta) > 0$ and $E_{pp}(P,\beta) \geq 0$. We also assume the 'Single-Crossing' condition which means that if a given type is more efficient than another type at one level of pollution, the same will be true for other levels of pollution:

$$E_{\beta P}(P,\beta) \leq 0. \tag{1}$$

The firm incurs costs (monetary and otherwise) equal to $\psi(e)$ when exercising effort in pollution control. Assume that these costs are strictly increasing: $\psi' > 0$ and $\psi'' > 0$. In order to exclude stochastic tax schemes and to ensure concavity of the objective-function, we assume that $\psi''' \geq 0$.

The firm's utility is determined by the costs incurred and the taxes paid. For simplicity, we do not consider the firm's production or pricing decisions. As is conventional, we can think of the cost of effort as a fixed cost rather than as a marginal cost,in which case production decisions are not affected. For expositional convenience we model taxes as monetary transfers from the regulator *to* the firm. Of course a negative transfer amounts to a tax. We assume that if the firm's utility falls below a certain level, which we normalize to zero, society bears a cost S, e.g. because the firm will stop producing or relocate production. The utility of the firm is defined as $U \equiv t - \psi(e)$ where t is the transfer to the firm. Hence the firm is risk-neutral with respect to income.

The benefit to society from pollution (which of course is negative) is given by the concave function $B(P)$ where P is the current level of pollution: $B_P < 0$, $B_{PP} \leq 0$.

2.2. The Regulator's Problem

The level of pollution is the only variable containing information on the behavior of the firm which the regulator can observe and verify. Hence, it is the only variable on which taxes can depend. The regulator chooses a tax scheme $t(P)$ which links the transfer to the firm with the firm's pollution. Given $t(P)$, the regulator can infer the amount of pollution which each type will find it optimal to emit, the corresponding effort and utility to each type and society's loss.

The privately optimal level of pollution for a type β firm is given by the solution to:[3]

$$\max_P [t(P) - \psi(E(P, \beta))]. \tag{2}$$

By monotonicity of the effort-function $E(\cdot)$ every pollution level for a particular type corresponds exactly to an optimal effort level $e(\beta)$. The utility of each type can then be easily found from the expression $U = t(P) - \psi(e)$. For each type the utility to society is $B(P(\beta)) - (1 + \lambda)t(P(\beta))$, since when the rest of society is taxed by the amount t, its utility decreases by t directly, but to this must be added the distortionary impact of taxes, measured by λ.

The social welfare function is assumed to be utilitarian:

$$W = S + B(P) - (1 + \lambda)t + U, \tag{3}$$

where S is assumed to be large if the firm continues to produce, while S equals 0 if this is not the case. In the main part of the following, we shall assume that S is so large as to make it worthwhile to ensure the survival of the firm. If we impose on the regulator's problem the constraint that no type of firm may obtain negative utility, we can ignore S in the social welfare function.[4]

Using that $U = t - \psi(e)$, the social welfare function can then be rewritten as

$$W = B(P) - (1 + \lambda)\psi(e) - \lambda U.$$

The regulator's problem is to design a tax scheme $t(P)$ which induces the set $(P(\beta), e(\beta), U(\beta))$ that maximizes expected welfare

$$\int_{\underline{\beta}}^{\overline{\beta}} [B(P(\beta)) - (1 + \lambda)\psi(e(\beta)) - \lambda U(\beta)] f(\beta) \mathrm{d}\beta$$

subject to the *Incentive Compatibility* constraint that each type of firm will choose the level of pollution $P(\beta)$ that maximizes its utility

$$t(P(\beta)) - \psi(E(P(\beta), \beta)) \geq t(P) - \psi(E(P, \beta)) \quad \forall P$$

and the *Individual Rationality* constraint that all types obtain at least zero utility, $U(\beta) \geq 0, \forall \beta$.[5]

In the appendix we show that these constraints are equivalent to:

$$\mathrm{d}P/\mathrm{d}\beta > 0, \tag{4}$$

$$U'(\beta) = -\psi'(e)E_\beta, \text{ for all } \beta \in [\underline{\beta}; \overline{\beta}], \tag{5}$$

$$U(\underline{\beta}) \geq 0. \tag{6}$$

Hence, the regulator should devise a mechanism to:

$$\max_{t(P)} \int_{\underline{\beta}}^{\overline{\beta}} [B(P(\beta)) - (1 + \lambda)\psi(E(P(\beta), \beta)) - \lambda U(\beta)] f(\beta) \mathrm{d}\beta$$

subject to the above constraints. Instead of maximizing social welfare by choosing the function $t(P)$ appropriately, the regulator can choose the function $e(\beta)$ appropriately, and then derive the optimal $t(P)$ function from the optimal $e(\beta)$ function. The problem can then be solved using optimal control theory, taking $e(\beta)$ as the control variable and $U(\beta)$ as the sale variable. The problem is to:

$$\max_{e(\beta)} \int [B(P(e(\beta), \beta)) - (1 + \lambda)\psi(e(\beta)) - \lambda U(\beta)] f(\beta) \mathrm{d}\beta$$

subject to the same constraints as above: (IC), (IR) and monotonicity. This is a standard optimal control problem if we ignore the monotonicity constraint. We will ensure monotonicity by checking proposed solutions for it. We also need the *monotone hazard-rate* condition.[6]

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{f(\theta)}{1 - F(\theta)}\right) \geq 0. \tag{7}$$

Using standard optimal control theory, we derive the following proposition:

PROPOSITION 1: *If the monotonicity (4) and the monotone hazard-rate condition (7) are fulfilled, and the function* e$(\beta) \geq 0$ *solves:*

$$\psi'(e) = \frac{1}{1 + \lambda} B'(P)P_e - \frac{\lambda}{1 + \lambda} \frac{F(\beta)}{f(\beta)} [\psi''(e)E_\beta + \psi'(e)E_{\beta P}P_e]$$

then $e(\beta)$ *is the solution to the maximization problem. If the solution to this equation for some* β *is negative, the solution at this point is* $e(\beta) = 0$.

