Implementing the Efficient Allocation of Pollution

By John Duggan and Joanne Roberts*

Markets to allocate pollution rights play an important role in current efforts to control pollution efficiently (the Environmental Protection Agency has allowed firms to trade pollution permits since 1977), and this role is likely to grow as pollution abatement efforts intensify. It is clear that a solution to the pollution control problem must involve decentralization of some sort: efficient pollution emissions depend on revenue and cost characteristics of firms, which are typically unknown to regulatory authorities. And with decentralization there arises the possibility of strategic behavior on the part of firms. The extensive literature on the theory of implementation is concerned with precisely this problem, but the mechanisms proposed there, while general in scope, are typically quite complex. In this paper, we construct a simple mechanism to solve the specific problem of efficiently allocating pollution emissions among a fixed set of firms, assuming the regulatory authority can observe pollution emissions, knows the social cost of pollution, but does not know the relevant characteristics of the firms. We assume complete information among the firms.[1]

In much of the existing pollution-control literature, it is assumed that the regulator knows the efficient level of total pollution, or at least an appropriate "target" level, and the problem is only to allocate that given quantity of pollution permits among firms. As discussed by John Dales (1968), a competitive market for pollution permits is well suited for this task: in equilibrium, firms equate the marginal benefit of pollution to a common price and, therefore, marginal benefits are equated across firms. If the impact of pollution is independent of its source, an efficient allocation of pollution permits is achieved. Evan Kwerel (1977) shows how a combination of licensing and effluent charges can induce firms to reveal their technological characteristics truthfully, allowing the regulator to determine the efficient level of total pollution, but his result relies on price-taking behavior in the market for permits, an untenable assumption if, as Robert W. Hahn (1984) supposes, some firms have market power. Tracy R. Lewis and David E. M. Sappington (1995) generalize the simplest problem by allowing firms to have incomplete information but still assume the regulator knows the socially optimal level of pollution.[2]

Our mechanism endogenously produces the efficient level of total pollution and allocates this total efficiently, while recognizing the strategic incentives of firms to exercise market power (i.e., to influence the prices they pay for pollution emissions). To address this problem, the mechanism replicates the most important feature of competitive markets: each firm purchases a quantity of pollution at a price that is independent of the firm's actions. In contrast to the above work, we allow the social impact of pollution to depend on the firm that produces it,[3] so efficiency cannot always be achieved with a uniform price. Nonetheless, by designing incentives for the firms to monitor each other, each firm's price is set appropriately and the allocation of pollution is determined efficiently in the unique pure-strategy Nash equilibrium of the mechanism.

The mechanism not only produces the efficient allocation of pollution as an equilibrium outcome, but (since the equilibrium is unique) it ensures that no other allocations can arise as a

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[1] See the working paper version of this paper, Duggan and Roberts (2001a), for consideration of the incomplete-information case.

[2] A. P. Xepapadeas (1991), Alexander S. Kritikos (1993), and Joseph A. Herriges et al. (1994) consider the problem of abatement monitoring in incomplete-information environments with budget balancing. They also assume that the regulator knows the optimal pollution level.

[3] The assumption of "anonymous" pollution is unrealistic; for example, if firms are geographically distinct and pollution is localized, the social cost of a medium amount of pollution, spread very thinly, may be insignificant; when concentrated at just one locality, however, it may be quite costly.

result of equilibrium behavior. Thus, we implement the efficient allocation in Nash equilibrium using a mechanism that is especially simple compared to those in the implementation literature.[4] Problematic constructions, such as integer games, modulo games, and other forms of "unwinnable competition," are not used. Firms simply select quantities of pollution (used to set prices) and are charged accordingly. The outcome function of the mechanism is continuous, and it is therefore robust to small mistakes in the strategic choices of firms. As long as the regulator can place an upper bound on the efficient level of total pollution (as we assume), the strategies of firms can also be restricted to compact sets. Furthermore, the equilibrium uniqueness result holds even if firms are allowed to use mixed strategies. Duggan and Roberts (2001b) show that, under these conditions, the equilibrium outcomes of the mechanism are robust to "small" departures from the complete information assumption. We show that the mechanism can be adapted to produce a balanced budget, both in and out of equilibrium, and to address voluntary participation constraints and the possibility of collusion. Moreover, we can extend the mechanism to allow for negative externalities across firms as the result of pollution emissions.

