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Imperfect observability of emissions and second-best emission and output taxes

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Abstract

This paper studies the second-best tax design problem when emissions are publicly unobservable but can be discovered through costly monitoring. A representative firm in industry chooses its emissions and declarations to maximize its expected profits. The paper shows: The Pigouvian rule of equating the marginal private benefit of emissions in production to the marginal social damage of emissions is modified to take account of the resource costs of monitoring and enforcement; emissions from different sectors must be taxed at different rates; every polluting good must face an effective emission tax that is less than the full marginal social damage of emissions; emission taxes do not reflect standard optimal tax objectives; output taxes have a Pigouvian role so that the targeting principle fails.

Keywords: Emissions; Imperfect observability; Taxes

1. Introduction

In most textbook models of externality the question of the choice between emission and output taxes does not arise; they are equivalent. Yet this is an important policy question. The equivalence of the tax instruments in these models

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is an artifact of an assumed one-to-one relationship between emissions and polluting goods. This is an unrealistic assumption. Input substitution, employing different technologies and abatement imply that a given level of output may result in different levels of emissions. Allowing for these possibilities breaks up the equivalence of output and emission taxes and allows one to study the role of these and other tax instruments in implementing different public policy objectives.

The recent burgeoning literature on environmental taxation has addressed the question of the role of tax instruments. The basic lesson that has emerged is one of instrument targeting: Correct externalities through emission taxes and use ''other'' tax instruments for other public policy objectives.[1] What other tax instruments are available depends on the structure of information in the economy. In implementing first-best outcomes, the government will have to levy differential lump-sum taxes coupled with emission taxes; no other taxes are required. Revenue and redistributive objectives are achieved through lump-sum taxes leaving emission taxes solely to correct for externalities. Output taxes cannot be substituted for emission taxes in that they cannot, in conjunction with differential lump-sum taxes, implement first-best outcomes. A recent paper by Cremer and Gahvari (2001) shows that the instrument targeting principle also holds in the second-best as long as there are no complementarity and/or substitutability relationships between emissions and consumption goods. A proportional emission tax corrects the impact of externalities, while output (and income) tax instruments are determined solely on the basis of optimal tax objectives.

The informational structure in the economy that lies behind these results postulate that emissions are publicly observable. If this assumption is not satisfied, actual emissions will diverge from the intended levels desired by policy makers. Under this circumstance, monitoring may be necessary to ensure the two will coincide. However, because monitoring is a costly activity, it would not typically be employed to deter all evasions. Monitoring strategies thus become one more crucial ingredient in the set of optimal environmental policies.[2] This observation has far reaching implications for the nature of second-best allocations (*including* emission levels) as well as the instruments the government should use in

[1]The pioneering study of second-best taxes in the presence of externalities is Sandmo (1975). He also originated the concept of the principle of targeting calling it ''the additive property''. Bovenberg and van der Ploeg (1994) also emphasized this principle. Cremer et al. (1998) have argued that the principle may break down if the direct instrument is more restrictive than the indirect instrument. However, their argument stems from a misreading of their own results. Using the formula for polluting good tax (t_y), given by their Eq. (21), in their Eq. (24) shows that the externality term does not appear in the formula characterizing the marginal income tax rate. Kopczuk (1999) has pointed this out.

[2]Some recent empirical studies attest to the effectiveness of such policies. See, e.g., Gray and Deily (1996), Nadeau (1997); and Foulon et al. (1999).

implementing such allocations. Does the Pigouvian rule (of equating the marginal private benefit of emissions in production to the marginal social damage of emissions) hold in this setting? Does the production efficiency result continue to be valid? Will the property of the targeting of tax instruments continue to hold? That is, should emission taxes be levied only to combat externalities or do they also incorporate optimal tax considerations? Similarly, will output taxes be levied only in light of optimal tax objectives or do they have a Pigouvian role as well? What are the tradeoffs between monitoring strategies and tax policies?

