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Further Results on Permit Markets with Market Power and Cheating

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This paper examines a market for pollution permits in which one firm has market power and one or more firms is noncompliant. I show that the firm with market power may choose to hold more permits than it needs, effectively retiring permits from the market. I also show that some noncompliance may be socially desirable because it can mitigate the distortion caused by market power. Similarly, some degree of market power may be socially desirable because it can, in turn, mitigate the distortion caused by noncompliance.

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1. Introduction

The properties of a system of marketable pollution permits are well known in the static "text-book" setting in which firms are price takers, the transactions costs associated with trading are small, and firms comply voluntarily with their permit limits. But this simplified setting ignores conditions that are likely to exist in actual permit markets. For example, firms may be able to influence permit prices, transactions costs may be high, and firms may choose to be noncompliant. A number of authors have examined the implications of these, and other, complications. The effects of market power are explored by Hahn (1984), Tietenberg (1985), Misolek and Elder (1989), van Egteren and Weber (1996), and Westskog (1996), among others. Tschirhart (1984) and more recently, Stavins (1995), formally examine the effects of transactions costs. Rubin (1996), Cornshaw and Kruse (1996), and Kling and Rubin (1997) evaluate the possibility of intertemporal trading of permits. Malik (1990, 1992), Keeler (1991), and van Egteren and Weber (1996) consider the implications of noncompliance.

This paper extends the work of both Hahn (1984) and van Egteren and Weber (1996) on the effects of market power and noncompliance in permit markets. It also qualifies a result obtained by van Egteren and Weber on the optimal initial endowment of permits.

Hahn examines the effects of market power in a setting where one firm has market power and all firms are compliant. He shows that in this setting the initial permit endowment of the firm with market power influences the equilibrium allocation of permits. He also shows that the equilibrium allocation generally does not coincide with the cost-effective one.

Van Egteren and Weber adapt Hahn's model of market power to allow for noncompliance, extending the work of Malik (1990), who examines the effects of noncompliance in a competitive permit market. They show that the initial permit endowment of the firm with market power influences the pattern of noncompliance among firms in a fairly predictable manner. They, therefore, argue that the initial endowment of permits can be used by the regulator to control not only the degree of market power in a permit market, but also the degree of noncompliance.

In the first part of this paper (Section 3), I consider a setting that differs from the one examined by Hahn in that noncompliance is allowed among the price-taking firms. I show that the firm with market power may choose to hold more permits than it needs (given its emissions level), in effect retiring some permits from the permit market. This behavior, which is not considered by Hahn, is not a function of the possible noncompliance of the price-taking firms. Nor is it related to permit "banking", since the model is a static one. Instead, it is an unusual form of monopoly restriction of output.

In Section 4 of the paper, I consider the social desirability of noncompliance among the price-taking firms. I show that noncompliance can be desirable, because it can mitigate the distortions caused by market power.

In Section 5, I go on to allow the firm with market power to be noncompliant. I find that this does not affect the firm's behavior if it retires permits: the exercise of monopoly power provides enough incentive for it to be compliant and hold an adequate number of permits. This compliance incentive of monopoly power also exists when the firm does not retire permits, though to a lesser degree.

In Section 6, I examine the optimal initial endowment of permits. I qualify a result presented by van Egteren and Weber on the desirability of increasing the initial permit endowment of the firm with market power. I show that the desirability of doing so is ambiguous even in the one case where they suggest it is not. I demonstrate, instead, that given noncompliance, it is generally not optimal to choose the initial endowment so that market power is eliminated. The final section of the paper contains some concluding remarks.

2. Basic Model

The basic model is identical to van Egteren and Weber's. Their notation is used to the extent possible, but it has been extended, and in a few cases modified, primarily because of the need to distinguish outcomes with and without noncompliance.[1]

The permit market consists of N firms, of which only one firm, Firm 1, has market power. The remaining (N-1) firms are price takers. Each firm's *gross* profits when emitting e_i units of pollutant are given by a strictly concave function $B_i(e_i)$. This function does not include the firm's permit expenditures (or revenues) or any penalties for possible noncompliance. Accordingly, $B_i(e_i)$ captures the social benefits generated by firm i's emissions. $B_i(e_i)$ is at first increasing in e_i, reaching a maximum at e_i^m, and then decreasing, as it becomes necessary for the firm to devote resources to generating pollution. Thus: $0 < B_i'(e_i) < \infty$ for $e_i < e_i^m$, $B_i'(e_i^m) = 0$, and $B_i'(e_i) < 0$ for $e_i > e_i^m$.[2] The firm would presumably never emit more than e_i^m since this would reduce its profits. We can, therefore, refer to e_i^m as firm i's maximum emissions level.

A total of $\bar{L}$ permits are issued by the regulator and allocated among the N firms, with l_i^0 denoting firm i's initial endowment of permits. The number of permits held by a firm after trading, l_i, may be larger or smaller than l_i^0, depending on whether the firm is a net buyer $(l_i^0 < l_i)$ or net seller $(l_i^0 > l_i)$ of permits at the prevailing permit price P. I use l_i to denote both a firm's permit holdings and its emissions when the two are equal.

If a firm is noncompliant, its emissions exceed its permit holdings, and $v_i = e_i - l_i > 0$. The expected penalty for noncompliance is given by a function $S_i(v_i)$, that is strictly increasing and strictly

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[1]The primary difference in notation is that a superscript c (indicating compliance) or a superscript n (indicating noncompliance) is used to denote optimal choices, rather than an asterisk.

[2]Hahn assumes that $B_i' > 0$ everywhere, which effectively implies that the firm has no finite maximum emissions level. This rules out the possibility of permit retirement.

convex when $v_i > 0$, with $S_i(v_i) = 0$ when $v_i = 0$.[3] In terms of van Egteren and Weber's notation, $S_i(v_i) \equiv \beta_i(v_i)F_i(v_i)$, where $\beta_i(v_i)$ represents the probability of the firm being audited and $F_i(v_i)$ represents the penalty for violations.

Malik (1990) shows that with this type of expected penalty function, the decision rule employed by a price-taking firm when choosing its emissions level is the same whether the firm is compliant or noncompliant. In both cases, at an interior optimum, emissions are chosen so that $B_i'(e_i) = P$. Thus, a price-taking firm's emissions choice function when noncompliant is identical to its emissions choice function when compliant. For both cases, this function will be written $e_i(P)$. In the case of a firm that is, by assumption, compliant, the emissions choice function coincides with the firm's permit demand function:[4]

$$e_i(P) \equiv l_i^c(P) \equiv B_i'^{-1}(P); \tag{1}$$

$B_i'^{-1}(\cdot)$ is simply the inverse of the marginal gross profit function $B_i'(\cdot)$. Given the strict concavity of $B_i(\cdot)$, emissions are decreasing in permit price: $de_i(P)/dP < 0$.

Noncompliance by a price-taking firm manifests itself in the firm's demand function for permits. It can be shown that[5]

$$l_i^n(P) \equiv l_i^c(P) - S_i'^{-1}(P), \tag{2}$$

where $S_i'^{-1}(\cdot) > 0$ is the inverse of the marginal expected penalty function. The strict convexity of the expected penalty function implies that $dS_i'^{-1}(P)/dP > 0$. This implies that a price-taking firm's demand for

[3]If, instead, $S_i(\cdot)$ were linear, the firm would generally be at one of two corner solution: if $S_i' > P$, it would choose $l_i = e_i$, and if $S_i' < P$, it would choose $l_i = 0$, even if $e_i > 0$.

[4]This is not true if the firm is at a corner solution when noncompliant, specifically, if it holds no permits even though its emissions are positive. I assume that the characteristics of the permit market and enforcement policy are such that all polluting firms hold some permits. Van Egteren and Weber also make this assumption.