Though our analysis takes place in the context of firms and pollution emissions, it applies equally well to the general problem of implementing social welfare optima in quasilinear environments. Thus, our mechanism is related to the "Nash-efficient" mechanisms surveyed by Theodore Groves (1979), and our extension to the case of negative externalities is reminiscent of Leonid Hurwicz's (1979) and Mark Walker's (1981) mechanisms for implementing Lindahl equilibria in public-good economies. Moore and Repullo (1988) and Hal R. Varian (1994) propose simple multistage mechanisms and, in contrast to other work cited here, use the refinement of subgame-perfect equilibrium to implement efficient outcomes.[5] While our approach is distinguished by the specific way in which announcements determine prices, our approach also differs from that taken in these papers by explicitly allowing the presence of social externalities (here, the cost of pollution) due to the agents' actions. Partha Dasgupta et al. (1980) consider the problem of efficient pollution control, allowing, as we do, for differential impact of pollution. Their mechanism, a simple adaptation of the mechanism of William Vickrey (1961), Edward H. Clarke (1971), and Groves (1973), has the advantage that firms have dominant strategies leading to efficient pollution. It is well known, however, that the Groves-Clarke-Vickrey (GCV) mechanisms are not generally budget-balanced, and the Dasgupta-Hammond-Maskin mechanism inherits that flaw. The Dasgupta-Hammond-Maskin mechanism also inherits the possibility of inefficient Nash equilibrium outcomes, which our mechanism addresses explicitly.

In Section I, we describe the model. In Section II, we present the mechanism and show that its unique Nash equilibrium yields the socially optimal allocation of pollution. In Section III, we discuss possible extensions of our model, some mentioned above. For another example, while we focus on the problem of negative social externalities in this paper, our mechanism works equally well in the "dual" problem of positive social externalities, where firms produce a social *good* as a by-product of their actions.

We close this section by mentioning two issues that, although beyond the scope of this work, merit future consideration. First, while we do not assume that the regulator knows the cost and revenue characteristics of firms, we do assume the regulator observes the pollution outputs of each firm. But in many situations, only the aggregate level of pollution may be observed and may not be easily attributed to the firms separately. Thus, we have focused on one important type of asymmetric information in regulatory problems (adverse selection) while abstracting away from another (moral hazard). Second, pollution control is a *dynamic* problem, with firms making output/pollution decisions repeatedly over time, and the analysis of

[4] See Eric Maskin (1977) and John Moore and Rafael Repullo (1990) for general analyses of Nash implementation.

[5] Johan Eyckmans (1997) adapts Varian's mechanism to implement a proportional solution to a complete-information pollution abatement problem. This solution requires that individuals bear abatement costs in proportion to their willingness to pay for abatement.

efficient pollution control should eventually incorporate the dynamic aspect of the problem explicitly.

I. The Model

We consider $n \geq 2$ firms, indexed by i. Denote i's level of pollution (or equivalently, i's quantity of pollution permits) by Q_i. The monetary benefit that i receives from producing Q_i units of pollution is denoted $B_i(Q_i)$, and $C(Q_1, \ldots, Q_n)$ is the social cost, measured in monetary terms, imposed on society by the firms' pollution. We assume that each $B_i(\cdot)$ is concave and differentiable, that $C(\cdot)$ is continuously differentiable,[6] and that $[\sum_{i=1}^{n} B_i(Q_i)] - C(Q_1, \ldots, Q_n)$ is strictly concave. The benefit and cost functions are common knowledge among the firms, while the regulator is assumed to know only the form of the cost function.

The regulator's problem is to implement the socially optimal allocation of pollution, that is, the solution to

$$(1) \qquad \max_{Q_1, \ldots, Q_n} \left[\sum_{i=1}^{n} B_i(Q_i) \right] - C(Q_1, \ldots, Q_n)$$

subject to

$$Q_1 \geq 0, \ldots, Q_n \geq 0.$$

We assume this problem has a solution, which, by strict concavity, must be unique. Denote it $(Q_1^*, \ldots, Q_n^*)$. We impose the appropriate Inada-type conditions on $C(\cdot)$ and each $B_i(\cdot)$ to ensure an interior solution,[7] so that the social optimum is characterized by the condition that each firm's marginal benefit equals the marginal social cost of pollution:

$$(2) \qquad \frac{dB_i}{dQ_i}(Q_i^*) = \frac{\partial C}{\partial Q_i}(Q_1^*, \ldots, Q_n^*)$$

for all i. Lastly, we assume the regulator is given some bound K such that $Q_i^* < K$ for all i.

Because social costs may depend differentially on the emissions of different firms, the efficient allocation cannot generally be obtained by fixing a price common to all firms. Optimality could be achieved if the regulator were to set price

$$\frac{\partial C}{\partial Q_i}(Q_1^*, \ldots, Q_n^*)$$

for each firm i, but this requires knowledge of the social optimum itself, and this in turn requires a familiarity with the firms' benefit functions that is unlikely to be found in practice. In the next section we construct a simple mechanism that implements the efficient allocation of pollution, without presuming such familiarity on the part of the regulator.