The proof of the proposition is in appendix 2.

The proposition tells us[7] to increase (any) type β's effort up to the point where a further increase has higher costs in terms both of effort on the part of the type itself and of increased rents to more efficient types than benefits in terms of an improved environment. To simplify the interpretation multiply through with $(1 + \lambda)f(\beta)$. Then

$$f(\beta)(B'(P)P_e - (1+\lambda)\psi'(e(\beta)))$$

expresses the marginal gain to society of type β exercising δe higher effort. There are $f(\beta)$ of this type and the immediate benefit to society is thus $f(\beta)B'(P)P_e$ whereas the extra cost to this type is $\psi'(e(\beta))$ which is multiplied by $(1 + \lambda)$ since the firm must be compensated with tax-payers' money to be as well off as before (and hence not have an incentive to choose an effort level designed for less efficient types).

Inducing type β to exercise δe higher effort involves the cost that higher rents must be paid out to more efficient types. The determinant of how much rents to the more efficient types must be *increased* is how much easier the more efficient type can manage the *same increase* in effort (performance) as type β. Already the extra rent which a type just slight more efficient than β earns in comparison to β is given by the incentive constraint $U'(\beta) = -\psi'(e(\beta))E_\beta$ which simply states that since the more efficient type can save E_β in effort simply by virtue of his type then his utility gain in comparison to β must be this amount of saved effort times the marginal disutility of effort $\psi'(e(\beta))$. When type β's effort level is increased, this extra rent increases and the increase is found by differentiating $\psi'(e(\beta))E_\beta$ with respect to $e(\beta)$. This gives $\psi''(e)E_\beta + \psi'(e)E_{\beta P}P_e$. The extra rent must be given to all types which are more efficient than β, of which there are $F(\beta)$. The cost in terms of extra rents thus equals

$$\lambda F(\beta)(\psi''(e)E_\beta + \psi'(e)E_{\beta P}P_e).$$

The first part of the following proposition follows from proposition 1.

PROPOSITION 2: *When the marginal benefits to society of pollution abatement is decreasing, the transfer function will be concave when rent extraction plays only a minor role (when e.g.* λ *is small) whereas the transfer function may be convex when the marginal benefit to society is not decreasing very fast and rent-extraction is important.*

If allocational efficiency dominates, which will be the case when λ is sufficiently small, the solution yields approximately $B'(P)P_e \approx \psi'(e(\beta))$. Assuming that the firm's maximization problem $\max_e[t(P(e,\beta)) - \psi(e)]$, has an interior solution, we

have $t'(P)P_e = \psi'(e(\beta))$, hence $t_P = B_P$. This holds for all P in the relevant range, implying that when B is concave, t will also be concave: $t_{PP} = B_{PP} \leq 0$.

The intuition is straightforward: When society's marginal disutility from pollution increases with the level of pollution and only allocational efficiency is important, marginal incentives for pollution abatement should also increase with the level of pollution.

The second part of the proposition follows from the Laffont-Tirole analysis. When marginal disutility from pollution is independent of the level of pollution, it is not difficult to construct examples in which the transfer function becomes convex. If we, e.g., assume that $P(\beta, e) = \beta - e$, and that $B'(P) = -1/(1 + \lambda)$, our model reduces to that of Laffont and tirole, who prove the transfer scheme to be convex.[8]

The conclusion is that under many circumstances the regulator cannot regulate through 'prices' (a menu of affine tax schemes) but must announce a concave transfer function or, if that is not feasible, announce a certain number of quantity-transfer pairs.

The interpretation is the following: If the regulator were able to tax $B(P)$ instead of P, i.e. if a tax could be levied on society's disutility of pollution and not on pollution itself, the results of the Laffont-Tirole model would apply. That is, the optimal transfer function would be convex in pollution, hence for high levels of pollution marginal incentives for abatement would be relatively small. This is due to the fact that the regulator is able to extract more rents from highly efficient types if he does not induce the least efficient types to abate very much. However, when taxing $B(P)$ is not possible, the convexity result no longer holds due to the concavity of the $B(\cdot)$ function. In this context another factor enters: allocational efficiency calls for *lower* marginal incentives at lower levels of pollution when society's disutility of pollution is increasing in the level of pollution.

3. An Example

It is not immediately obvious how or whether the general solution can be implemented in practice. In the following example with specific functional forms, we derive the optimal tax function and graphically show the resulting pollution levels, the rents and the effort levels of different types. Furthermore, by changing a parameter which expresses both absolute levels of abatement efficiency and the differences in productivity of abatement between different types, i.e. the regulator's uncertainty concerning the type he is facing, we gain some intuition on the importance of technology and technology differences (the regulator's uncertainty) for the optimal tax-system.