[5]This relationship can be derived by manipulating the first-order conditions for the noncompliant firm's problem, $P = S_i'(v_i)$ and $B_i'(e_i) = S_i'(v_i)$.

permits is more elastic when the firm is noncompliant:

$$\frac{dl_i^n}{dP} \equiv \frac{dB_i'^{-1}(P)}{dP} - \frac{dS_i'^{-1}(P)}{dP} < \frac{dB_i'^{-1}(P)}{dP} \equiv \frac{dl_i^c}{dP} < 0. \tag{3}$$

Let $L^k_{-1}(P) \equiv \sum_{i=2}^{N} l_i^j(P)$ denote the aggregate gross permit demand of the (N-1) price-taking firms. When all (N-1) firms are compliant, k = j = c; when some or all of them are noncompliant, k = n and j = n for at least some i. We can use (2) and (3) to write:

$$0 < L^n_{-1}(P) < L^c_{-1}(P), \quad L^n_{-1}{}'(P) < L^c_{-1}{}'(P) < 0. \tag{4}$$

These quantity relationships can be translated into price relationships by making use of the market-clearing condition $L^k_{-1}(P) = (\bar{L} - l_1)$, where k = c or n. As van Egteren and Weber note, the market-clearing condition can be inverted to obtain an equilibrium price function $P_k(\bar{L} - l_1)$. In terms of this price function, the inequalities in (4) imply:

Result 1. *When some or all of the price-taking firms are noncompliant, their aggregate demand for permits contracts and becomes more elastic:*

$$0 < P_n(\cdot) < P_c(\cdot), \quad P_c'(\cdot) < P_n'(\cdot) < 0. \tag{5}$$

The contraction in demand, but not the increase in elasticity, is derived and discussed by Malik (1990). The increase in elasticity is fairly intuitive, since noncompliance can be viewed as a substitute for the purchase of a permit.

As noted earlier, in Section 4, I consider the desirability of allowing some noncompliance by the price-taking firms. The notion of allowing *some* noncompliance can be formalized most easily by introducing a parameter, α, in (2):

$$l_i(P;\alpha) \equiv l_i^c(P) - \alpha S_i'^{-1}(P). \tag{6}$$

If $\alpha = 0$, $l_i(P;0) \equiv l_i^c(P)$; and if $\alpha = 1$, $l_i(P;1) \equiv l_i^n(P)$. The qualitative effect of allowing some noncompliance can be determined by evaluating the sign of the appropriate comparative static derivative with respect to α at $\alpha = 0$. For example, the effect on the permit holdings of a price-taking firm of allowing some noncompliance would be given by $\partial l_i/\partial\alpha|_{\alpha=0} = -S_i'^{-1}(P) < 0$. For this case, the comparative static derivative does not depend on the magnitude of α, but in general it would.

Given (1), a change in α would not directly affect a price-taking firm's emissions level. But it would indirectly affect the firm's emissions level through its effect on the equilibrium permit price. This can be seen from the market-clearing condition, which would now be written

$$\sum_{i=2}^{N_c} l_i^c(P) + \sum_{i=N_c+1}^{N} l_i(P;\alpha) = \bar{L} - l_1, \tag{7}$$

because we need to distinguish between compliant and noncompliant, price-taking firms. Firms 2 through N_c are the compliant firms and firms N_c+1 through N are the potentially noncompliant ones. Examining this condition, it is evident that a change in α would alter the equilibrium permit price.

3. Permit Retirement

Let us begin by considering the possibility that Firm 1 retires permits. Following Hahn, I will assume, for the moment, that Firm 1 is compliant. But, unlike him, I will allow for the possibility that one or more of the price-taking firms is noncompliant.

Explicitly allowing Firm 1's permit holdings to exceed its emissions, Firm 1's problem is:[6]

$$\max_{e_1 \geq 0,\, l_1 \leq \bar{L}} \quad B_1(e_1) \;+\; P_n(\bar{L} - l_1)\cdot(l_1^0 \;-\; l_1)$$
$$s.t. \quad l_1 \;\geq\; e_1. \tag{8}$$

Letting $\mu \geq 0$ be the Lagrange multiplier associated with the constraint, the Kuhn-Tucker conditions for an interior solution to this problem ($e_1^{cn} > 0$, $l_1^{cn} < \bar{L}$) are:

$$B_1'(e_1) \;=\; \mu, \tag{9}$$

$$M_n(\bar{L} - l_1, l_1^0 - l_1) \;=\; \mu, \tag{10}$$

$$\mu\cdot(l_1 - e_1) \;=\; 0, \tag{11}$$

where

$$M_n(\bar{L} - l_1, l_1^0 - l_1) \;\equiv\; P_n(\bar{L} - l_1) \;+\; P_n'(\bar{L} - l_1)\cdot(l_1^0 - l_1) \tag{12}$$

captures Firm 1's marginal revenues from the sale of permits when it has a monopoly power, or its marginal expenditures on the acquisition of permits when it has monopsony power. Note that $M_n(\cdot) < P_n(\cdot)$ in the first case, and $M_n(\cdot) > P_n(\cdot)$ in the second. If $l_1^0 = l_1$, $M_n(\cdot) = P_n(\cdot)$ and the firm does not exercise market power.

Given the strict concavity of $B_1(\cdot)$, a sufficient condition for the solution to (9)-(11) to be unique is for marginal revenues to be declining in permit sales (reductions in l_1):

$$\frac{-\,\partial M_n}{\partial l_1} \;\equiv\; 2P_n'(\bar{L} - l_1) \;+\; P_n''(\bar{L} - l_1)\bullet(l_1^0 - l_1) \;\; < 0. \tag{13}$$

The first term on the RHS of this expression is negative. The sign of the second term depends on the curvature of the equilibrium price function and on whether Firm 1 has monopoly or monopsony power. I will assume that P_n'' is sufficiently small for (13) to hold in both cases.

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[6] A non-negativity constraint is not required for l_1 because $l_1 \geq 0$ is ensured by the included constraints $l_1 \geq e_1 \geq 0$.

If Firm 1 retires permits, $l_1 > e_1$ holds. The complementary slackness condition in (11) implies that in this case, μ must equal zero. Condition (9) then reduces to $B_1'(e_1) = 0$, which implies that the firm emits e_1^m. Note that for the firm to be able to emit e_1^m while remaining compliant, e_1^m must be smaller than $\bar{L}$. Assuming this is true, the relevant question is whether the firm would actually choose to retire permits. The following proposition answers this question; its proof is provided in Appendix A.

Proposition 1. *Firm 1 will retire permits if, and only if, its marginal revenues are negative when* $l_1 = e_1^m$*:*

$$M_n(\bar{L} - e_1^m, l_1^0 - e_1^m) \equiv P_n(\bar{L} - e_1^m) + P_n'(\bar{L} - e_1^m)\cdot(l_1^0 - e_1^m) < 0. \tag{14}$$

This condition can hold only if Firm 1's initial endowment of permits, l_1^0*, exceeds* e_1^m*.*

The possibility of Firm 1 finding it optimal to retire permits is illustrated in Figure 1. For simplicity, the result is illustrated for the case where all firms are compliant (as was assumed by Hahn). The emissions level and permit holdings of Firm 1 are measured from the right edge of the "trading box." The aggregate emissions and permit holdings of the (N-1) price-taking firms, denoted E_{-1} and L_{-1}, are measured from the left edge of the box. The width of the trading box corresponds to the total number of permits issued, $\bar{L}$.

The simplest case to consider in the figure is one where Firm 1 is initially granted all $\bar{L}$ permits, $l_1^0 = \bar{L}$. Firm 1 then becomes a monopoly supplier of permits. Its marginal cost of supplying permits is given by $B_1'(l_1)$, and its marginal revenues are related in the usual manner to the aggregate demand function:

$$M_c(\bar{L} - l_1, \bar{L} - l_1) \equiv \frac{d[P_c(\bar{L} - l_1)(\bar{L} - l_1)]}{d(\bar{L} - l_1)}, \tag{15}$$

where the subscript "c" reflects the assumed compliance of the price-taking firms. Figure 1 shows the marginal revenue curve corresponding to the linear aggregate demand curve drawn. This type of demand curve is obtained if the (N-1) price-taking firms are identical and have a quadratic gross profit function, B(·). As indicated in the figure, in this case, $P_c(\bar{L} - l_1) = B'((\bar{L} - l_1)/(N-1))$.

The equilibrium allocation of permits is given by the point labeled A^{cc} in the figure. The corresponding equilibrium permit price is P^{cc}. The number of permits retired by Firm 1 at this equilibrium is given by the distance between A^{cc} and e_1^m. This outcome is nothing other than the standard monopoly outcome at which marginal revenues are equated to marginal costs: $M_c(\bar{L} - l_1^c, \bar{L} - l_1^c) = B_1'(e_1^m) = 0$. What is unusual, is that both are equal to zero.