II. The Mechanism

The mechanism is defined as follows. Firm i purchases a quantity $\hat{Q}_i \in [0, K]$ for itself and reports a quantity $\bar{Q}_{i-1} \in [0, K]$ for its "neighbor," firm $i - 1$, where we treat n as firm 1's neighbor. As a function of these reports, firm i pays

$$(3) \qquad \hat{Q}_i \frac{\partial C}{\partial Q_i}(\hat{Q}_1, \ldots, \hat{Q}_{i-1}, \bar{Q}_i, \hat{Q}_{i+1}, \ldots, \hat{Q}_n) + |\bar{Q}_{i-1} - \hat{Q}_{i-1}|.$$

Thus, firm i faces the price

$$\frac{\partial C}{\partial Q_i}(\hat{Q}_1, \ldots, \hat{Q}_{i-1}, \bar{Q}_i, \hat{Q}_{i+1}, \ldots, \hat{Q}_n)$$

which is independent of its own reports. The second term in the firm's payment is a penalty for misrepresenting the demand of its neighbor.

PROPOSITION 1: *The unique pure strategy Nash equilibrium of the above mechanism is given by* $(\bar{Q}_{i-1}, \hat{Q}_i) = (Q_{i-1}^*, Q_i^*)$ *for all* i,

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[6] Continuity of the derivative is used only to ensure that the mechanism defined in Section II is continuous.

[7] For example,

$$\lim_{Q_i \to 0} \left[\frac{dB_i}{dQ_i}(Q_i) - \frac{\partial C}{\partial Q_i}(Q_1, \ldots, Q_n) \right] = \infty$$

for all $Q_1, \ldots, Q_n$.

and this yields the socially optimal allocation $(Q_1^*, \ldots, Q_n^*)$ *of pollution.*

It is straightforward to verify, using concavity of each $B_i(\cdot)$, that the above specification of strategies is indeed a Nash equilibrium. To verify that it is unique, consider an arbitrary pure-strategy equilibrium, and note that, because firm i cannot affect its own price, $\hat{Q}_i$ solves

$$\max_{Q_i \in [0,K]} B_i(Q_i) - Q_i \frac{\partial C}{\partial Q_i}(\hat{Q}_1, \ldots, \hat{Q}_{i-1}, \bar{Q}_i, \hat{Q}_{i+1}, \ldots, \hat{Q}_n). \tag{4}$$

By our assumptions, we have $\hat{Q}_i > 0$ and

$$\frac{dB_i}{dQ_i}(\hat{Q}_i) \geq \frac{\partial C}{\partial Q_i}(\hat{Q}_1, \ldots, \hat{Q}_{i-1}, \bar{Q}_i, \hat{Q}_{i+1}, \ldots, \hat{Q}_n) \tag{5}$$

are satisfied. Assume for now that the first-order condition holds with equality for each i.[8] Also note that $(\bar{Q}_i, \hat{Q}_{i+1})$ is a best response for firm $i + 1$ only if $\hat{Q}_i = \bar{Q}_i$, so we have

$$\frac{dB_i}{dQ_i}(\hat{Q}_i) = \frac{\partial C}{\partial Q_i}(\hat{Q}_1, \ldots, \hat{Q}_{i-1}, \hat{Q}_i, \hat{Q}_{i+1}, \ldots, \hat{Q}_n) \tag{6}$$

for all i, which is satisfied only at the social optimum.

Now suppose that the first-order condition of some firm i is not met with equality. Writing

$$W(Q_1, \ldots, Q_n) = \left[\sum_{i=1}^{n} B_i(Q_i)\right] - C(Q_1, \ldots, Q_n) \tag{7}$$

we see that this is equivalent to $(\partial W/\partial Q_i)(\hat{Q}_1, \ldots, \hat{Q}_n) \neq 0$ and implies that $(\partial W/\partial Q_i)(\hat{Q}_1, \ldots, \hat{Q}_n) > 0$ and $\hat{Q}_i = K > Q_i^*$. Thus, $\nabla W(\hat{Q}_1, \ldots, \hat{Q}_n) > 0$ and

$$0 > \nabla W(\hat{Q}_1, \ldots, \hat{Q}_n) \cdot [(Q_1^*, \ldots, Q_n^*) - (\hat{Q}_1, \ldots, \hat{Q}_n)] \tag{8}$$

$$\geq W(Q_1^*, \ldots, Q_n^*) - W(\hat{Q}_1, \ldots, \hat{Q}_n) \tag{9}$$

where the weak inequality follows from concavity of $W(\cdot)$ and Rangarajan K. Sundaram's (1996) theorem 7.9. But then $W(\hat{Q}_1, \ldots, \hat{Q}_n) > W(Q_1^*, \ldots, Q_n^*)$, a contradiction.