The monitoring of emissions has been examined previously, particularly in the literature on regulation. However, it has never been studied as an ingredient in the second-best tax design problem. The problem has an obvious parallel in the tax evasion literature, and one can bring many of the lessons of that literature to bear upon it. To our knowledge, our paper is the first one to address the monitoring issue in this light. The closet precursors to our paper are Schmutzler and Goulder (1997), Smulders and Vollebergh (1999), and Fullerton et al. (1999). Schmutzler and Goulder have examined the implications of imperfect observability of emissions and monitoring for the choice between emission and output taxes. Their study is limited to one polluting good in isolation ignoring other tax instruments altogether. Smulders and Vollebergh consider many polluting goods. However, in their model, as in Schmutzler and Goulder's, the sole purpose of taxation is to correct for externalities. The revenue and redistributive objectives are assumed away. Fullerton et al., allow for tax instruments to have multiple objectives (Pigouvian as well as optimal tax considerations). The question they study, however, is not one of the design of an integrated tax and monitoring system or the issues pertaining to the role of different instruments. In their setup, there is *either* an emission tax *or* an output tax. Their aim is to compare the two alternatives which, in combination with a labor income tax, are levied to achieve all government objectives. Consequently, they cannot address the issues that arise in connection with the role and the properties of different tax instruments. In addition, the monitoring strategy does not enter their analysis.

We consider a framework similar to Cremer and Gahvari's (2001). In particular, we model the production side by assuming that the unit production costs of polluting goods are inversely related to their emissions. This is a ''short cut'' reflecting the fact that technologies that cut emissions (through a different production technique, abatement, or the use of different polluting inputs) are more expensive to employ. However, we now drop Cremer and Gahvari's crucial assumption that per unit emissions are publicly observable. To tax polluting firms, the government will have to rely on the firms' own self-reports. The truthfulness of the reports can however be discovered through costly examinations and monitoring.

We depart from Cremer and Gahvari (2001) in one other aspect. We ignore the differences between individual types and set aside the question of income taxation.[3] This allows us to concentrate on the earlier questions raised. For simplicity, we also restrict the set of tax instruments to linear commodity taxes on polluting and nonpolluting goods and linear emission taxes. To focus on the importance of the observability assumption, we assume that preferences are weakly separable in emissions and other goods. Under this circumstance, Cremer and Gahvari (2001) have shown that were emissions publicly observable, the first-best Pigouvian rule holds; and that the second-best tax design calls for a proportional emission tax to correct for externalities, leaving output taxes to be determined solely on the basis of optimal tax considerations.

Each polluting good is produced by an industry which is comprised of a fixed but sufficiently large number of identical firms. A firm's emission taxes depend on its reported emissions. The firm may thus attempt to lower its tax payments by reporting only a fraction of its emissions. The government inspects and monitors a fraction of the firms in industry; the inspections are costly and reveal the firms' emissions accurately. Firms that are caught cheating will have to pay a fine in addition to taxes already paid. The expected fines increase with the fractions of emissions that go unreported, with the statutory tax rate and with the intensity of the regulatory control.

A representative firm in industry chooses its emissions and declarations to maximize its expected profits. From this setup, we establish how a firm responds to a change in the tax rates and the intensity of the regulatory control. We then formulate the government's optimization problem in light of the firms' responses. Our main results are: First, the Pigouvian rule of equating the net marginal private benefit of a firm's emission to the marginal social damage of emissions is modified to take account of the resource costs of monitoring and enforcement. Second, we characterize statutory and effective emission taxes and prove that they differ across different sectors. Third, every polluting good must face an effective emission tax that is less than the full marginal social damage of emissions. Fourth, we show that any given effective emission tax can be achieved through different mixes of the statutory emission tax and the regulatory control parameter. We characterize the tradeoff between the two instruments. Fifth, optimal tax terms do not enter emission tax formulas; that is, emission taxes (statutory and effective) do not reflect standard optimal tax objectives. On the other hand, output taxes must supplement emission taxes to correct for emission. That is, the targeting of instruments principle fails and output taxes have a Pigouvian role. We also show that output taxes have a Pigouvian role even in the presence of lump-sum taxes.