This permit retirement outcome can be explained quite easily. The firm's objective is to maximize profits, that is, the difference between revenues and costs. Revenues are maximized by increasing output, i.e., selling permits, until marginal revenues equal zero. Generally, this output level does not maximize profits because increasing output raises the costs incurred by the firm; in our context, selling more permits requires lowering emissions, which is costly. However, in the situation considered, the firm's initial permit endowment is such that it does not have to lower its emissions in order to sell the revenue maximizing number of permits.

As this explanation indicates, Firm 1's initial permit endowment must be generous for it to engage in permit retirement. As Proposition 1 states, $l_1^0 > e_1^m$ must hold. This condition is not as stringent as it appears. In particular, the condition does not require Firm 1's initial endowment to exceed its emissions level prior to regulation. Rather, it requires the initial endowment to exceed Firm 1's maximum emissions level after it has undertaken measures to comply with the permit program. This distinction is best conveyed with an example. Let e_1^{pre} denote the firm's maximum emissions level prior to regulation. Upon introduction of the permit market, the firm is given $l_1^0 < e_1^{pre}$ permits. Now suppose that it is cheaper for the firm to achieve compliance by adopting an inherently less polluting production technology, than by purchasing additional permits. The firm's maximum emissions level with this new technology in place is the quantity captured by e_1^m. Because of the lumpiness of technologies, e_1^m could well be smaller than l_1^0 even though $l_1^0 < e_1^{pre}$.[7]

Inspecting Figure 1, we can see that for the permit retirement condition in (14) to be satisfied, the aggregate permit demand of the price-taking firms must be small relative to the total number of permits issued ($\bar{L}$), and relative to Firm 1's maximum emissions level (e_1^m). If the aggregate demand of the price-taking firms were much higher than that depicted, $M_c(\cdot)$ would intersect $B_1'(\cdot)$, and Firm 1 would not choose to retire permits. For permit retirement to occur in this case, Firm 1's marginal profits would have to be smaller, shifting $B_1'(\cdot)$ in and reducing e_1^m. To the extent that permit retirement is simply an unusual form of monopoly restriction of output, these observations imply that this form of monopoly restriction occurs only if output demand is sufficiently low and the marginal cost of output provision by the monopolist is sufficiently small.

Interestingly, in this model, permit retirement cannot occur as a result of Firm 1 buying up permits and retiring them (which is a seemingly reasonable possibility). It can occur only if Firm 1's initial endowment is larger than its maximum emissions level once in compliance. If Firm 1 were a buyer of permits, it would attempt to exercise monopsony power instead, and restrict its permit purchases.

Monopsony behavior by Firm 1 raises the possibility of the price-taking firms retiring permits. This would occur if Firm 1 restricts its permit purchases to such an extent that, collectively, the price-taking firms are left holding excess permits. It can be shown that this would only occur in the unlikely case where the equilibrium permit price equals zero.[8] Thus, when Firm 1 exercises monopsony power, each firm's emissions are generally equal to its permit holdings.

[7]For example, a coal-fired powered can reduce its emissions of sulfur dioxide by: (1) installing a scrubber that traps the sulfur dioxide before it enters the atmosphere, or (2) modifying its generating unit (at considerable capital cost) to burn low-sulfur coal (Moore, 1995). The first option presumably would not affect the plant's maximum emissions level, but the second option would reduce it substantially.

[8]This can be verified using the Kuhn-Tucker conditions in (9)-(11).

4. Desirability of Noncompliance among Price-Taking Firms

The exercise of market power by Firm 1 imposes social costs. As Hahn points out, the aggregate costs of reducing emissions to the desired level are not minimized because marginal profits are not equated between Firm 1 and the price-taking firms. This can be seen from the first-order conditions for Firm 1's problem, (9)-(11). In general, these conditions imply that: $M_n(\cdot) = B_1'(\cdot) \neq B_i'(\cdot) = P_n(\cdot)$, i = 2,3,...,N, given that the price-taking firms set their marginal profits equal to the permit price. In the case where Firm 1 exercises monopoly power, marginal profits are lower for Firm 1: $M_n(\cdot) = B_1'(\cdot) < B_i'(\cdot) = P_n(\cdot)$. In the case where Firm 1 exercises monopsony power, the opposite is true.

As noted in Result 1, noncompliance by the price-taking firms renders their demand for permits more elastic. Therefore, when Firm 1 exercises monopoly power, noncompliance by the price-taking firms reduces its monopoly power and reduces the divergence between its marginal profits and the marginal profits of the price-taking firms. This suggests that there may be social benefits to noncompliance.[9] These benefits have to be weighed against the obvious costs of noncompliance, namely higher emissions levels and environmental damages. Obviously, if noncompliance is severe, its costs will outweigh any possible benefits. In this section, I will show that under certain conditions *some* noncompliance is, in fact, socially desirable.

A. Preliminaries

To establish this result, an assumption must be made about the manner in which the total number of permits issued, $\bar{L}$, was chosen by the regulator. Following van Egteren and Weber, I will assume that $\bar{L}$ was chosen assuming a first-best setting, i.e., it was chosen to maximize net social benefits assuming

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[9] The divergence between Firm 1's marginal profits and the marginal profits of the price-taking firms represents Firm 1's markup. It should be noted that a lower markup does not necessarily imply a lower welfare loss. In the conventional monopoly model, the welfare loss due to monopoly is not an increasing function of the markup (e.g., see Tirole, p. 67). In the setting considered here, noncompliance further attenuates the relationship between these two quantities. The potential desirability of noncompliance is established below by identifying changes in welfare.

universal compliance and price-taking behavior.[10] Thus, $\bar{L}$ is the value of $L \equiv \sum_{i=1}^{N} l_i$ that maximizes the difference between aggregate gross profits and environmental damages, $\sum_{i=1}^{N} B_i(l_i) - D(L)$, under the assumption that marginal profits are equated across firms. It can be verified that the following relationships would then hold at a competitive equilibrium:

$$P_c(\bar{L} - \hat{l}_1) = B_1'(\hat{l}_1) = D'(\bar{L}), \tag{16}$$

where $\hat{l}_1$ denotes Firm 1's first-best permit holdings and $D'(\cdot)$ captures marginal damages, which are assumed, as usual, to be positive and non-decreasing in aggregate emissions.

This first-best equilibrium is depicted in Figure 1. The equilibrium allocation of permits is given by $\hat{A}$ and the corresponding permit price by $\hat{P}$. Comparing this equilibrium to the equilibrium when Firm 1 exercises monopoly power $\{A^{cc}, P^{cc}\}$, we can see that Firm 1's permit holdings are larger at the monopoly outcome, and that the price of permits is higher. This is a special case of a more general result derived by Hahn:

Result 2 (Hahn). *When all firms are compliant and Firm 1 has monopoly power, Firm 1's permit holdings exceed the first-best level and the equilibrium permit price is higher than the first-best price:* $l_l^{cc} > \hat{l}_1$ *and* $P^{cc} > P_c(\bar{L} - \hat{l}_1)$. *The opposite is true when Firm 1 has monopsony power:* $l_l^{cc} < \hat{l}_1$ *and* $P^{cc} < P_c(\bar{L} - \hat{l}_1)$.

We can now turn to demonstrating the potential desirability of noncompliance by the price-taking firms.

In the presence of noncompliance, net social benefits are captured *in part* by the difference between aggregate (gross) profits and environmental damages:

$$PNSB \equiv B_1(e_1^{cn}) + \sum_{i=2}^{N} B_i(e_i(P^{cn})) - D(e_1^{cn} + \sum_{i=2}^{N} e_i(P^{cn})), \tag{17}$$

[10] As van Egteren and Weber note, determining the consequences of market power (and noncompliance) is more difficult if the regulator takes enforcement costs into account when choosing $\bar{L}$.

where, as before, e_1^{cn} is Firm 1's emissions level, P^{cn} is the equilibrium permit price, and the $e_i(\cdot)$ are the price-taking firms' emissions choice functions (see (1)). For the RHS of (17) to fully capture net social benefits, the (social) costs of enforcement have to be subtracted from it. However, to avoid introducing additional notation, I have chosen not to do so. It is sufficient to note that the costs of enforcement are presumably lower when the regulator condones some noncompliance, than when it ensures perfect noncompliance. Thus, any increases in PNSB from allowing noncompliance would understate the full social benefits of doing so.