III. Extensions

The result of the previous section can be extended in several interesting ways.

A. *Mixed Strategies*

Proposition 1 restricts firms to pure strategies, but the result can be extended if we assume each $B_i(\cdot)$ is *strictly* concave. In this case, consider an arbitrary mixed-strategy equilibrium. Then $\partial C/\partial Q_i$ is a random variable, and firm i's strategy can put positive probability only on solutions to

$$\max_{Q_i \in [0,K]} B_i(Q_i) - Q_i E\left[\frac{\partial C}{\partial Q_i}(\hat{Q}_1, \ldots, \hat{Q}_{i-1}, \bar{Q}_i, \hat{Q}_{i+1}, \ldots, \hat{Q}_n)\right]. \tag{10}$$

Since $B_i(\cdot)$ is strictly concave, this problem has a unique solution, say $\hat{Q}_i$. Thus, i's mixed strategy is to play $\hat{Q}_i$ with probability 1. A similar observation holds true for the other firms, so our original argument applies.

B. *Budget Balancing*

When $n \geq 3$, the mechanism is easily modified to achieve budget balance in equilibrium:

[8] If the firms were not restricted to the compact set $[0, K]$, equality would obviously obtain. Dropping that restriction would simplify the proof but would result in a less "well-behaved" mechanism.

simply subtract from firm i's payment to the regulator the amount

$$(11)\quad \hat{Q}_{i+1} \frac{\partial C}{\partial Q_{i+1}} (\hat{Q}_1, \ldots, \hat{Q}_{i-1}, \bar{Q}_i, \bar{Q}_{i+1}, \hat{Q}_{i+2}, \ldots, \hat{Q}_n)$$

where we use $\bar{Q}_i$ here instead of $\hat{Q}_i$ and we drop the term $|\bar{Q}_i - \hat{Q}_i|$, so firm i cannot affect this adjustment. Since $\bar{Q}_i = \hat{Q}_i$ and $\bar{Q}_{i+1} = \hat{Q}_{i+1}$ in equilibrium, this amount equals firm $i + 1$'s equilibrium payment, yielding a balanced budget.

To balance the budget out of equilibrium as well, we modify the original mechanism somewhat. In addition to purchasing quantity $\hat{Q}_i$, firm i reports quantities $\bar{Q}_{i-1}$ and $\bar{\bar{Q}}_{i-2}$ for two neighbors. As a function of these reports, firm i pays the "base amount":

$$(12)\quad \hat{Q}_i \frac{\partial C}{\partial Q_i} (\hat{Q}_1, \ldots, \hat{Q}_{i-2}, \bar{\bar{Q}}_{i-1}, \bar{Q}_i, \hat{Q}_{i+1}, \ldots, \hat{Q}_n) + |\bar{Q}_{i-1} - \hat{Q}_{i-1}| + |\bar{\bar{Q}}_{i-2} - \hat{Q}_{i-2}|.$$

As with the original mechanism, firm i's reports of $\bar{Q}_{i-1}$ and $\bar{\bar{Q}}_{i-2}$ will match $\hat{Q}_{i-1}$ and $\hat{Q}_{i-2}$ in equilibrium. The difference that allows us to fully balance the budget is that now the first part of the base payment is independent of firm $i - 1$'s reports. Budget balance is achieved by subtracting from firm i's payment the amount

$$(13)\quad \hat{Q}_{i+1} \frac{\partial C}{\partial Q_{i+1}} (\hat{Q}_1, \ldots, \hat{Q}_{i-2}, \bar{\bar{Q}}_i, \bar{Q}_{i+1}, \hat{Q}_{i+1}, \ldots, \hat{Q}_n) + |\bar{Q}_{i+1} - \hat{Q}_{i+1}| + |\bar{\bar{Q}}_{i-1} - \hat{Q}_{i-1}|$$

which is independent of firm i's reports. The first term above is exactly the first term in firm $i + 1$'s base payment, whereas the second and third terms above are the second term in firm $i + 2$'s base payment and the third term in firm $i + 1$'s base payment.