As mentioned, the example is characterized by the assumption that society's disutility of pollution is linear in the level of pollution. This means that we can interpret the example as covering the regulation of an industry with many firms, since the pollution of one firm does not affect the optimal tax scheme for another firm under linearity. On the other hand, when society's marginal disutility of pollution is not constant, the optimal tax schemes for different firms are interdependent, and the present analysis does not apply. Interpreting the types as actual firms in an industry, we can compare this method of regulation to other methods which are generally employed to regulate an industry, such as the system of tradeable permits. We show that the regulatory method of Laffont-Tirole is superior to the system of tradeable permits, basically because it allows different marginal tax-rates for different types.

3.1. Assumptions

We assume: $B(P) = -P, P(e,\beta) = \beta - \frac{1}{\beta k}e, \psi(e) = e^2$, S is large.

Notice the society's disutility of pollution is assumed to be linear in the level of pollution. As mentioned previously, this allows us to interpret the example as the regulation of an entire industry in which the fraction of firms of type β is $f(\beta)$. The constant k is a measure of the differences in marginal abatement productivity between the different types. We thus assume that the more efficient types pollute less even without exercising any effort. One can imagine that they have access to cleaner technologies. We initially assume that survival of the firm is essential.

The specification has little meaning if e is negative. We hence require that $e(\beta) \geq 0, \forall\beta$. This yields the least-effort function $E(\beta, P) = \max(0, \beta^2 k - \beta k P)$. We choose a uniform distribution of the types, $\beta \in [\underline{\beta}; \overline{\beta}] = [1; 2]$: $F(\beta) = \beta - 1 \Rightarrow f(\beta) = 1$. This distribution meets the monotone-hazard-rate requirement.

3.2. The Solution

The example meets the assumptions: $E_p \leq 0, E_\beta \geq 0, \psi' \geq 0, \psi''' \geq 0, f(\beta) > 0$. From the first-order-condition

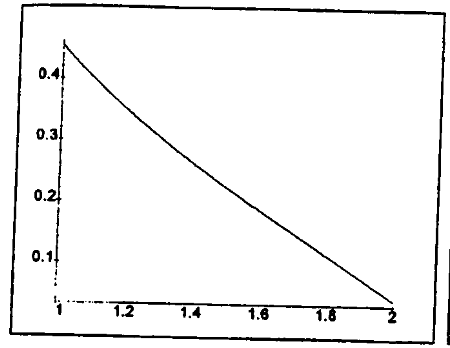
$$\psi'(e) = \frac{1}{1+\lambda}B'(P)P_e - \frac{\lambda}{1+\lambda}\frac{F(\beta)}{f(\beta)}[\psi''(e)E_\beta + \psi'(e)E_{\beta P}P_e]$$

we obtain the optimal effort of type β:

$$e(\beta) = \frac{\frac{1}{2k} + \lambda\beta^2 k(1-\beta)}{\beta + 3\lambda\beta - 2\lambda)}, e \geq 0. \tag{8}$$

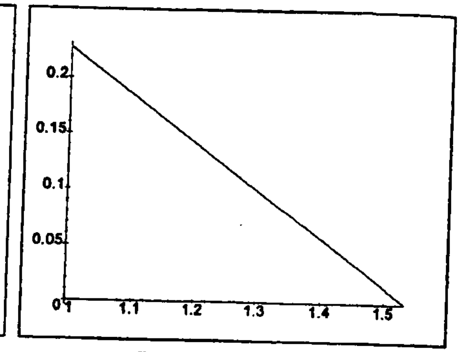
Figures 1(a) and 1(b) clearly show the effect of changing the efficiency parameter k. When k is raised from 1 to 2, the marginal efficiency of all types are lowered. This lowers the optimal effort levels of all types. Actually, when $k = 2$ all types $\beta \in [1.5323; 2]$ will exercise zero effort due to their low efficiency![9]

The *rent* or utility of a type β firm can be computed by integrating the (IC)-condition:



Effort and type for $k = 1$.

Figure 1 (a)



Effort for $k = 2$.

Figure 1 (b)

$$U(\beta) = U(\overline{\beta} + \int_{\underline{\beta}}^{\beta} \left(U'(\tilde{\beta})\right) d\tilde{\beta} = \int_{\beta}^{\overline{\beta}} \left[\psi'(e)E_{\tilde{\beta}}\right] d\tilde{\beta} =$$

$$\int_{\beta}^{\overline{\beta}} 2e(\tilde{\beta} \left(2\tilde{\beta}k - k\left(\tilde{\beta} - \frac{e(\tilde{\beta})}{\tilde{\beta}k}\right)\right) d\tilde{\beta} \tag{9}$$

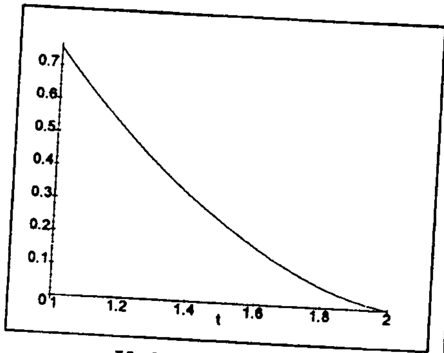
and the transfer is determined by:

$$t(\beta) = U(\beta) + \psi(e(\beta)) =$$

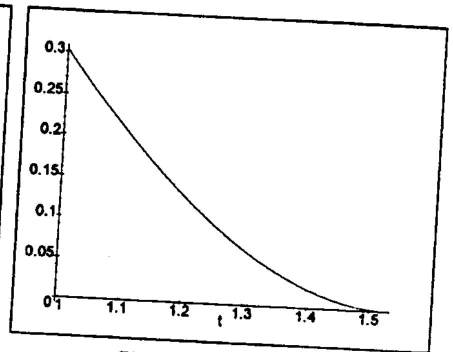
$$\int_{\beta}^{\overline{\beta}} 2e(\tilde{\beta}) \left(2\tilde{\beta}k - k\left(\tilde{\beta} - \frac{e(\tilde{\beta})}{\tilde{\beta}k}\right)\right) d\tilde{\beta} + e(\beta)^2. \tag{10}$$