[3] As in Cremer and Gahvari (2001), as long as the tax instruments on the consumption side are unrestricted, income redistribution issues have no impact on the production side of the model. No further insights, relevant to the issues we are raising here, will thus be gained by complicating the model.

Sixth, if preferences are weakly separable in labor supply and consumption goods, with the consumption good subutility being homothetic, all private goods must be taxed at a uniform proportionate rate, while polluting goods are taxed non-uniformly at rates that exceed the uniform tax rate on private goods.

2. The model

Consider a community of identical individuals where a representative consumer has preferences over consumer goods, labor supply and *total* emissions of pollutants in the atmosphere. The production sector is modeled as in Cremer and Gahvari (2001). There are $n+m$ consumer goods. The first n goods, denoted by $\underline{x}=(x_1,x_2,\ldots,x_n)$, are nonpolluting in that their production is associated with no externality. The second m goods are polluting goods whose production entails emissions of pollutants into the atmosphere thus result in a consumption externality. Denote the vector of polluting goods by $\underline{y}=(y_1,y_2,\ldots,y_m)$, and emission per unit of output in the polluting industry s by e_s. Assume the resource cost of producing one unit of y_s, $C_s(e_s)$, is a continuously differentiable, decreasing and convex function of e_s.[4]

The appearance of a firm's per unit emissions in our model is a ''short cut''. In effect, we are modeling situations where polluting goods can be produced in different ways each resulting in a different emission level. These include the use of different production techniques, abatement, or different polluting inputs where each particular input entails a different emission level. Whatever is the source, these aspects of the firms' operations are not readily observable by the government's ''regulatory agency''. Thus we assume that the emission levels of firms producing polluting goods are not publicly observable. However, they can be discovered through costly inspection and monitoring. Inspecting a firm and observing its emission directly and/or observing its input combinations, reveals its ''true'' emissions.

Denote the consumer prices of $\underline{x}$ and $\underline{y}$ by $\underline{p}=(p_1,p_2,\ldots,p_n)$ and $\underline{q}=(q_1,q_2,\ldots,q_m)$. There are two types of policy instruments: Taxes and ''regulatory control''. Tax instruments consist of linear commodity taxes on $\underline{x}$ and $\underline{y}$, denoted by $\underline{\eta}=(\eta_1,\eta_2,\ldots,\eta_n)$ and $\underline{\tau}=(\tau_1,\tau_2,\ldots,\tau_m)$, and linear emission taxes, $\underline{t}=(t_1,t_2,\ldots,t_n)$. The ''intensity'' of the regulatory control is captured by $\underline{\beta}=(\beta_1,\beta_2,\ldots,\beta_m)$, where the audit probability and the level of fine facing a polluting firm in industry s are subsumed in the parameter β_s.

[4]More precisely the assumption is that $C_s'<0$ for all e_s up to some limit $\bar{e}_s$, and that $C_s'(\bar{e}_s)=0$; $s=1,2,\ldots,m$.

2.1. The individuals

Let E denote total emissions generated by the production of y_s's so that

$$E=\sum_{s=1}^{m} e_s y_s. \tag{1}$$

Assume the representative consumer's preferences are weakly separable in emissions and all other goods.[5] Denote labor supply by L and let the utility function

$$U=\boldsymbol{U}\big(\boldsymbol{F}(\underline{x},\underline{y},L),E\big), \tag{2}$$

represent the preferences. Assume $\boldsymbol{U}$ is strictly quasi-concave, twice continuously differentiable, strictly increasing in $\underline{x}$ and $\underline{y}$, and strictly decreasing in L and E. The consumer treats E as given and maximizes (2) subject to the budget constraint

$$\sum_{k=1}^{n} p_k x_k+\sum_{s=1}^{m} q_s y_s=wL-T, \tag{3}$$

where w is the wage, assumed untaxed, and T is the lump-sum tax, if any. The maximization problem yields the demand functions for $\underline{x}$, $\underline{y}$ and the supply function for L, as functions of consumer prices, w and T. The weak separability of preferences implies that these functions are independent of emissions. One may then define the individual's indirect utility function as