C. *Voluntary Participation*

In an established market, the political viability of a regulatory mechanisms may rely on acceptance by firms in the market, which means the firms must be able to expect post-regulation profits at least equal to their pre-regulation profits. Letting $\tilde{Q}_i$ denote firm i's pollution level prior to regulation, the mechanism must give firm i at least $B_i(\tilde{Q}_i)$ in equilibrium. We can modify the mechanism to give firm i a rebate in the amount

$$(14)\quad \tilde{Q}_i \frac{\partial C}{\partial Q_i} (\hat{Q}_1, \ldots, \hat{Q}_{i-1}, \bar{Q}_i, \hat{Q}_{i+1}, \ldots, \hat{Q}_n).$$

This essentially charges firm i the price

$$\frac{\partial C}{\partial Q_i} (\hat{Q}_1, \ldots, \hat{Q}_{i-1}, \bar{Q}_i, \hat{Q}_{i+1}, \ldots, \hat{Q}_n)$$

for each unit of pollution $\hat{Q}_i - \tilde{Q}_i$ in excess of its emissions benchmark. The firm can clearly obtain $B_i(\tilde{Q}_i)$ by setting $\hat{Q}_i = \tilde{Q}_i$, regardless of the pollution levels of the other firms. It follows that, in equilibrium, the firm's "voluntary participation constraint" is satisfied.

D. *Collusion*

We have employed the concept of Nash equilibrium in the analysis of our mechanism, but we can also account for the possibility of collusive behavior. Ideally, we could modify the mechanism to implement the efficient allocation of pollution not only in Nash equilibrium but also in strong Nash equilibrium, which captures the incentives of coalitions to engage in cooperative behavior.[9] Since strong Nash equilibria (or any equilibria that allow for coalitional deviations) form a subset of the Nash equilibria, the uniqueness argument in the proof of Proposition 1 applies. The problem left is to show that the efficient strategies (Q^*_{i-1}, Q^*_i) are immune to coalitional deviations: though individual firms cannot influence their own prices, a group of firms may be able to influence each other's prices, perhaps lowering them and increasing their

[9] See Maskin (1979) for a general analysis of implementation in strong Nash equilibrium.

profits. Because the issue becomes complex quickly, we will limit our analysis to joint deviations by *pairs* of firms. Because communication and coordination difficulties would be minimal for pairwise deviations, this is an especially interesting case to consider.

Specifically, assuming $n \geq 3$, we will modify the mechanism so that no two firms can jointly deviate from their Nash equilibrium strategies in a way that increases the profits of both firms. In addition to purchasing a quantity $\hat{Q}_i$, we have firm i report $\bar{Q}_{i-1}$ and $\bar{\bar{Q}}_{i-2}$ for two neighbors. Let

$$(15)\qquad \Delta_i = \max_{j \neq i} |\bar{Q}_j - \hat{Q}_j| + |\bar{\bar{Q}}_j - \hat{Q}_j|$$

measure the greatest disparity between the pollution of any firm other than i and the reports of its two neighbors. Having each firm monitored by two others will make it impossible for two firms to deviate from the efficient strategy profile without one of them being detected. Assuming C is increasing convex and twice continuously differentiable, set

$$(16)\qquad \alpha_i = \max \left| K \frac{\partial^2 C}{\partial Q_i \partial Q_j} (Q_1, \ldots, Q_n) \right| + \max \frac{\partial C}{\partial Q_i} (Q_1, \ldots, Q_n)$$

where the first maximum is over $j \neq i$ and $Q_1, \ldots, Q_n$ and the second over $Q_1, \ldots, Q_n$. We add $\alpha_i \Delta_i$ to firm i's required payment in the mechanism of Section III, making it difficult for two firms to profitably collude. There is yet the complication that firms i and $i + 1$ may collude, where $i + 1$ sets $\bar{Q}_i$ low to depress the price faced by firm i. In doing so, however, $i + 1$'s reported quantity $\bar{Q}_i$ will necessarily differ from $i + 2$'s report $\bar{\bar{Q}}_i$, a discrepancy that can be used to punish firm i. Define

$$(17)\qquad \beta_i = \max K \frac{\partial^2 C}{\partial Q_i^2} (Q_1, \ldots, Q_n)$$

where the maximum is taken over $Q_1, \ldots, Q_n$. We then also add $\beta_i |\bar{Q}_i - \bar{\bar{Q}}_i|$ to firm i's payment. Since no firm can influence the extra terms in its payment, the argument in the proof of Proposition 1 shows that the unique Nash equilibrium of the modified mechanism is $(\bar{\bar{Q}}_{i-2}, \bar{Q}_{i-1}, \hat{Q}_i) = (Q^*_{i-2}, Q^*_{i-1}, Q^*_i)$ for each firm i.