From the graphs of the utility and transfer functions (Figures 2(a), (b) and 3(a), (b)) it is clear that the transfer function is convex. Pollution abatement is hence rewarded progressively. Convexity implies that the function can be approximated by a set of linear tax-schemes. Note that the utility levels are everywhere higher for more efficient firms, e.g. when $k = 1$ as opposed to $k = 2$.

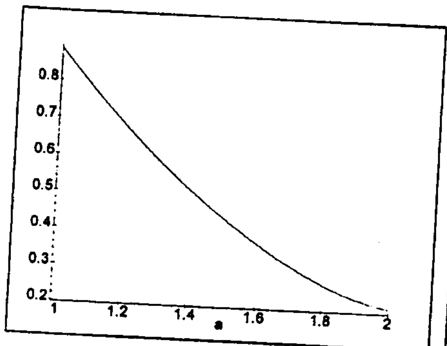
One might conjecture that the transfer scheme becomes concave for sufficiently high values of k, since in this case very efficient types need almost no inducement to limit pollution. This would, however, be incorrect. It can be seen from the graphs which depict the case $k = 2$ that the optimal tax scheme remains convex and that the main effect of increasing k is to diminish incentives of the least efficient types (this was confirmed for other values of k). However, as k increases, more types are given zero incentives for pollution abatement and hence do not exercise any



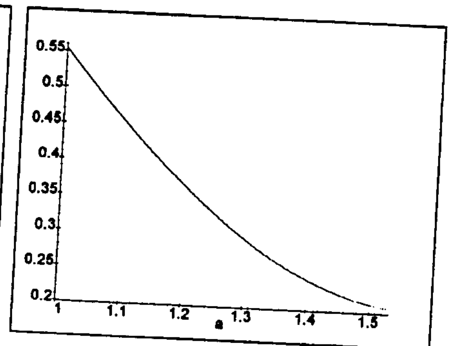
Utility for $k = 1$.
Figure 2 (a)



Utility for $k = 2$.
Figure 2 (b)



Transfer for $k = 1$.
Figure 3 (a)



Transfer for $k = 2$.
Figure 3 (b)

effort. Compare the graph of effort for $k = 1$, where all types exercise positive effort, with the figure for $k = 2$, types above 1.53 do not exercise any effort at all. Since the high types exercise no effort when $k = 2$, rents for the efficient firms can be lowered.

Concerning the incentives provided for the inefficient types, we note that it is costly in terms of rents paid to efficient types (due to the incentive-compatibility condition) to induce the inefficient types to limit pollution while ensuring their survival. Incentives increase as we move from less efficient to more efficient firms because the number of even more efficient firms, whose rents must increase as a consequence, falls.

3.3. Allowing Shutdown of Heavy Polluters

It has been assumed so far that the survival of the firm is so important that even the firms who exercise no effort in pollution abatement should be allowed to survive. In the case $k = 2$, firms were essentially given the option not to participate in the tax mechanism. The question arises when the regulator should instead require that all firms participate in the mechanism, even when this implies that some types will not continue their operations.

We denote the cut-off type β^*. If we shut out all types higher than β^*, the participation constraint is that type β^* obtains at least zero utility. The maximization problem then becomes identical to the one we have already analyzed, only the interval of possible types has changed and with it the probability distributions. If we restrict attention to types in the interval $[\beta, \beta^*]$ the new density function $f^*(\beta)$ becomes $f(\beta)/F(\beta^*)$ (the conditional probability) while the new cumulative distribution function becomes $F(\beta)/F(\beta^*)$. Derivation of the Hamiltonian yields the expression

$$\psi'(e) = \frac{1}{1+\lambda} B'(P) P_e - \frac{\lambda}{1+\lambda} \frac{F(\beta)/F(\beta^*)}{f(\beta)/F(\beta^*)} [\psi''(e) E_{\beta\beta} + \psi'(e) E_{\beta P} P_e]$$

where the only change lies in the distribution functions where $F(\beta^*)$ cancels out. Hence, for any $\beta \geq \beta^*$, the effort level will be unchanged. However, the utility of type β will change. It will be given by

$$U(\beta) = \int_{\beta}^{\beta^*} \psi'(\tilde{e}(\tilde{\beta})) E_{\tilde{\beta}} \mathrm{d}\tilde{\beta}$$

by (5)

We can hence calculate the optimal cut-off level. Social welfare is a function of β^*:

$$\begin{aligned} W(\beta^*) = & \int_{\underline{\beta}}^{\beta^*} (S + B(\tilde{P}(\beta) - (1+\lambda)\psi(\tilde{e}(\beta)) f(\beta) \mathrm{d}\beta - \\ & \int_{\underline{\beta}}^{\beta^*} \left(\int_{\beta}^{\beta^*} \lambda \psi'(\tilde{e}(\tilde{\beta})) E_{\tilde{\beta}} \mathrm{d}\tilde{\beta} \right) f(\beta) \mathrm{d}\beta. \end{aligned}$$