The desirability of allowing *some* noncompliance can be determined by differentiating PNSB with respect to α, the compliance parameter introduced in (6), and evaluating the resulting derivative at $\alpha = 0$. The derivative is quite messy, but it simplifies considerably when evaluated at $\alpha = 0$. The price-taking firms are then all compliant, so their emissions equal their permit holdings, $e_i = l_i^c$. As shown in Appendix B, this implies, not surprisingly, that the solution to Firm 1's problem, (8), is no different than the solution when all price-taking firms are, by assumption, compliant. As before, this solution is $\{e_1^{cc}, l_1^{cc}\}$ with the corresponding equilibrium permit price of P^{cc}. Using these results, together with the earlier result that price-taking firms set their marginal profits equal to the permit price (see (1)), we can write:[11]

$$\frac{\partial PNSB}{\partial \alpha}\Big|_{\alpha=0} \equiv [B_1'(e_1^{cc}) - D'(\bar{L} - l_1^{cc} + e_1^{cc})]\frac{\partial e_1^{cn}}{\partial \alpha} + [P^{cc} - D'(\bar{L} - l_1^{cc} + e_1^{cc})] \cdot \sum_{i=2}^{N} \frac{de_i}{dP} \cdot \frac{\partial P^{cn}}{\partial \alpha}. \quad (18)$$

I will first analyze this derivative under the assumption that Firm 1 does not engage in permit retirement. Its permit holdings are then equal to its emissions, and (18) simplifies to:[12]

[11]In writing this expression, I have made use of the fact that the aggregate emissions of the price-taking firms equal their aggregate permit holdings, $(\bar{L} - l_1^{cc})$, when compliant.

[12]For simplicity, I will ignore the possibility that the increase in α induces Firm 1 to switch from not retiring permits to retiring permits.

$$\frac{\partial PNSB}{\partial \alpha}\Big|_{\alpha=0} \equiv [B_1'(l_1^{cc}) - D'(\bar{L})]\frac{\partial l_1^{cn}}{\partial \alpha} + [P^{cc} - D'(\bar{L})] \cdot \sum_{i=2}^{N} \frac{de_i}{dP} \cdot \frac{\partial P^{cn}}{\partial \alpha}. \tag{19}$$

The first term on the RHS reflects the effect of introducing some noncompliance on Firm 1's emissions; the second term reflects the effect on the price-taking firms' emissions. The signs of these terms depend on whether noncompliance shifts the emissions of the relevant firm(s) closer to the first-best level or further away from it. If the former is true, the term has a positive sign; if the latter is true, the term has a negative sign.

Determining the sign of the two terms in (19) requires knowledge of the signs of $\partial l_1^{cn}/\partial \alpha$ and $\partial P^{cn}/\partial \alpha$. The sign of $\partial P^{cn}/\partial \alpha$ is characterized in the following result, which is derived in Appendix B:

Result 3. *When Firm 1 exercises monopoly power, introducing noncompliance unambiguously lowers the equilibrium permit price: $\partial P^{cn}/\partial \alpha < 0$ regardless of the value of α. But when Firm 1 exercises monopsony power, it can lower or raise the equilibrium permit price.*

The monopoly result is not surprising. As Result 1 indicates, noncompliance increases the elasticity of the price-taking firms' aggregate demand for permits. This reduces Firm 1's monopoly power, leading to a lower permit price. Result 1 also indicates that the price-taking firms' aggregate demand for permits is reduced, this also leads to a lower permit price.

The monopsony result can be understood by observing that the increased elasticity of the price-taking firms' aggregate (gross) demand for permits implies that their net supply of permits, $\sum_{i=2}^{N} l_i^0 - L_{-1}^k(P)$, is more elastic (see (4)). This reduces Firm 1's monopsony power, which, by itself, would lead to a higher permit price. However, noncompliance also reduces the price-taking firms' aggregate demand for permits, thereby increasing the aggregate supply of permits, which leads to a lower permit price. Thus, the overall effect on price of introducing noncompliance is, in general, ambiguous in the monopsony case.

Result 3, along with the market-clearing condition in (7), can be used to evaluate the sign of $\partial l_1^{cn}/\partial\alpha$. Substituting P^{cn} for P in (7) and differentiating with respect to α, one can write:

$$\frac{\partial l_i^{cn}}{\partial\alpha}\Big|_{\alpha=0} = -\sum_{i=2}^{N}\frac{\partial l_i^{c}}{\partial P}\cdot\frac{\partial P^{cn}}{\partial\alpha} + \sum_{i=N_c+1}^{N} S_i'^{-1}(P). \tag{20}$$

The second term on the RHS of this expression is positive (see (2)). The sign of the first term is determined by the sign of $\partial P^{cn}/\partial\alpha$. If $\partial P^{cn}/\partial\alpha < 0$, as is always true when Firm 1 exercises monopoly power, the first term is negative; in this case, the sign of (20) is ambiguous. Therefore, when Firm 1 exercises monopoly power, introducing noncompliance can raise or lower its permit holdings. This can be explained as follows. The lower permit price induced by noncompliance would, by itself, increase the quantity of permits demanded by the price-taking firms. But noncompliance also lowers their aggregate demand for permits, which reduces the quantity of permits demanded. The former effect is captured by the first term on the RHS of (20), and the latter effect by the second term.

If $\partial P^{cn}/\partial\alpha > 0$, as is possible when Firm 1 exercises monopsony power, the first term is positive and introducing noncompliance unambiguously raises Firm 1's permit holdings. As shown in Appendix C, this is always true when Firm 1 has monopsony power, i.e., the effect captured by the second term dominates the effect captured by the first term. These findings are summarized in:

Result 4. *When Firm 1 exercises monopoly power, introducing some noncompliance can raise or lower its permit holdings. However, when Firm 1 exercises monopsony power, introducing some noncompliance unambiguously raises its permit holdings,* $\partial l_1^{cn}/\partial\alpha > 0$.

B. Monopoly Power

We are now in a position to evaluate the sign of (19) and determine the desirability of introducing some noncompliance. Let us first consider the case where Firm 1 exercises monopoly power. Result 2 together with the relationships in (16) then imply that the first bracketed expression in (19) is negative and the second bracketed expression is positive. Given Result 3, we can conclude that the second term in (19) is positive. The sign of the first term in (19) depends on the sign of $\partial l_1^{cn}/\partial\alpha$. If $\partial l_1^{cn}/\partial\alpha < 0$, the first term is also positive, but if $\partial l_1^{cn}/\partial\alpha > 0$, it is negative. These observations imply that when Firm 1 exercises monopoly power, introducing some noncompliance is socially desirable if it lowers Firm 1's permit holdings, since (19) is then unambiguously positive.

Noncompliance could be desirable even in the case where it raises Firm 1's permit holdings. In this case, the first term in (19) is negative but the second term is positive. The second term could outweigh the first one, that is, the losses from moving Firm 1's emissions further away from the first-best level could be outweighed by the gains from moving the price-taking firms' emissions closer to the first best level. This possibility is illustrated in Figure 2, which depicts a case in which Firm 1 has monopoly power but does not retire permits. As in the previous figure, Firm 1 is assumed to receive all L permits initially, and the price-taking firms are assumed to be identical. The dashed curves in the figure, P_n and M_n, are the equilibrium price and marginal revenue functions when the price-taking firms are noncompliant. The curves are drawn under the assumption that the price-taking firms face the same quadratic expected penalty function, with $S_i'(0) = 0$.[13]

The equilibrium allocation of permits when the price-taking firms are noncompliant is given by A^{cn}. The corresponding permit price is P^{cn}. Since Firm 1 is compliant, A^{cn} also gives its equilibrium emissions level. The aggregate emissions of the price-taking firms are given by the point labeled E_{-1}^{cn}. The distance between E_{-1}^{cn} and A^{cn} captures the amount by which total emissions exceed L because of noncompliance.