Consider two firms j and k that deviate to $\hat{Q}_j = Q'_j \neq Q^*_j$ and $\hat{Q}_k = Q'_k \neq Q^*_k$. We claim that, assuming B is increasing, the profits of at least one firm decrease or stay the same. Since there are at least three firms, either $j + 1 \neq k$ or $j + 2 \neq k$ must hold, so $\Delta_k \geq |Q'_j - Q^*_j| > 0$. Suppressing $Q_i = Q^*_i$ for all $i \neq j, k$ in the following, the increase in firm k's profits is

$$\begin{aligned}(18)\quad & B_k(Q'_k) - Q'_k \frac{\partial C}{\partial Q_k}(Q'_j, \bar{Q}_k) \\ &\quad - \alpha_k \Delta_k - \beta_k |\bar{Q}_k - \bar{\bar{Q}}_k| \\ &\quad - B_k(Q^*_k) + Q^*_k \frac{\partial C}{\partial Q_k}(Q^*_j, Q^*_k) \\ &= B_k(Q'_k) - B_k(Q^*_k) \\ &\quad + Q^*_k \left[\frac{\partial C}{\partial Q_k}(Q^*_j, Q^*_k) - \frac{\partial C}{\partial Q_k}(Q'_j, Q^*_k) \right] \\ &\quad + \frac{\partial C}{\partial Q_k}(Q'_j, Q^*_k)(Q^*_k - Q'_k) \\ &\quad + Q'_k \left[\frac{\partial C}{\partial Q_k}(Q'_j, Q^*_k) - \frac{\partial C}{\partial Q_k}(Q'_j, \bar{Q}_k) \right] \\ &\quad - \alpha_k \Delta_k - \beta_k |\bar{Q}_k - \bar{\bar{Q}}_k|. \end{aligned}$$

By the mean value theorem, there exists Q''_j between Q^*_j and Q'_j and there exists Q''_k between Q^*_k and $\bar{Q}_k$ such that this difference can be written as

$$\begin{aligned}(19)\quad & B_k(Q'_k) - B_k(Q^*_k) \\ &\quad + Q^*_k \frac{\partial^2 C}{\partial Q_k \partial Q_j}(Q''_j, Q^*_k)(Q^*_j - Q'_j) \\ &\quad + \frac{\partial C}{\partial Q_k}(Q'_j, Q^*_k)(Q^*_k - Q'_k) \\ &\quad + Q'_k \frac{\partial^2 C}{\partial Q_k^2}(Q'_j, Q''_k)(Q^*_k - \bar{Q}_k) \\ &\quad - \alpha_k \Delta_k - \beta_k |\bar{Q}_k - \bar{\bar{Q}}_k|. \end{aligned}$$

Note that, if $j = k + 1$, then $\bar{\bar{Q}}_k = Q_k^*$, so $|\bar{Q}_k - \bar{\bar{Q}}_k| \geq Q_k^* - \bar{Q}_k$; if $j \neq k + 1$, then $\bar{Q}_k = Q^*$, so again $|\bar{Q}_k - \bar{\bar{Q}}_k| \geq Q_k^* - \bar{Q}_k$ (since the latter is zero). Then,

$$(20) \quad \alpha_k \Delta_k \geq K \max \left| \frac{\partial^2 C}{\partial Q_k \partial Q_j} \right| |Q_j' - Q_j^*| + \max \frac{\partial C}{\partial Q_k} |Q_j' - Q_j^*|$$

$$(21) \quad \geq Q_k^* \frac{\partial^2 C}{\partial Q_k \partial Q_j} (Q_j'', Q_k^*)(Q_j^* - Q_j') + \frac{\partial C}{\partial Q_k} (Q_j', Q_k^*)(Q_k^* - Q_k')$$

and

$$(22) \quad \beta_k |\bar{Q}_k - \bar{\bar{Q}}_k| \geq Q_k' \frac{\partial^2 C}{\partial Q_k^2} (Q_j', Q_k'')(Q_k^* - \bar{Q}_k).$$

If firm k's profits do not decrease as a result of the joint deviation, it follows that $B_k(Q_k') \geq B_k(Q_k^*)$, implying $Q_k' \geq Q_k^*$. A similar argument establishes that $Q_j' \geq Q_j^*$ (i.e., the pollution emissions of both firms increase). Assume without loss of generality, since either $j \neq k + 1$ or $k \neq j + 1$, that the former holds. Thus, $\bar{Q}_k = Q_k^*$. Then, $Q_j' \geq Q_j^*$ and convexity of C imply

$$(23) \quad \frac{\partial C}{\partial Q_k} (Q_1^*, \ldots, Q_{j-1}^*, Q_j', Q_{j+1}^*, \ldots, Q_n) \geq \frac{\partial C}{\partial Q_k} (Q_1^*, \ldots, Q_n^*).$$

That is, the price of pollution for firm k weakly increases as a result of the joint deviation. Since Q_k^* is chosen optimally given the lower price, the profits of k cannot be higher given the higher price. (In fact, profits will be strictly lower.) Thus, no two firms can jointly deviate from the unique Nash equilibrium in a way that increases the profits of both firms.