Social welfare for $\beta \geq \beta^*$ is zero, reflecting no pollution, no effort, no transfer and no S. From the expression above, we obtain:

$$\begin{aligned} W'(\beta^*) = & (S + B(P(\beta^*)) - (1+\lambda)\psi(\tilde{e}(\beta^*)) f(\beta^*) - \\ & \lambda F(\beta^*) \psi'(\tilde{e}(\beta^*) E_{\beta^*}. \end{aligned}$$

The derivative of

$$\int_{\underline{\beta}}^{\beta^*} \left(\int_{\beta}^{\beta^*} \lambda\psi'(\tilde{e}(\tilde{\beta}))E_{\tilde{\beta}}\mathrm{d}\tilde{\beta} \right) f(\beta)\mathrm{d}\beta$$

with respect to β^* equals $\lambda F(\beta^*)\psi'(\tilde{e}(\beta^*)E_{\beta*}$ from an integration by parts.[10]

Assuming an interior maximum, i.e. assuming $\beta^* \leq \overline{\beta}$, a necessary condition for β^* to be the optimal cut-off type is hence:

$$(S + B(\tilde{P}(\beta^*)) - (1+\lambda)\psi(\tilde{e}(\beta^*))f(\beta^*) - \lambda F(\beta^*)\psi'(\tilde{e}(\beta^*)E_{\tilde{\beta}} = 0.$$

There is a clear interpretation of this expression. When considering whether or not to shut down type β^*, the regulator must weigh the value lost if the firm does not survive with the disutility of the pollution of this type given the optimal tax function and the disutility of effort of the firm, multiplied by $(1 + \lambda)$ since the disutility of the firm means that it must be given a transfer (or that it can be taxed less). These benefits and costs must be weighed by their probability $f(\beta^*)$. On top of this, the regulator needs to take into account a less obvious benefit of shutting down the type β^*. Fewer rents have to be paid to types more efficient than β^*. The type who is marginally more efficient than β^* can be given $\psi'(\tilde{e}(\beta^*)E_{\tilde{\beta}}$ less in rent if β^* is shut down. Incentive compatibility will be maintained if rents to all types who are more efficient than β^* is diminished by this amount when β^* is shut down, since the relative attractiveness of alternative points on the tax function then remains unchanged. There are $F(\beta^*)$ of these more efficient types. The expected rent $F(\beta^*)\psi'(\tilde{e}(\beta^*)E_{\tilde{\beta}}$ is a transfer, the cost of which to society is only λ times the amount. There is thus a simple rule to decide whether or not to shut down a given type. Compare the value of survival to the direct cost $B(\tilde{P}(\beta^*)) - (1+\lambda)\psi(\tilde{e}(\beta^*))$ and as an extra cost the rent earned by the adjacent, slightly more efficient firm multiplied by λ. The latter is multiplied by the frequency of types who are more efficient than β^*, and the former are multiplied by the frequency of the type itself.

In our example, the condition for shut-down becomes:

$$S - \left(\beta^* - \frac{\tilde{e}(\beta^*)}{\beta^* k}\right) - (1+\lambda)\tilde{e}(\beta^*)^2 - \lambda(\beta^* - 1)2\tilde{e}(\beta^*) = 0$$

where

$$\tilde{e}(\beta^*) = \frac{-1/2k - \lambda(\beta^* - 1)\beta^{*2}k}{\beta^*(1+2\lambda)}.$$

From this, the optimal cut-off type β^* can be expressed as a function of λ and k.

More importantly from the operational viewpoint, that level of pollution which should cause the authorities to shut down the firm can be calculated. It is given

by $P(\tilde{e}(\beta^*), \beta^*) = \beta^* - \frac{1}{\beta^* k}\tilde{e}(\beta^*)$. When $e(\beta^*) = 0$, that is when the optimal tax function gives the type β^* no incentive for pollution control, the condition for shut down reduces to $S = \beta^*$. This means that the value of keeping the firm alive should simply be compared with the disutility of its pollution. The fact that the type survives has no implications for the rents paid to other firms, since the option of choosing to do nothing and receiving no transfers yields zero utility, and we have already assumed that all types must receive at least zero utility.

4. Tradeable Permits

In the case where β is one of several actually existing firms, one can compare the regulatory system analyzed above with other mechanisms. One mechanism consists of granting tradeable permits to the firms. The tradeable permit system may take many forms; we shall define the *simple tradeable system* to be a system in which each firm is given an *equal* number of permits (since it is assumed that the regulator cannot distinguish firms ex-ante, i.e. there is no information, no historical data, for example, on which to base an unequal distribution of permits), and perhaps an *equal* lump-sum transfer or tax.

The simple tradeable system will not in general do as well as the tax system of the model. The problem can be explained as follows. The system needs to be organized so as to address the issue of survival of the high-cost abaters. The regulator must ensure that a given number of permits, distributed among the firms in the industry, will not force some high-cost abater into bankruptcy, i.e. into a situation in which the firm has no means to buy more permits and cannot afford to bring its own pollution down within the permissible limit. This problem can always be solved by increasing the lump-sum transfer to the high-cost abater or by increasing its number of permits. However, this means increasing the lump-sum transfer to the other firms in the industry by an equal amount or increasing their number of permits by an equal amount, by the definition of the simple tradeable permit system above. This may become very expensive in terms of tax revenue. The idea of the tax system of the model was to provide different types with different marginal incentives for pollution abatement. This enabled us to bring down the rent earned by the low-cost types (firms), as explained in the introduction. However, this is not possible in the simple tradeable permit system since marginal incentives for different firms will always be equal, namely equal to the market price of permits. In the third appendix we demonstrate this intuition using the specific example analyzed above.