$$\begin{aligned} V &= \boldsymbol{V}\big(\boldsymbol{\Phi}(\underline{p},\underline{q},w,T),E\big)\\ &\equiv \boldsymbol{U}\big(\boldsymbol{F}(\underline{x}(\underline{p},\underline{q},w,T),\underline{y}(\underline{p},\underline{q},w,T),L(\underline{p},\underline{q},w,T)),E\big). \end{aligned} \tag{4}$$

2.2. The firms

The firms producing nonpolluting goods operate in a competitive environment and face a linear technology subject to constant returns to scale. Consequently, normalizing the producer prices of these goods at one, we have $p_i=1+\eta_i$. As with nonpolluting goods, firms producing polluting goods also operate in a competitive environment. The polluting good y_s is produced by an industry that is comprised of a fixed but sufficiently large number of identical firms. It is produced, for a given $C_s(e_s)$, also by a linear technology subject to constant returns to scale. Consequently, the average and the marginal cost of producing y_s are equal to $C_s(e_s)$.

A polluting firm's emission taxes depend on its reported emissions. The firm

[5] Cremer and Gahvari's (2001) results concerning the validity of the Pigouvian rule for emissions, the optimality of Pigouvian (marginal) emission taxes, and the targeting of instruments (emission taxes for Pigouvian considerations and output taxes for optimal tax considerations) under the second best, are derived for *these* preferences.

may thus attempt to lower its tax payments by concealing a fraction, ω_s, of its emissions. It will then pay a tax totaling to $t_s(1-\omega_s)e_s y_s$. The firm, if inspected and found in violation, will also have to pay a fine. Denote the *expected* fine, per unit of emissions, for industry s by f^s.[6] Assume that f^s is an increasing function of ω_s, t_s, and β_s.[7] That is, the expected fine increases as the firm under-reports more, faces a higher statutory tax rate, or as the regulatory control intensifies. We will also assume that $f^s_{\omega\omega} > 0, f^s_{\omega\beta} > 0$ and $f^s_{\omega t} < 1$. In words, f^s_ω increases with ω_s and β_s but that it does not increase ''too much'' with t_s. The first assumption ensures that the firm's profit maximization problem has a maximum; the second assumption ensures that the firm will report more accurately as it is inspected and monitored more often (the outcome one would expect); the third assumption ensures that the firm will under-report more at higher statutory tax rates (again the outcome one would expect).

A representative firm in industry s chooses its emissions and declarations to maximize its expected profits, π^e_s. (For simplicity in notation, we drop the subscript s in the rest of this section). Introduce θ to denote the ''effective'' tax per unit of emissions

$$\theta \equiv t(1-\omega) + f. \tag{5}$$

Expected profits can then be written as

$$\pi^e = [q - C(e) - \tau - \theta e]y. \tag{6}$$

If $y > 0$, Eq. (6) implies that the firm chooses e and ω such that

$$z \equiv C(e) + \theta e, \tag{7}$$

is minimized. Note that z is independent of y. Consequently, the per unit emissions, e, and the fraction of unreported emissions, ω, are also independent of y (provided that $y > 0$). The separability between the level of output, per unit emissions, and evasion decision arises because production costs and fines are assumed proportional to output.

The first-order conditions for this problem are[8]

$$f_\omega(\omega, t, \beta) = t, \tag{8a}$$

$$-C'(e) = \theta. \tag{8b}$$

Note ω is determined by (8a) independently of e indicating that a firm's choice of

[6]The expected fines depend on statutory fines and the intensity of the regulatory control. Our formulation here does not require an explicit accounting of the precise relationships. See Cremer and Gahvari (1992, 1993) for a specific modelling of this.

[7]The functional form of f^s may differ across, or be identical for, all firms.