As shown, noncompliance lowers the equilibrium permit price from P^{cc} to P^{cn}, moving the price-taking firms' aggregate emissions from A^{cc} to E_{-1}^{cn}, which is closer to the first-best level, $\hat{A}$. Firm 1's

[13]This assumption is made to simplify the diagram. It implies that the demand curves with and without noncompliance intersect at the horizontal axis.

emissions, on the other hand, move further away from the first-best level, from A^{cc} to A^{cn}. To quantify the welfare effects of these movements, we need to specify a marginal damage curve, $D'(\cdot)$, in the figure. It is simplest to assume that marginal damages are constant ($D'' = 0$). Given (16), D' would then coincide with $\hat{P} \equiv P_c(\bar{L} - \hat{l}_1)$, as indicated by the dashed horizontal line in the figure. Given this marginal damage curve, the (net) benefits of moving the price-taking firms' emissions toward the first-best level are given by area (dgho), and the (net) costs of moving Firm 1's emissions away from the first-best level are given by area (afdb). The first area is clearly larger than the second, hence noncompliance yields positive net social benefits.

C. Monopsony Power

We now turn to the case where Firm 1 exercises monopsony power. For this case, Result 2 and the relationships in (16) imply that the first bracketed expression in (19) is positive and the second bracketed expression is negative. It follows from Result 4 that the first term in (19) is positive. As for the second term, its sign is determined by the sign of $\partial P^{cn}/\partial\alpha$. If $\partial P^{cn}/\partial\alpha > 0$, the second term is also positive, but if $\partial P^{cn}/\partial\alpha < 0$, the second term is negative. This implies that when Firm 1 exercises monopsony power, some noncompliance is unambiguously desirable if it raises the equilibrium permit price, since (19) is then unambiguously positive. If noncompliance lowers the equilibrium permit price, it may still be socially desirable if the beneficial effect on Firm 1's emissions outweighs the detrimental effect on the price-taking firms' emissions, that is, if the first term is larger in absolute value than the second.

D. Allowing for Permit Retirement

The above assessment of the desirability of noncompliance has assumed that Firm 1 does not engage in permit retirement. Recall that eq. (19) was obtained from eq. (18) by assuming that Firm 1's emissions were equal to its permit holdings: $e_1^{cc} = l_1^{cc}$. If Firm 1 engages in permit retirement, then its permit holdings are larger than its emissions, which are at their maximum level: $l_1^{cc} > e_1^{cc} = e_1^m$. If we assume, for simplicity, that a change in α does not induce Firm 1 to move from retiring permits to not retiring permits, this implies that the first term in (18) vanishes, because Firm 1's emissions would stay constant at e_1^m. Thus, (18) would reduce to:

$$\frac{\partial PNSB}{\partial \alpha}\Big|_{\alpha=0} \equiv [P^{cc} - D'(\bar{L} - l_1^{cc} + e_1^{m})] \cdot \sum_{i=2}^{N} \frac{de_i}{dP} \cdot \frac{\partial P^{cn}}{\partial \alpha}. \tag{21}$$

From Result (2) and (16), we know that $P^{cc} > D'(\bar{L})$. This implies, given $(- l_1^{cc} + e_1^m) < 0$ and non-decreasing marginal damages, that $P^{cc} > D'(\bar{L} - l_1^{cc} + e_1^m)$. Thus, the bracketed expression in (21) is positive, which implies that (21) is unambiguously positive, since $\partial P^{cn}/\partial \alpha < 0$ when Firm 1 has monopoly power, as must be true if it retires permits. Thus, introducing noncompliance is unambiguously desirable when Firm 1 engages in permit retirement. The sole effect of noncompliance in this setting is to raise the price-taking firms' emissions, bringing them closer to the first-best level.

The above results are summarized in the following proposition:

Proposition 2. *(i) When Firm 1 exercises monopoly power, some noncompliance is unambiguously desirable if Firm 1 engages in permit retirement or if noncompliance lowers Firm 1's permit holding; if Firm 1's permit holdings rise, some noncompliance may still be desirable. (ii) When Firm 1 exercises monopsony power, some noncompliance is unambiguously desirable if it lowers the equilibrium permit price; if it raises the equilibrium permit price, it may still be desirable.*

The potential desirability of noncompliance in this setting is an illustration of the theory of second best. The permit market considered is one in which there are two market failures–market power and noncompliance. The theory of second best implies that correcting one of these market failures without correcting the other need not improve social welfare. The potential desirability of allowing noncompliance when Firm 1 exercises market power confirms this.

5. Noncompliance by Firm 1

I have thus far only allowed the price-taking firms to be noncompliant. If Firm 1 is noncompliant as well, its decision problem changes to the extent that: (i) the constraint $l_1 \geq e_1$ must be replaced with a simple non-negativity constraint, $l_1 \geq 0$; and (ii) the expected penalties for noncompliance, $S_1(v_1)$, must be subtracted from the objective function in (8). The changes to the Kuhn-Tucker conditions for an interior optimum ($0 < l_1^{nn} < L$, $e_1^{nn} > 0$) are quite straightforward: μ is replaced by $S_1'(v_1)$ in conditions (9) and (10), and condition (11) is eliminated. The resulting changes in Firm 1's emissions and permit holdings are those we would expect to observe:

Result 5. *Noncompliance by Firm 1 lowers its permit holdings and raises its emissions level:* $l_1^{nn} < l_1^{cn}$ *and* $e_1^{nn} > e_1^{cn} = l_1^{cn}$. *The lower permit holdings imply a lower equilibrium permit price, which in turn implies higher emissions by the price-taking firms:* $P^{nn} < P^{cn}$ *and* $\sum_{i=2}^{N} e_i^{nn} > \sum_{i=2}^{N} e_i^{cn}$.

The proof of the key result, $l_1^{nn} < l_1^{cn}$, is provided in Appendix D.

Noncompliance by Firm 1 obviously rules out the possibility of permit retirement, since Firm 1's permit holdings must be smaller than its emissions level for it to be noncompliant. A question we can ask is whether noncompliance would ever be attractive to Firm 1 if it chooses to retires permits when it is constrained to being compliant. Intuition suggests that noncompliance would not be attrac-tive, since the firm voluntarily chooses to hold excess permits when it engages in permit retirement. This intuition can be confirmed quite easily (see Appendix E). It has an obvious, yet interesting, implication:

Proposition 3. *If Firm 1 chooses to retire permits, no enforcement is necessary to ensure its compliance. The exercise of monopoly power provides sufficient incentive for it to be compliant.*

This compliance incentive of monopoly power is also present, albeit to a lesser degree, in the case where Firm 1 does not retire permits. The incentive manifests itself in terms of the magnitude of the marginal expected fine needed to ensure Firm 1's compliance. It can be shown that a smaller marginal expected fine is needed to ensure Firm 1's compliance when it has monopoly power than when it is a price taker.[14]

Noncompliance on the part of Firm 1 does not eliminate the potential desirability of noncompliance among the price-taking firms: the shifts in emissions towards the first-best levels noted in Section 4 would still occur. However, the desirability of the price-taking firms' noncompliance could be diminished because noncompliance by Firm 1 further raises total emissions above the target level, $\bar{L}$. If $D'' > 0$, this would result in higher marginal damages from the price-taking firms' emissions.

The desirability of Firm 1's noncompliance is also of interest. However, unlike noncompliance by the price-taking firms, there are no readily identified cases in which noncompliance by Firm 1 is unambiguously desirable. It appears to invariably have some undesirable effect.