Admittedly, it can be maintained that if we allowed more sophisticated forms of tradeable permit systems allowing, e.g., the number of permits granted to a firm to depend on its level of pollution then the superiority of the Laffont–Tirole tax system could disappear.

Conclusion

We have shown that the regulation set-up in Tirole and Laffont (1993) can be applied to pollution control. The idea of offering more than one contract to a firm whose type is unknown is clearly important in the area of pollution control. However, society's decreasing marginal benefit of abatement is likely to render a menu of simple (affine) tax schemes sub-optimal. When the marginal benefit to society of abatement is strongly decreasing and rent extraction is not very important, the optimal tax function is concave. In this case, the regulator should either announce the concave tax function, or announce a certain number of pollution-tax pairs.

In an example with constant social benefit from abatement, we derived specific transfer functions and showed how a change in productivity, which affected both absolute productivity and increased productivity differences between the different types, made it more likely that the inefficient types should either be given the option of doing nothing or shut down. This result is intuitive in the sense that when productivity differences are large, it becomes too costly in terms of rents paid to the efficient types to induce the inefficient types to abate.

We derived a concrete formula for when to leave the firm alone and when to shut it down.

The transfer function for the remaining types turned out to be convex in our example, independent of the productivity differences, which means that the regulator can use tax schemes of the form $t(P) = a - bP$. It may be worth reminding, however, that this result is far from general, even when as in the example marginal social benefit is constant. For example, it does not hold when the regulator knows the firm to be one of two possible types.[11]

In a reinterpretation of the model, the different types were seen as actually existing firms in an industry. The results mentioned above apply in this reinterpretation, only actual firms are shut down or given certain incentives, not hypothetical types of a given firm. In this interpretation, the regulatory system can be compared to other regulatory methods such as tradeable permits. The present approach was shown to be superior to the system of tradeable permits, since the latter does not pay attention to the question of survival of firms. If it is made to do so, it will be equivalent to offering one tax scheme to all types, which is an option in the present approach but generally sub-optimal.

Notes

1. The terms 'tax' and 'transfer' will be used interchangeably; a transfer will denote simply a negative tax, i.e. a payment from the regulator to the firm.
2. Hence the solution we obtain should be viewed as a first step in maximizing social welfare. In the second step the regulator must consider whether it is optimal to shut some types of firms down. The shut-down decision is discussed later in the paper.
3. The function $t(P)$ will be defined in the following.
4. Of course the 'correct' way to model this would be to include the firm's pricing decision explicitly. Doing this would add even more ambiguity to the model, as well as obscuring the main point.

Some authors include this decision at the cost of assuming specific functional forms in order to solve the model, see e.g., (Baron (1985) and Spulber (1988).
5. We also require non-negative effort, that is $e(\beta) \geq 0$.
6. A form of monotone hazard-rate is assumed in all models of this type, see e.g., Fudenberg and Tirole, ch. 7.
7. The interpretation is also given in Tirole and Laffont (1993), p. 170.
8. Page 68 in Tirole and Laffont (1993).
9. The value of λ is set to 0.1 in all the graphs.
10. Perform an integration by parts with $F = F(\beta)$ and $g = \int_{\underline{\beta}}^{\beta^*} \lambda\psi'(\tilde{e}(\tilde{\beta})E_{\tilde{\beta}}d\tilde{\beta}$. Then
$$\int_{\underline{\beta}}^{\beta^*} F'(\beta)g(\beta)d\beta = \int_{\underline{\beta}}^{\beta^*} \left(\int_{\underline{\beta}}^{\beta^*} \lambda\psi'(\tilde{e}(\tilde{\beta})E_{\tilde{\beta}}d\tilde{\beta})\right) f(\beta)d\beta$$
By the rule of integration, this equals $\left[F(\beta)\int_{\underline{\beta}}^{\beta^*} \lambda\psi'(\tilde{e}(\tilde{\beta})E_{\tilde{\beta}}d\tilde{\beta}\right]_{\underline{\beta}}^{\beta^*} + \int_{\underline{\beta}}^{\beta^*} F(\beta)\lambda\psi'(\tilde{e}(\beta)E_{\beta}d\beta$.
The expression $\left[F(\beta)\int_{\underline{\beta}}^{\beta^*} \psi'(e(\tilde{\beta})E_{\tilde{\beta}}d\tilde{\beta}\right]_{\underline{\beta}}^{\beta^*}$ equals zero as can be verified by a simple calculation, using that $F(\underline{\beta}) = 0$. The derivative of $\int_{\underline{\beta}}^{\beta^*} F(\beta)\lambda\psi'(\tilde{e}(\beta)E_{\beta}d\beta$ with respect to β^* equals $\lambda F(\beta^*)\psi'(\tilde{e}(\beta^*)E_{\beta^*}$.
11. See Tirole and Laffont (1993), p. 73.

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Appendix 1

THE (IC) AND (IR) CONDITIONS

PROPOSITION 3: *(i) Under the assumptions in section 2.2, any piecewise continuously differentiable function* $\tilde{P}(\beta) = P(\beta, e(\beta))$ *is implementable, if and only if, it is strictly increasing,* $dP/d\beta > 0$.