[8]The second-order sufficient conditions are $f_{\omega\omega} > 0$, and $C''(e) > 0$. These inequalities are satisfied by our assumptions on the properties of $f(.)$ and $C(.)$.

ω and e are *not* interdependent. This follows because in the expression for z, ω appears in θ only, and that e does not enter in the expression for θ.[9] With ω being determined in (8a) as a function of t and β, Eq. (8b) determines e also as a function of t and β. (It is interesting to note that e may be expressed as a function of θ only).

Finally, the zero profit condition in the $\underline{y}$ industries implies that market equilibrium occurs at

$$q = C(e) + \tau + \theta e, \tag{9}$$

where C and θ are evaluated at the firm's optimal value of ω and e.[10] This is because at any other price the firms would want to supply either $y = 0$, or $y = \infty$ and an equilibrium cannot be achieved. The determination of the consumer prices of polluting goods independently of output reflects the separability of decisions pertaining to per unit emissions and evasions on the one hand, and output on the other. Note also that if the price is given by (9), a firm's expected profit is independent of its output. The market output is determined by the demand function.

3. Comparative statics

Tax rates and the regulatory control parameters are policy instruments for the government, but the firms take them as given. This section investigates how the firms and the markets respond to the changes in these instruments, leaving the study of the properties of optimal policy rules to the next section. Specifically, we will examine how the equilibrium values of emissions, unreported emissions, effective emission taxes and prices respond to changes in statutory emission taxes, commodity tax rates and regulatory control parameters.

We begin by stating and proving three lemmas. Lemma 1 establishes that the fundamental separability that exists between the markets with linear technologies continues to hold in our model, given our specifications for the production of polluting goods, and monitoring and enforcement costs. In particular, Lemma 1 implies that whereas the emission and commodity taxes in a given market may affect the equilibrium *quantities* in all markets, they have no impact on the equilibrium *prices* in other markets. The same is true for the regulatory control parameter in a market.

[9] This also implies that θ is being minimized by the choice of ω.

[10] If emissions were publicly observable, the firms will face no uncertainty in their tax payment and Eq. (9) simplifies to

$$q = C(e) + \tau + te.$$

The uncertainty associated with detection implies that the statutory emission tax t is replaced by the expected emission tax θ.

Lemma 1. *$\forall\, l \neq k = 1,2,\ldots,m$, and $i \neq j = 1,2,\ldots,n$, varying t_k,β_k or τ_k does not affect the equilibrium values of (i) e_l,ω_l,θ_l and q_l, in the markets for polluting goods, and (ii) p_i in the markets for nonpolluting goods. Similarly, varying η_i does not affect p_j, and leaves $e_l,\omega_l,\theta_l,q_l$ unaffected.*

Proof. The proof follows from the fact that the expressions for $p_i,\omega_l,\theta_l,e_l$ and q_l are independent of the tax rates and regulatory control parameters in the other markets. To see this, first note that in the market for nonpolluting goods, $p_i = 1 + \eta_i$. Next, consider firms producing polluting goods. There, Eq. (8a) determines ω_l as a function of t_l and β_l. Eqs. (5) and (8b) then determine θ_l and e_l as functions of t_l and β_l also. Finally, Eq. (9) determines q_l as a function of t_l,β_l and τ_l. □

The second lemma establishes that per unit emission levels, emission reports and expected emission taxes in any given market are independent of the commodity tax in that market. These variables depend only on the emission tax and the regulatory control parameter in that market. We have

Lemma 2. *The choice of ω_l and e_l by firms producing y_l, and the resulting value of the expected emission tax faced by these firms, θ_l, are independent of τ_l, $\forall\, l = 1,2,\ldots,m$.*

Proof. The lemma follows from Eqs. (8a,b) and (5) where the values of ω,e and θ are determined independently of τ, the commodity tax. □

Finally, we observe, from $p_i = 1 + \eta_i$, Eq. (9), and Lemmas 1 and 2, that a change in the *commodity tax* levied on any given good will change the consumer price of that good by the full amount of the tax and will have no impact on the prices of other goods. For completeness, we summarize this observation as