6. Optimal Permit Endowments

Van Egteren and Weber observe that a regulator can influence both the extent of noncompliance in the permit market and the extent of Firm 1's market power by varying Firm 1's initial endowment of permits. Accordingly, they examine the effects of changes in Firm 1's initial endowment of permits on net social benefits. These consist of firms' gross profits less the damages from pollution and the costs of enforcement. The latter are given by $G\beta_i(v_i)$, where G is the social cost of conducting an audit and $\beta_i(v_i)$ is the probability of an audit. They derive the following derivative (in their eq. (21)) which describes the

[14] As van Egteren and Weber show, for Firm 1 to be compliant, the marginal expected fine it faces when at a zero violation must be no smaller than its marginal revenue from the sale of a permit: $S_1'(0) \geq M_n(\bar{L} - l_1, l_1^0 - l_1)$. This condition is both necessary and sufficient given the assumption that $-\partial M_n / \partial l_1 < 0$. The corresponding condition for a price-taking firm to be compliant is $S_1'(0) \geq P_n(\bar{L} - l_1)$. To the extent that $M_n(\bar{L} - l_1^{cn}, l_1^0 - l_1^{cn}) < P_n(\bar{L} - l_1')$, where l_1' denotes Firm 1's permit holdings when it is a price taker, the stringency of the compliance condition is reduced.

effect of an increase in Firm 1's initial endowment on net social benefits:

$$\frac{\partial NSB}{\partial l_1^0} = P_n' \cdot (l_1^0 - l_1^{nn}) \cdot \frac{\partial l_1^{nn}}{\partial l_1^0} + [P_n(\bar{L} - l_1^{nn}) - D'(E^{nn})] \cdot \left(\frac{\partial e_1^{nn}}{\partial l_1^0} + \sum_{i=2}^{N} \frac{\partial e_i}{\partial l_1^0} \right) - G \sum_{i=1}^{N} \beta_i' \frac{\partial v_i^n}{\partial l_1^0}, \quad (22)$$

where E^{nn} represents total emissions in the presence of noncompliance by both Firm 1 and price-taking firms.

Van Egteren and Weber argue that the sign of this derivative is ambiguous, except for the case where Firm 1 is compliant and has monopoly power (then l_1^{nn} and E^{nn} should be replaced by l_1^{cn} and E^{cn}). They argue that in this case the derivative is negative, i.e., increasing Firm 1's initial permit endowment reduces net social benefits (see their Proposition 3). However, the sign of (22) is ambiguous even in this case. The first term in (22) is negative and the third term is positive, as they show, but the second term is not equal to zero, contrary to their argument; its sign is ambiguous.[15] This can be verified from Figure 2, which depicts a permit market in which Firm 1 is a compliant monopolist and the price-taking firms are noncompliant. The bracketed expression is positive for the case depicted since $P^{cn} > D'(\bar{L})$. Inspecting Figure 2, it is not difficult to see that if noncompliance were more severe, $P^{cn} < D'(\bar{L})$) could hold. Thus, the sign of (22) is ambiguous even when Firm 1 is a compliant monopolist. As a result, no simple prescriptions can be derived regarding the direction in which the regulator should vary Firm 1's initial permit endowment to increase net social benefits.

The derivative in (22) can be used to show that it is generally not socially optimal to eliminate Firm 1's market power. As can be verified from (12), Firm 1's market power can be eliminated by setting its initial endowment of permits equal to its equilibrium permit holdings ($l_1^0 = l_1^{nn}$ or $l_1^0 = l_1^{cn}$). If this were

[15]A possible explanation of the analysis in van Egteren and Weber is that they assume $\partial e_i^n/\partial l_1^0$ is constant across firms. This allows them to rewrite the second term in an alternative form which they present in equation (23) of their paper. They argue that this alternative expression equals zero. However, this expression need not equal zero because the pattern of emissions differs when price-taking firms are noncompliant and thus, total emissions may exceed $\bar{L}$.

socially optimal, the derivative in (22) would equal zero when $l_1^0 = l_1^{nn}$. The first term in (22) does vanish when $l_1^0 = l_1^{nn}$, but the third term is (still) positive. This reflects the fact that setting $l_1^0 > l_1^{nn}$ and conferring monopoly power on Firm 1 results in lower enforcement costs because it reduces the magnitudes of firms' violations. For the price-taking firms, this occurs because their violations are decreasing in the permit price, and monopoly power raises the equilibrium permit price. The sign of the second term in (22) is, in general, still ambiguous when $l_1^0 = l_1^{nn}$. It can be verified that the bracketed expression in this term is negative if we assume, as before, that the total number of permits issued, $\bar{L}$, was chosen assuming a first-best setting.[16] The expression in parentheses is also negative if the increase in Firm 1's emissions resulting from its larger endowment is offset by the reduction in the price-taking firms' emissions because of the higher permit price (as van Egteren and Weber show, $\partial e_1^{nn}/\partial l_1^0 > 0$ and $\partial e_i/\partial l_1^0 < 0$). If this offset holds, then $\partial NSB/\partial l_1^0 > 0$, which implies that conferring some monopoly power on Firm 1 is socially optimal. If the offset does not hold, the expression in parentheses is positive, rendering the sign of $\partial NSB/\partial l_1^0$ ambiguous. If $\partial NSB/\partial l_1^0 < 0$, it would be optimal to set $l_1^0 < l_1^{nn}$ and confer some monopsony power on Firm 1. We therefore have:

Proposition 4. *In the presence of noncompliance, it is generally not socially optimal to eliminate Firm 1's market power.*

This result can be viewed as another illustration of the theory of second best, complementing the earlier result on the potential desirability of noncompliance.

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[16]Given this assumption, $\hat{P} = D'(\bar{L})$, as in (16). As Malik (1990) shows, noncompliance leads to a lower equilibrium permit price in a competitive permit market, thus $P_n(\bar{L} - l_1^{nn}) < \hat{P}$. Noncompliance also implies $E^{nn} > \bar{L}$ in a competitive market. If $D'' \geq 0$, this in turn implies that $D'(E^{nn}) \geq D'(\bar{L})$. We can therefore write: $P_n(\bar{L} - l_1^{nn}) - D'(E^{nn}) < \hat{P} - D'(\bar{L}) = 0$.

7. Concluding Remarks

The possibility that a firm with monopoly power could choose to retire permits is a fairly striking one. This result might appear to hinge on the implicit assumption that Firm 1 is unable to engage in price discrimination. If price discrimination were feasible, permit retirement would be reduced, if not eliminated. Firm 1 would then be able to raise its profits by selling excess permits at a reduced price to buyers with low permit valuations, without affecting the price it receives for permits sold to high valuation buyers. However, scope for price discrimination is virtually nonexistent in the setting considered given the ease with which permits can be resold. This would undercut any attempt by Firm 1 to price discriminate.

Price discrimination, albeit of an involuntary nature, may be relevant in a multi-period setting in which permits are valid for more than one period. In this case, Firm 1 would effectively be a durable goods monopolist. As is well known, the ability of such a firm to exercise monopoly power is diminished if it engages in intertemporal price discrimination by lowering its price over time (e.g., see Tirole, 1989, pp. 72-74). However, this form of price discrimination is detrimental to the monopolist, and it would seek devices that would enable it to commit to a price and retain its monopoly power.

Like Hahn and van Egteren and Weber, I have assumed that only one firm in the permit market is able to behave strategically and influence permit price. Westskog (1997) has extended Hahn's model of market power with perfect compliance to allow for multiple firms behaving strategically, each engaging in Cournot behavior. He finds that Hahn's results carry over to this more complex setting. In terms of the model presented here, this extension would result in two changes to the conditions characterizing market equilibrium: (i) additional conditions such as those in (9)-(11) would be introduced for each of the other firms with market power, and (ii) the equilibrium price function, $P_k(\cdot)$ would now have as its argument $(\bar{L} - \sum_{j \in J} l_j)$, where J denotes the set of firms with market power. These changes would complicate the analysis (and render infeasible graphical exposition), but they should not affect the basic results of the paper, namely, the possibility of permit retirement and the potential desirability of noncompliance among the price-taking firms. The condition for one of the market power firms to engage in permit retirement would be identical to (14), except for the change in the argument of the equilibrium price function. Identifying the conditions under which noncompliance by the price-taking firms is desirable would be more complicated, especially if some firms had monopoly power and others monopsony power. However, consistent with the theory of second best, there would invariably be situations in which noncompliance would be desirable, given its ability to shift emissions toward first-best levels.

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Appendices

A. Proof of Proposition 1

The sufficiency of the condition in (14) can be established by observing that it rules out $l_1 = e_1 = e_1^m$ satisfying (10) and being a solution to the firm's problem when $\mu = 0$ (as must be true when $e_1 = e_1^m$). Given (13), condition (14) also rules out $e_1 = l_1 < e_1^m$ satisfying (10) and being a solution when $\mu \geq 0$. This leaves $1_1 > e_1 = e_1^m$ as the only possible solution to the firm's problem. [The existence of a solution is guaranteed by the continuity of the objective function and the compactness of the feasible set defined by the constraints in (8).] Necessity can be established by observing that, given (13), (10) will hold for $1_1 > e_1 = e_1^m$ only if (14) holds. Finally, examining (14), it is evident that for it to hold, the second term must be negative and sufficiently large in absolute value. Since $P_n'(\cdot) < 0$, the second term is negative only if $l_1^0 > e_1^m$.