(ii) The (IC) condition is equivalent to

$$U'(\beta) = -\psi'(e)E_{\beta}$$

for all $\beta \in [\underline{\beta}; \overline{\beta}]$, *and* $dP/d\beta > 0$.

(iii) The (IR) condition is then equivalent to

$$U(\beta_h) \geq 0.$$

PROOF: *ad (i)*: Since the preferences are quasi-linear, (i) follows directly from standard results, cf. Guesnerie-Laffont (1984).

ad (ii) and (iii): Suppose first that the (IC) condition is satisfied. The indirect utility of type β is

$$U(\beta) \equiv \max_{\hat{\beta}\in[\underline{\beta};\overline{\beta}]} \phi(\hat{\beta},\beta) = \max_{\hat{\beta}\in[\underline{\beta};\overline{\beta}]} [t(\hat{\beta}) - \psi(E(\beta, P(\hat{\beta})))].$$

In equilibrium truth-telling is optimal: $U(\beta) = \phi(\beta,\beta)$. Differentiate this: $U'(\beta) = \phi_1(\beta,\beta) + \phi_2(\beta,\beta)$. Using the first-order condition $\phi_1(\beta,\beta) = 0$ to obtain the local (IC) condition:

$$U'(\beta) = \psi_1(\beta,\beta) = -psi'(e)E_\beta \forall\beta \in [\underline{\beta};\overline{\beta}].$$

Hence $U(\beta)$ is decreasing and the (IR) condition is met for all types if and only if it is met for type β_h:

$$U(\beta_h) \geq 0.$$

Suppose now that the local, but not the global (IC) condition is satisfied. Then there must be a pair $\hat{\beta},\beta \in [\underline{\beta};\overline{\beta}]$ such that $\phi(\hat{\beta},\beta) - \phi(\beta,\beta) > 0 \Leftrightarrow \int_\beta^{\hat{\beta}} \phi_1(a,\beta)\,da > 0 \Leftrightarrow \int_\beta^{\hat{\beta}}(\phi_1(a,\beta) - \phi_1(a,a))da > 0 \Leftrightarrow \int_\beta^{\hat{\beta}}\int_a^\beta(\phi_{12}(a,b)dbda > 0$. Where $\phi_{12}(\hat{\beta},\beta) = -(\psi''(e)E_\beta E_P + \psi'(e)E_{P\beta})\frac{dP}{d\beta} > 0$. But $\hat{\beta} < \beta \Rightarrow a < \beta \Rightarrow \int\int \psi_{12}(\cdot)dbda < 0$ and $\hat{\beta} > \beta \Rightarrow a > \beta \Rightarrow \int\int \phi_{12}(\cdot)dbda < 0$, which is a contradiction. Hence the global (IC) condition must be satisfied.

Appendix 2

PROOF OF PROPOSITION 1

The Hamiltonian of the problem is:

$$H = (B(\bar{P}(e(\beta),\beta) - (1+\lambda)\psi(e(\beta)) - \lambda U(\beta))f(\beta) - u(\beta)\psi'(e(\beta))E_\beta.$$

We can define the derived Hamiltonian $H^0(U(\beta),\mu(\beta),\beta)$ as

$$\max_{e(\beta)} H(U(\beta), e(\beta), \mu(\beta), (\beta).$$

It is shown in Sethi and Thompson that if $(\tilde{e}(\beta), \tilde{U}(\beta), \tilde{\mu}(\beta))$ fulfil the following necessary conditions:

$$\tilde{U}'(\beta) = -\psi'(\tilde{e}(\beta))E_\beta$$

$$\tilde{U}(\overline{\beta}) = 0$$

$$\tilde{\mu}'(\beta) = -dH((\tilde{e}(\beta), \tilde{U}(\beta), \tilde{\mu}(\beta))/dU$$

$$\tilde{\mu}(\underline{\beta}) = 0$$

$$H((\tilde{e}(\beta), \tilde{U}(\beta), \tilde{u}(\beta)) \text{ maximizes } H((e(\beta), \tilde{U}(\beta), \tilde{\mu}(\beta)) \text{ for all } e(\beta) \text{ in } \Omega$$

and if, furthermore, $H^0(U(\beta),\mu(\beta),\beta)$ is concave in U for all β, then $(\tilde{e}(\beta), \tilde{U}(\beta), \tilde{\mu}(\beta))$ is a solution. Since $-\partial H((\tilde{e}(\beta), \tilde{U}(\beta))/\partial U = \lambda f(\beta)$, $H^0(U(\beta),\mu(\beta),\beta)$ is linear and hence concave in U for all β. Thus, the necessary conditions are also sufficient.

Given our assumptions, including that $\psi''' \geq 0$, H is a concave function of $e(\beta)$, hence the solution to the first-order condition

$$dH((e(\beta), \tilde{U}(\beta), \tilde{\mu}(\beta))/d(e(\beta)) = 0 \tag{11}$$

maximizes H. If the solution to this first-order condition is non-negative, we have found a solution to the overall problem. If it is negative, H is maximized by $e(\beta) = 0$, since $d^2H/de^2 \leq 0$, by the concavity of H with respect to $e(\beta)$.