Lemma 3. *We have, $\forall\, i,j = 1,2,\ldots,n$, and $l,k = 1,2,\ldots,m$,*

$$\frac{\partial p_i}{\partial \eta_j} = \frac{\partial q_l}{\partial \tau_k} = 1, \quad \text{if } i = j \text{ and } l = k,$$
$$= 0, \quad \text{if } i \neq j \text{ and } l \neq k.$$

We are now in a position to state the following results which are proved in Appendix A. Proposition 1 addresses the question of how the equilibrium is affected if the emission tax rate, or the regulatory control parameter, in one of the markets changes.

Proposition 1. *As t or β increases, ω,e,θ and q change according to:*

$$\frac{\partial\omega}{\partial t}>0 \quad \frac{\partial e}{\partial t}<0 \quad \frac{\partial\theta}{\partial t}>0 \quad \frac{\partial q}{\partial t}>0$$
$$\frac{\partial\omega}{\partial\beta}<0 \quad \frac{\partial e}{\partial\beta}<0 \quad \frac{\partial\theta}{\partial\beta}>0 \quad \frac{\partial q}{\partial\beta}>0.$$

The signs accord with intuition. An increase in the statutory emission tax rate leads to an increase in the proportion of unreported emissions, the effective per unit emission tax and the consumer price of polluting goods, and to a reduction in per unit emissions. Similarly, intensifying the regulatory control, lowers the proportion of unreported emissions and per unit emissions, but it increases the effective per unit emission tax and the consumer price of polluting goods.

4. Optimal emission taxes

If emissions are publicly observable at no cost, the question of under-reporting of emissions does not arise. The government policy then centers around its tax instruments only. With unobservable emissions, inspection strategies become another ingredient in the government's policy tools for welfare maximization. This paper does not discuss such strategies; however. Our study is directed at the implication of the existence of costly monitoring for optimal tax policy. For our purpose, it will suffice to allow the regulatory agency to monitor different firms at varying degrees of intensity. This is the approach we follow.

The government's problem is to choose its tax instruments,[11] $\underline{\eta}$, $\underline{\tau}$, $\underline{t}$, and regulatory control parameters $\underline{\beta}$, to maximize the representative consumer's utility

$$\boldsymbol{V}\big(\boldsymbol{\Phi}(\underline{p},\underline{q},w,T),E\big),$$

subject to its budget constraint

$$\sum_{k=1}^{n}\eta_k x_k(\underline{p},\underline{q},w,T)+\sum_{s=1}^{m}(\tau_s+\theta_s e_s)y_s(\underline{p},\underline{q},w,T)+T-d(\underline{\beta},\underline{t},\underline{\omega})\geq\bar{R}. \tag{10}$$

where $d(.)$ is the resource cost of enforcement, monitoring and fine collections and $\bar{R}$ is the government's external revenue requirement. We assume that the regulatory agency's monitoring costs are an increasing function of the fines to be collected. It thus follows that, as with the expected fine itself, $d(.)$ is an increasing function of $\underline{\omega}$, $\underline{t}$ and $\underline{\beta}$. Note also that we have included T for the generality of exposition; lump-sum taxation is not feasible in the second-best. However, its inclusion has no impact on the characterization of *emission taxes* and *regulatory*

[11] In the Ramsey tax tradition, and without any loss of generality, we set the tax rate on w equal to zero.

control parameters. (See the first-order conditions of the government's problem in Appendix A). Lump-sum taxes will of course affect the structure of optimal commodity taxes. We will take up this issue in the next section.