B. Effect of Introducing Noncompliance on the Equilibrium Permit Price

The effect of introducing noncompliance on the equilibrium permit price is most easily determined by analyzing the dual of Firm 1's problem in (8).[17] Using the market-clearing condition in (7), this problem can be written as:

$$\max_{e_1 \geq 0,\, P \geq 0} \; B_1(e_1) + P[l_1^0 - l_1] \qquad s.t. \quad l_1 \geq e_1, \tag{A-1}$$

where

$$l_1 \equiv \bar{L} - \sum_{i=2}^{N_c} l_i^c(P) - \sum_{i=N_c+1}^{N} l_i(P;\alpha). \tag{A-2}$$

Letting $\lambda \geq 0$ be the Lagrange multiplier associated with the constraint, the Kuhn-Tucker conditions for an interior solution ($e_1^{cn} > 0$, $P^{cn} > 0$) to this problem are:

$$B_1'(e_1) = \lambda, \tag{A-3}$$

$$[P - \lambda] \cdot \left[\sum_{i=2}^{N_c} \frac{dl_i^c(P)}{dP} + \sum_{i=N_c+1}^{N} \frac{\partial l_i(P;\alpha)}{\partial P} \right] + [l_1^0 - l_1] = 0, \tag{A-4}$$

$$\lambda \cdot (l_1 - e_1) = 0. \tag{A-5}$$

[17] Hahn considered this dual problem.

Examining (A-2)-(A-5) and (6), it is not difficult to see that if $\alpha = 0$, the Kuhn-Tucker conditions would be identical to those when *all* the price-taking firms are by assumption compliant. This implies: $e_1^{cn}|_{\alpha=0} = e_1^{cc}$ and $P^{cn}|_{\alpha=0} = P^{cc}$.

I now turn to determining the effect of introducing noncompliance on the equilibrium permit price, $\partial P^{cn}/\partial\alpha$. Let us first consider the case where Firm 1 does not engage in permit retirement ($l_1 = e_1$). In this case, (A-3)-(A-5) can be collapsed into a single condition:

$$[P - B_1'(l_1)] \cdot \left[\sum_{i=2}^{N_c} \frac{dl_i^c(P)}{dP} + \sum_{i=N_c+1}^{N} \frac{\partial l_i(P;\alpha)}{\partial P} \right] + [l_1^0 - l_1] = 0, \qquad \text{(A-6)}$$

Note that the first bracketed expression must be positive if Firm 1 exercises monopoly power, that is, if $l_1^0 > l_1$, and negative if it exercises monopsony power, that is, if $l_1^0 < l_1$.

Let Z denote the LHS of (A-6). It can be verified that given the second-order sufficient conditions for the problem in (A-1), if Z is decreasing in α, then introducing noncompliance unambiguously reduces the equilibrium permit price: $\partial P^{cn}/\partial\alpha < 0$ regardless of the value of α. Differentiating Z with respect to α yields:

$$\frac{\partial Z}{\partial \alpha} \equiv -B_1''(l_1) \cdot \left[\sum_{i=2}^{N_c} \frac{dl_i^c(P)}{dP} + \sum_{i=N_c+1}^{N} \frac{\partial l_i(P;\alpha)}{\partial P} \right] \cdot \left[\sum_{i=N_c+1}^{N} S_i^{\prime -1}(P) \right]$$
$$- [P - B_1'(l_1)] \cdot \left[\sum_{i=N_c+1}^{N} \frac{dS_i^{\prime -1}}{dP} \right] - \left[\sum_{i=N_c+1}^{N} S_i^{\prime -1}(P) \right]. \qquad \text{(A-7)}$$

The first term in this expression is negative, while the last term is positive, as is the large bracketed expression in the second term (see the discussion of eq. (2)). The sign of the second term depends on that of $[P - B_1'(l_1)]$. As noted in the text, when Firm 1 exercises monopoly power, it has a positive sign. Together, these observations imply that when Firm 1 exercises monopoly power, $\partial Z/\partial\alpha$ is unambiguously negative, which in turn implies that $\partial P^{cn}/\partial\alpha < 0$. On the other hand, when Firm 1 exercises monopsony power, the second term has a negative sign because $[P - B_1'(l_1)] < 0$ then; this implies that the sign of $\partial Z/\partial\alpha$ is ambiguous and, as a result, the sign of $\partial P^{cn}/\partial\alpha$ is ambiguous.

When Firm 1 retires permits, $e_1 = e_1^m < l_1$. The relevant first-order condition is then (A-6) with $B_1'(l_1)$ replaced by zero, since $B_1'(e_1^m) = 0$. Equation (A-7) would change to the extent that the first term would drop out, and in the second term, $B_1'(l_1)$ would be replaced by zero. Hence, $\partial Z/\partial\alpha$ and $\partial P^{cn}/\partial\alpha$ would be unambiguously negative.

C. Proof that noncompliance by the price-taking firms raises Firm 1's permit holdings when it has monopsony power, $l_1^{cn} > l_1^{cc}$

This result, which holds for any level of noncompliance, can be established by contradiction. By definition: $M_c(\bar{L} - l_1^{cc}, l_1^0 - l_1^{cc}) = B_1'(l_1^{cc})$ and $M_n(\bar{L} - l_1^{cn}, l_1^0 - l_1^{cn}) = B_1'(l_1^{cn})$ given the Kuhn-Tucker conditions in (9) and (10), and remembering that $e_1 = l_1$ holds when Firm 1 has monopsony power (and is compliant). Result 1 implies that $M_n < M_c$ when Firm 1 has monopsony power. We can therefore write $M_n(\bar{L} - l_1^{cc}, l_1^0 - l_1^{cc}) < B_1'(l_1^{cc})$. If $l_1^{cn} \leq l_1^{cc}$, this inequality would imply $M_n(\bar{L} - l_1^{cn}, l_1^0 - l_1^{cn}) < B_1'(l_1^{cn})$, given (13) and the strict concavity of $B_1(\cdot)$. This contradicts the definition of l_1^{cn}.

D. Proof of Result 5

Given that Firm 1 is noncompliant, $e_1^{nn} > l_1^{nn}$ must hold. Assuming that Firm 1 does not retire permits when compliant,[18] there are five possibilities to consider: (i) $e_1^{nn} > l_1^{nn} \geq l_1^{cn}$, (ii) $l_1^{cn} \geq e_1^{nn} > l_1^{nn}$, (iii) $e_1^{nn} = l_1^{cn} > l_1^{nn}$, (iv) $e_1^{nn} > l_1^{cn} = l_1^{nn}$, and (v) $e_1^{nn} > l_1^{cn} > l_1^{nn}$. The first possibility can be ruled out by contradiction. Given the Kuhn-Tucker conditions in (9) and (10), $M_n(\bar{L} - l_1^{cn}, l_1^0 - l_1^{cn}) = B_1'(l_1^{cn})$ and $M_n(\bar{L} - l_1^{nn}, l_1^0 - l_1^{nn}) = B_1'(e_1^{nn})$. If $e_1^{nn} > l_1^{nn} \geq l_1^{cn}$, the first equality would imply $M_n(\bar{L} - l_1^{nn}, l_1^0 - l_1^{nn}) > B_1'(e_1^{nn})$, given $-\partial M_n/\partial l_1 < 0$ and $B_1'' < 0$. The second, third, and fourth possibilities can be ruled out in a similar manner. Only the fifth one is feasible.

E. Proof of Proposition 3

Replacing μ with $S_1'(v_1)$, the Kuhn-Tucker conditions in (9) and (10) imply $M_n(\bar{L} - l_1, l_1^0 - l_1) = B_1'(e_1) = S_1'(v_1) > 0$ must hold when Firm 1 is noncompliant, with $l_1 < e_1 < e_1^m$. The last inequality follows from the assumption that $S_1'(v_1) > 0$ when $v_1 > 0$. If the permit retirement condition holds, that is, if $M_n(\bar{L} - e_1^m, l_1^0 - e_1^m) < 0$ (see (14)), then $M_n(\bar{L} - l_1, l_1^0 - l_1) > 0$ cannot hold for $l_1 < e_1^m$, given $-\partial M_n/\partial l_1 < 0$. An analogous argument applies if the price-taking firms are all compliant—one need only replace $M_n(\cdot)$ with $M_c(\cdot)$.