The equations

$$\tilde{\mu}(\beta) = 0$$

$$\tilde{\mu}'(\beta) = -\partial H(\tilde{e}(\beta), \tilde{U}(\beta), \tilde{\mu}(\beta))/\partial U$$

can be used to find the $\tilde{\mu}(\beta)$ function. Since $-\partial H((\tilde{e}(\beta), \tilde{U}(\beta), \tilde{\mu}(\beta))/\partial U = \lambda f(\beta)$ and $\tilde{\mu}(\beta) = 0$, we have $\tilde{\mu}(\beta) = F(\beta)$. This expression is inserted into the H-function, and the first-order-condition then reads:

$$\psi'(e) = \frac{1}{1+\lambda}B'(P)P_e - \frac{\lambda}{1+\lambda}\frac{F(\beta)}{f(\beta)}[\psi''(e)E_\beta + \psi'(e)E_{\beta P}P_e].$$

This is monotonicity constraint is not necessarily satisfied, so we must verify for each solution we find whether it displays monotonicity. If the solution to this equation fulfils the constraint that $e(\beta) \geq 0$, it is the solution to the maximization problem. If the solution to this equation for some β is negative, we know from what was derived above that the solution is $e(\beta) = 0$.

Appendix 3

AN EXAMPLE OF THE INFERIORITY OF TRADEABLE PERMITS

The problem inherent in the simple tradeable permit system can be illustrated in our example.

Assume as in the example that society's disutility of pollution is P, i.e. $B(P) = -P$, that $P = \beta - e(\beta)/\beta$, as when $k = 1$ and that the other assumptions of the example are also maintained. Assume that each firm is given q tradeable permits. Each firm may then pollute q units without paying taxes. Alternatively a firm may of course sell some of its rights or buy rights from others. The decision variable for each firm can be seen as how much effort to exercise in pollution abatement. This effort decision then determines the amount of permits which the firm must trade. If the equilibrium price of a permit is ν, the maximization problem for firm β is to choose $e(\beta)$ so as to:

$$\max_{e(\beta)}[(q - P)\nu - e(\beta)^2]$$

where $P = \beta - e(\beta)/\beta$. The first-order condition which is necessary and sufficient due to concavity of the criterion-function, yields

$$e(\beta) = \nu/2\beta.$$

Assume now that the regulator attempts to implement the first-best allocation. In this allocation the marginal cost of pollution to the firms must equal society's marginal disutility. Since $B(P) = -P$, the marginal disutility is 1. Hence, in market equilibrium, for the first-best to be realized, we must have $\nu = 1$. If we insert this into the expression for firm β's effort, we see that type β will exercise effort equal to $1/2\beta$ which means that he will pollute at the level $P = \beta - e(\beta)/\beta = \beta - 1/2\beta^2$. The total level of pollution will hence equal

$$\int_1^2 \left(\beta - \frac{1}{2\beta^2}\right) f(\beta)\mathrm{d}\beta$$

which, since firms are uniformly distributed on the interval (Baron 1985; Baron and Myerson 1982) equals 6/4. In other words, the total number of permits is 5/4 equal to the level of pollution. When firms are uniformly distributed on the unit interval, this means that 'each (infinitesimally small) firm' is given 5/4 units. The utility of type β will then be $(q - P)\nu - e(\beta)^2 = 5/4 - (\beta - 1/2\beta^2) - 1/4\beta^2$. This expression equals zero or less for $\beta \geq 1.38$.

Hence, recalling that β lies between 1 and 2, and is uniformly distributed, 62% of all types will not survive the tradeable permit system which allocates efficiently.

To make all firms survive, if they must all be given the same amount as a lump-sum transfer, this transfer equals 11/16. The outcome is then equal to the one which would result if in our mechanism, we offered all firms the transfer-scheme

$$t(P) = 11/16 - P$$

recalling that the price of a permit is 1 in equilibrium. Since this was not the optimal transfer scheme, sub-optimality of the system of tradeable permits, when survival and rent extraction matter, is apparent.

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Pollution Accumulation and Firm Incentives to Accelerate Technological Change Under Uncertain Private Benefits

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Abstract. The paper explores the relationships between the design of public incentives and the policy-maker's desired timing of abandonment of a polluting technology, when this requires an irreversible private investment and the firm faces uncertain appropriable benefits from the technological change. Two regulatory approaches are examined. Firstly, we consider the quite common one of lowering the private investment cost, through a subsidy, in order to bridge the gap between the private and the policy-maker's desired timing of environmental innovation. Secondly, we consider a policy scenario where the regulator, instead of simply lowering the investment's rental price, also stimulates abandonment of the polluting technology by reducing – through appropriate announcements – the uncertainty surrounding the technological switch's private profitability. We then compare the two approaches and show the latter's benefits, in terms of the policy's effectiveness and/or budgetary savings.

Key words: environmental policy, technological change, irreversibility

1. Introduction

In recent years, alongside the traditional attitude of simply responding to the standards imposed by current legislation, firms have begun to show an anticipatory attitude by spontaneously adopting "green technologies" or by overmeeting environmental standards.

The literature offers various explanations for this phenomenon: firms may deliberately curtail emissions to anticipate (stricter) regulations, or to induce regulatory authorities to tighten up standards so as to raise the cost of compliance for their competitors (Azzone and Bertelé 1992; Sassone 1992; Arora and Cason 1995). Moreover, the effort to reformulate products or re-design production processes may be also explained by the aim to gain the reputation as an environmentally friendly company, so as to exploit consumers' preference to buy from a company with a better environment record. In fact, evidence of environmental consciousness exists, at least in affluent societies, where an increasing number of individuals do not merely demand a better environment through the ballot-box, but also in the market-place: "[...] polls have indicated a willingness to pay a higher price for a