E. *Negative Externalities Across Firms*

We have assumed each firm's benefit from polluting is independent of the levels of pollution of other firms. A more general model would allow for externalities: firms may experience either market externalities (as when high levels of pollution by other firms may reflect high levels of production and a competitive output market) or production externalities as pollution levels rise. We now allow for externalities among the firms, using $B_i(Q_1, \ldots, Q_n)$ to denote the benefit of firm i corresponding to pollution quantities $Q_1, \ldots, Q_n$. We assume that externalities are negative (i.e., $\partial B_i / \partial Q_j < 0$ for $i \neq j$), and we impose the appropriate Inada-type conditions to guarantee interior solutions. The regulator's problem is defined as before, with unique solution $(Q_1^*, \ldots, Q_n^*)$ given by the first-order conditions

$$(24) \quad \sum_{i=1}^{n} \frac{\partial B_i}{\partial Q_j} (Q_1^*, \ldots, Q_n^*) = \frac{\partial C}{\partial Q_j} (Q_1^*, \ldots, Q_n^*)$$

for $j = 1, \ldots, n$.

We modify the mechanism as follows. We have each firm i purchase a vector $\hat{\mathbf{Q}}^i = (\hat{Q}_1^i, \ldots, \hat{Q}_n^i)$ of pollution quantities, one quantity for each firm,[10] where $\hat{Q}_j^i$ is interpreted as an amount added to firm j's pollution by firm i. Note that Q_i^i is the amount of firm i's pollution purchased by itself. If $\hat{Q}_j^i < 0$, which we allow, then firm j's outputs are decreased by $-\hat{Q}_j^i$. Along with these purchases, firm i reports a vector $\bar{\mathbf{Q}}^{i-1} = (\bar{Q}_1^{i-1}, \ldots, \bar{Q}_n^{i-1})$, where $\bar{Q}_j^{i-1}$ represents the increment to firm j's pollution purchased by firm $i - 1$. Firm i is then allocated the total amount of pollution purchased for it,

$$(25) \quad q_i = \max\left\{0, \sum_{j=1}^{n} \hat{Q}_i^j\right\}$$

[10] If externalities among firms are limited, we can simplify the mechanism by having firm i only purchase pollution quantities for firms imposing externalities on i.

and is charged for its own purchases according to a vector of prices. Once again, it is important that firm i not be able to influence its prices, so for each k we let

$$\bar{q}_k = \max\left\{0,\ \bar{Q}_k^i + \sum_{j \neq i} \hat{Q}_k^j\right\} \tag{26}$$

which is the total amount of firm k's pollution with firm i's increment replaced by $\bar{Q}_k^i$, reported by firm $i + 1$. Firm i then faces price

$$2\,\frac{\partial C}{\partial Q_i}\,(\bar{q}_1, \ldots, \bar{q}_n) \tag{27}$$

for its own pollution output, and faces prices

$$-\left(\frac{1}{n-1}\right)\frac{\partial C}{\partial Q_j}\,(\bar{q}_1, \ldots, \bar{q}_n) \tag{28}$$

$j = 1, \ldots, n$, for other firms' pollution outputs. Note that the latter prices are negative, so firm i pays for the reduction of other firms' pollution outputs and is compensated for increases in their pollution levels. In addition, i pays the penalty $\|\bar{\mathbf{Q}}^{i-1} - \hat{\mathbf{Q}}^{i-1}\|$ for misrepresenting its neighbor's pollution purchases.

The argument that this mechanism implements the socially optimal allocation of pollution is similar to our earlier one. Ignoring corner solutions, every equilibrium pollution allocation must satisfy

$$\frac{\partial B_i}{\partial Q_i}\,(q_1, \ldots, q_n) = 2\,\frac{\partial C}{\partial Q_i}\,(\bar{q}_1, \ldots, \bar{q}_n) \tag{29}$$

$$\frac{\partial B_i}{\partial Q_j}\,(q_1, \ldots, q_n) = -\left(\frac{1}{n-1}\right)\frac{\partial C}{\partial Q_j}\,(\bar{q}_1, \ldots, \bar{q}_n) \tag{30}$$

for all i and all $j \neq i$. Because of the strict incentive for each firm i to report its neighbor's purchases accurately, we also have $(q_1, \ldots, \bar{q}_n) = (\bar{q}_1, \ldots, \bar{q}_n)$ in equilibrium. Making this substitution and summing up the firm's first-order conditions with respect to each Q_j, we see that the first-order conditions for the socially optimal allocation are met.

