Two Decision Variables

Jean-Pierre Benoît

Let us now expand our basic model of tort law by supposing that the injurer and victim each have *two* parameters to choose, only one of which is observable by the courts. Specifically, in addition to care levels i and v, we now posit unobservable "activity" levels z and x. Thus, for example, the probability a driver causes injury depends not only on how carefully he or she drives (the care level), but also on how often (the activity level).

We now write the expected damage as $D(i, v, z, x)$ where D is strictly decreasing in the first two arguments and strictly increasing in the last two. Denote the benefit to the injurer by $w(i, z)$ and the benefit to the victim by $b(v, x)$, where each function is decreasing in the first argument. Suppose that for each i, the benefit $w(i, z)$ is maximized by a unique z and that for each v, the benefit $b(v, x)$ is maximized by a unique x.

For any i,v, and x, and $0 \leq \alpha \leq 1$, let $z_\alpha(i, v, x)$ maximize $[w(i, z) - \alpha D(i, v, z, x)]$. Given any (i, v, x), $z_0(i, v, x)$ maximizes I's benefit, $w(i, z)$, while $z_1(i, v, x)$ maximizes I's benefit net of damages , $[w(i, z) - D(i, v, z, x)]$. Similarly, for any v, i, and z, and $0 \leq \alpha \leq 1$, let $x_\alpha(v, i, z)$ maximize $[b(v, x) - \alpha D(i, v, z, x)]$.

Let i^*, v^*, z^*, x^* be the social optimum. That is,

$$\begin{aligned} & w(i^*, z^*) + b(v^*, x^*) - D(i^*, v^*, z^*, x^*) \\ = \; & \max_{i,v,z,x} [w(i, z) + b(v, x) - D(i, v, z, x)] \end{aligned}$$

Note that $z^* = z_1(i^*, v^*, x^*)$ and $x^* = x_1(v^*, i^*, z^*)$. Assume that all functions are differentiable and that all maxima are interior.[1]

Let $\beta > \alpha$. A revealed preference argument yields:

[1] Question: Assumptions on primitives?

$$
\begin{aligned}
w(i, z_\alpha) - \alpha D(i, v, z_\alpha, x) &\geq w(i, z_\beta) - \alpha D(i, v, z_\beta, x) \\
w(i, z_\alpha) - \beta D(i, v, z_\alpha, x) &\leq w(i, z_\beta) - \beta D(i, v, z_\beta, x) \Rightarrow \\
(\beta - \alpha) D(i, v, z_\alpha, x) &\geq (\beta - \alpha) D(i, v, z_\beta, x) \\
z_\alpha &\geq z_\beta
\end{aligned}
$$

Thus, z_α is decreasing in α.

For any given i, v, and x, the first order condition for z_α is

$$\frac{\partial w(i, z_\alpha)}{\partial z} - \alpha \frac{\partial D(i, v, z_\alpha, x)}{\partial z} = 0 \tag{1}$$

For fixed z, the left hand side of (1) is strictly decreasing in α. Thus z_α is not constant as α changes, and the fact that z_α is decreasing in α implies that z_α is actually strictly decreasing in α. In particular, for any $\alpha < 1$

$$z_\alpha(i, v, x) > z_1(i, v, x) \tag{2}$$

Thus, given any (i, v, x), if I does not pay for all the damages, he chooses too high an activity level (from a social point of view).[2] Similarly, given any (i, v, z), if V does not pay for all the damages, she chooses too high an activity level.

1 Unilateral Case

Suppose the only decision agent is I, so that we can write $D(i, v, z, x) \equiv D(i, z)$ and $z_\alpha(i, v, x) \equiv z_\alpha(i)$.

Now consider a negligence rule with $X_r = i^*$. I's problem is

$$
\begin{aligned}
&\max_{i,z} f(i, z), \\
\text{where } f(i, z) &= \begin{cases} w(i, z) & \text{if } i \geq i^* \\ w(i, z) - D(i, z) & \text{if } i < i^* \end{cases}
\end{aligned}
$$

Since $[w(i, z) - D(i, z)]$ reaches an unconstrained maximum at (i^*, z^*), I chooses the level of care i^*. However, given the choice of i^*, the argument of

[2] What if we do not assume differentiability? interior solution?

the previous selection establishes that I then selects an activity level $z_0(i^*) > z^*$. That is, I chooses too high an activity level.

Clearly, no negligence rule with $X_r < \infty$ induces efficient behavior. On the other hand, a strict liability rule ($X_r = \infty$) *does* induce I to take the efficient amount of care, since his maximization problem is then the same as society's. However, as we will see in the next section, in the two-agent case efficiency cannot be obtained.

2 Bilateral Case

We now show that no loss assignment rule R yields the efficient outcome when there are two agents.

Given the rule R, let $(\hat{\imath}, \hat{v}, \hat{z}, \hat{x})$ be an equilibrium. Then I's payoff is

$$w(\hat{\imath}, \hat{z}) - R(\hat{\imath}, \hat{v}; X_I, X_V) D(\hat{\imath}, \hat{v}, \hat{z}, \hat{x})$$

while V's payoff is

$$b(\hat{\imath}, \hat{z}) - (1 - R(\hat{\imath}, \hat{v}; X_I, X_V)) D(\hat{\imath}, \hat{v}, \hat{z}, \hat{x})$$

Either $R(\hat{\imath}, \hat{v}; X_I, X_V) < 1$ or $[1 - R(\hat{\imath}, \hat{v}; X_I, X_V)] < 1$. Suppose that

$$R(\hat{\imath}, \hat{v}; X_I, X_V) < 1$$

Note that $\hat{z} = z_{R(\hat{\imath},\hat{v};X_I,X_V)}(\hat{\imath}, \hat{v}, \hat{x})$. But from (2),

$$z_{R(\hat{\imath},\hat{v};X_I,X_V)}(\hat{\imath}, \hat{v}, \hat{x}) > z_1(\hat{\imath}, \hat{v}, \hat{x}),$$

so that the equilibrium cannot be efficient. Similarly, if $[1 - R(\hat{\imath}, \hat{v}; X_I, X_V)] < 1$, then

$$x_{R(\hat{\imath},\hat{v};X_I,X_V)}(\hat{\imath}, \hat{v}, \hat{z}) > x_1(\hat{\imath}, \hat{v}, \hat{z}),$$

and again the equilibrium is not efficient.