[18] As show in Appendix E below, if Firm 1 retires permits, it would never choose to be noncompliant. We can therefore ignore the case where Firm 1 retires permits when noncompliant.

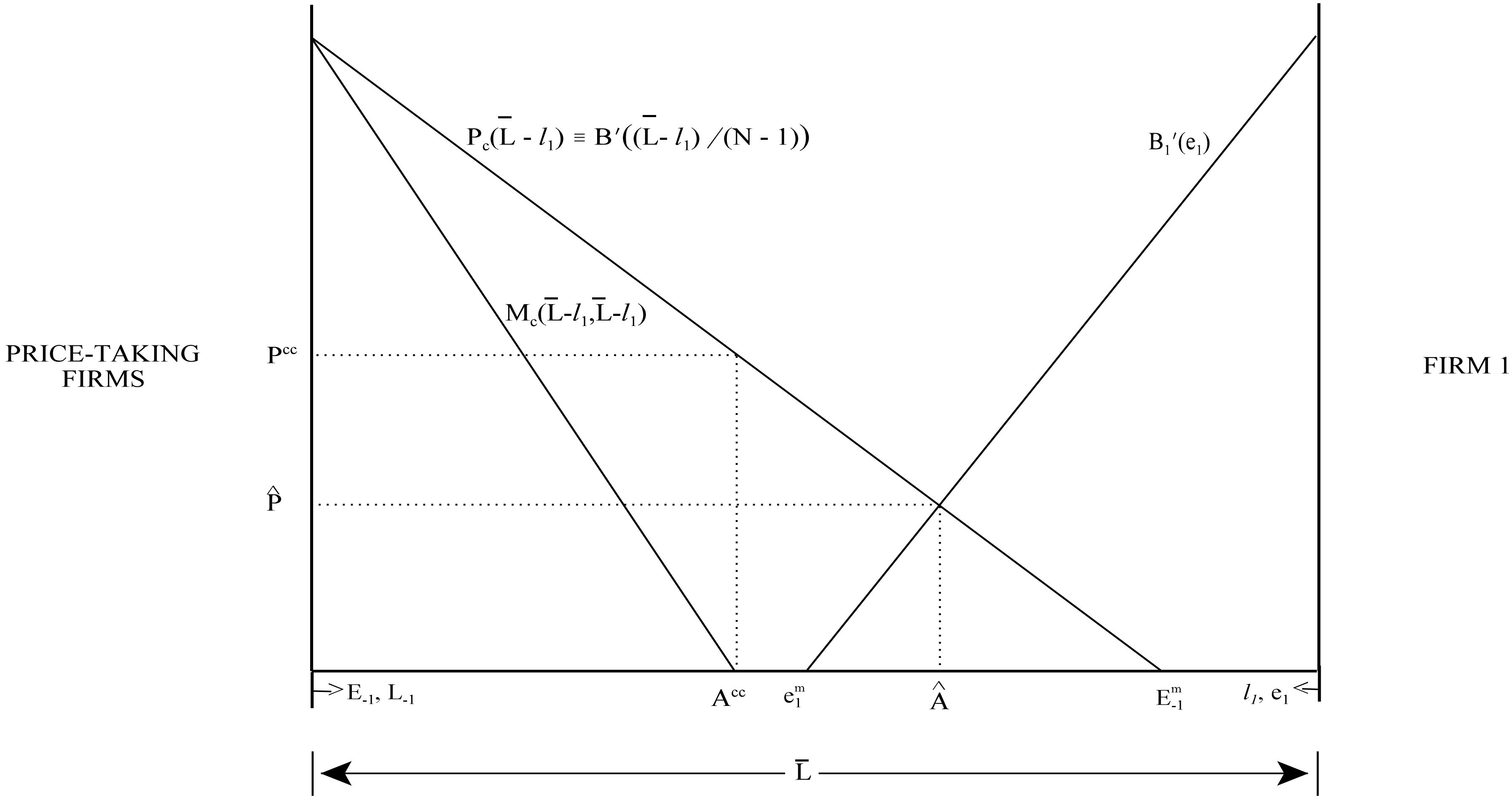

Figure 1. Monopoly Power and Permit Retirement

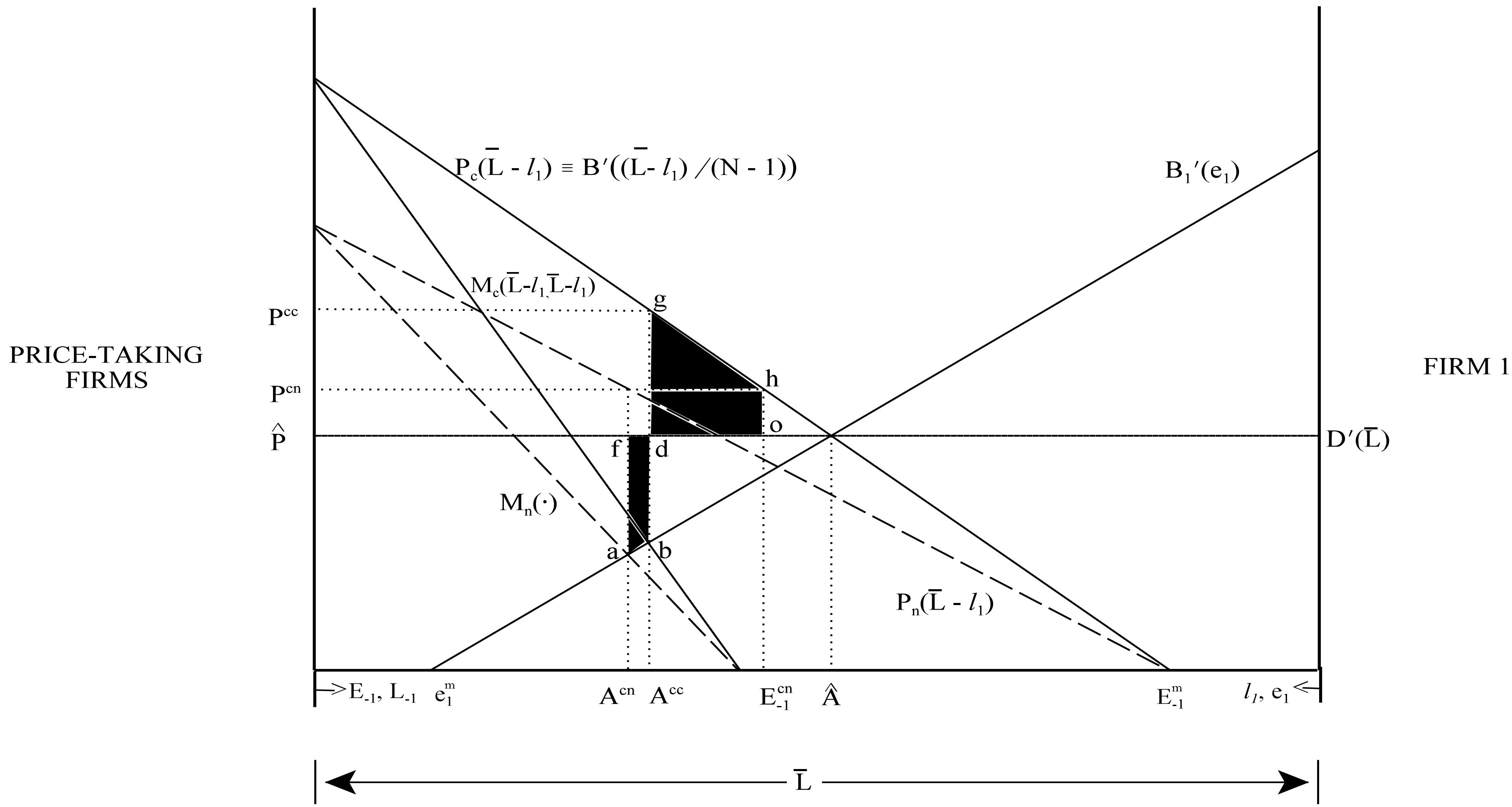

Figure 2. Monopoly Power and the Effects of Noncompliance (no permit retirement)