F. *Positive Social Externalities*

We have analyzed the problem of negative social externalities, of which pollution is a special case, but our mechanism also implements solutions to problems of *positive* social externalities. We now interpret Q_i as an activity of firm i that costs $C_i(Q_i)$, where $C_i(\cdot)$ is differentiable and convex, and the social benefit of activity is $B(Q_1, \ldots, Q_n)$, where $B(\cdot)$ is also differentiable. We now assume that $B(Q_1, \ldots, Q_n) - \sum_{i=1}^n C_i(Q_i)$ is strictly concave and that the social optimization problem has an interior solution. The mechanism is unchanged, except that payments from firms become payments to firms. For an example of positive social externalities, suppose that Q_i measures the attractiveness of storefronts in a downtown area, $C_i(Q_i)$ is the cost to firm i of maintaining a storefront of quality Q_i, and $B(Q_1, \ldots, Q_n)$ is the corresponding social benefit. Of course, the agents under consideration need not be firms. They may be workers in a factory, where Q_i denotes i's contribution of effort, $C_i(Q_i)$ a cost of effort, and $B(Q_1, \ldots, Q_n)$ the monetary worth of output, as a function of the vector of efforts expended by workers.

REFERENCES

Clarke, Edward H. "Multipart Pricing of Public Goods." *Public Choice*, Fall 1971, *11*, pp. 17–33.

Dales, John. *Pollution, property and prices: An essay in policy-making and economics*. Toronto: University of Toronto Press, 1968.

Dasgupta, Partha; Hammond, Peter and Maskin, Eric. "On Imperfect Information and Optimal Pollution Control." *Review of Economic Studies*, October 1980, *47*(5), pp. 857–60.

Duggan, John and Roberts, Joanne. "Implementing the Efficient Allocation of Pollution." Wallis Institute of Political Economy Working paper, University of Rochester, 2001a.

______. "Robust Implementation." Working paper, University of Rochester, 2001b.

Eyckmans, Johan. "Nash Implementation of a Proportional Solution to International Pollution Control Problems." *Journal of Environmental Economics and Management*, July 1997, *33*(3), pp. 314–30.

Groves, Theodore. "Incentives in Teams." *Econometrica*, July 1973, *41*(4), pp. 617–31.

______. "Efficient Collective Choice when Compensation Is Possible." *Review of Economic Studies*, April 1979, *46*(2), pp. 227–41.

Hahn, Robert W. "Market Power and Transferable Property Rights." *Quarterly Journal of Economics*, November 1984, *99*(4), pp. 753–65.

Herriges, Joseph A.; Govindasamy, Ramu and Shogren, Jason F. "Budget-Balancing Incentive Mechanisms." *Journal of Environmental Economics and Management*, November 1994, *27*(3), pp. 275–85.

Hurwicz, L. "Outcome Functions Yielding Walrasian and Lindahl Allocations at Nash Equilibrium Points." *Review of Economic Studies*, April 1979, *46*(2), pp. 217–25.

Kritikos, Alexander S. "Environmental Policy under Imperfect Information: Comment." *Journal of Environmental Economics and Management*, July 1993, *25*(1), Part 1, pp. 89–92.

Kwerel, Evan. "To Tell the Truth: Imperfect Information and Optimal Pollution Control." *Review of Economic Studies*, October 1977, *44*(3), pp. 595–601.

Lewis, Tracy R. and Sappington, David E. M. "Using Markets to Allocate Pollution Permits and Other Scarce Resource Rights under Limited Information." *Journal of Public Economics*, July 1995, *57*(3), pp. 431–55.

Maskin, Eric. "Nash Implementation and Welfare Optimality." Mimeo, Harvard University, 1977.

______. "Implementation and Strong Nash Equilibrium," in Jean-Jacques Laffont, ed., *Aggregation and revelation of preferences*. Amsterdam: North-Holland, 1979, pp. 433–40.

Moore, John and Repullo, Rafael. "Subgame Perfect Implementation." *Econometrica*, September 1988, *56*(5), pp. 1191–220.

______. "Nash Implementation: A Full Characterization." *Econometrica*, September 1990, *58*(5), pp. 1083–99.

Sundaram, Rangarajan K. *A first course in optimization theory*. Cambridge: Cambridge University Press, 1996.

Varian, Hal R. "A Solution to the Problem of Externalities When Agents are Well-Informed." *American Economic Review*, December 1994, *84*(5), pp. 1278–93.

Vickrey, William. "Counterspeculation, Auctions, and Competitive Sealed Tenders." *Journal of Finance*, March 1961, *16*(1), pp. 8–37.

Walker, Mark. "A Simple Incentive Compatible Scheme for Attaining Lindahl Allocations." *Econometrica*, January 1981, *49*(1), pp. 65–71.

Xepapadeas, A. P. "Environmental Policy under Imperfect Information: Incentives and Moral Hazard." *Journal of Environmental Economics and Management*, March 1991, *20*(2), pp. 113–26.