Define the marginal social damage of emissions by $-V_E/\mu$ where $V_E \equiv \partial \boldsymbol{V}/\partial E < 0$ is the marginal utility of emissions and μ is the Lagrange multiplier associated with the government's budget constraint in its optimization problem. Introduce A_t^j and A_β^j, for all $j = 1,2,\ldots,m$, to denote the regulatory agency's marginal monitoring cost, per unit of emissions, induced by the emission tax and by the regulatory control parameter

$$A_t^j \equiv \frac{\partial d/\partial t_j + (\partial d/\partial \omega_j)(\partial \omega_j/\partial t_j)}{-y_j(\partial e_j/\partial t_j)}, \tag{11a}$$

$$A_\beta^j \equiv \frac{\partial d/\partial \beta_j + (\partial d/\partial \omega_j)(\partial \omega_j/\partial \beta_j)}{-y_j(\partial e_j/\partial \beta_j)}. \tag{11b}$$

Optimal emissions in industries $j = 1,2,\ldots,m$, are then characterized by the following rules proved in Appendix A (using the first-order conditions of the government's problem),

$$-C_j'(e_j) = -V_E/\mu - A_t^j, \tag{12a}$$

$$-C_j'(e_j) = -V_E/\mu - A_\beta^j. \tag{12b}$$

Eq. (12a) characterizes the optimal emission rule when unobservable emissions are controlled through an emission tax. Its left-hand side shows the marginal cost of *reducing* emissions to a firm j producing y_j (through an *increase* in the tax rate). The firm loses $-C_j'(e_j)$ in the form of an increase in its unit cost of production. The right-hand side consists of two terms. The first term, $-V_E/\mu$, is the marginal social damage of emissions. This is what the society gains as emissions are cut. The second term, A_t^j, is, as defined above, the regulatory agency's marginal monitoring cost (induced by the emission tax and expressed per unit of emissions). It is an additional source of social cost due to enforcement necessitated by the unobservability of emissions. Note that, from the signs of its components, $A_t^j > 0$. This means that in balancing the marginal costs and benefits of emissions, A_t^j works as a partial offset to the benefit of cutting the emissions. The rule makes perfect sense as, at the optimum, one wants to equate the net costs and benefits of marginal emissions. Eq. (12b) has a similar interpretation. It also indicates that at the optimum, net marginal benefit and cost of emissions are equalized. In this case though, emissions are controlled through the regulatory control parameter.[12]

Conditions (12a,b) have two important implications. First, they replace the Pigouvian rule condition (equating the net marginal private benefit of a firm's emission, $-C_j'$, to the marginal social damage of emissions, $-V_E/\mu$), that Cremer and Gahvari (2001) found assuming emissions are perfectly observable. Imperfect observability thus modifies the Pigouvian rule. The modification is in terms of an additional cost to the society due to the regulatory agency's monitoring costs as shown by A_t^j and A_β^j.

The second implication follows from the fact that in general A_t^j and A_β^j vary with j. To see this, rewrite these expressions as,[13]

$$A_t^j = \frac{C_j''\left[(\partial d/\partial t_j) f^j_{\omega\omega} + (\partial d/\partial \omega_j)(1 - f^j_{\omega t})\right]}{y_j f^j_{\omega\omega}(1 - \omega_j + f^j_t)}, \tag{13a}$$

$$A_\beta^j = \frac{C_j''\left[(\partial d/\partial \beta_j) f^j_{\omega\omega} - (\partial d/\partial \omega_j) f^j_{\omega\beta}\right]}{y_j f^j_{\omega\omega} f^j_\beta}. \tag{13b}$$

Thus, in the absence of perfect symmetry between different sectors, $A_t^j \neq A_t^s$ and $A_\beta^j \neq A_\beta^s$, for all $j \neq s = 1,2,\ldots,m$. In turn, this implies that

$$-C_j'(e_j) \neq -C_s'(e_s), j \neq s = 1,2,\ldots,m.$$

This tells us that the effect of a marginal increase in emissions on the unit cost of production varies across industries. That is, with imperfect observability of emissions, the production efficiency rule in the firms' usage of polluting inputs no longer holds. As with the Pigouvian rule, this is in sharp contrast with the result under perfect observability derived by Cremer and Gahvari (2001). The intuition for this result is straightforward. Observe from conditions (12a,b) that at the optimum marginal private benefits of emissions, $-C_j'(e_j)$'s, *net* of monitoring

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[12] Conditions (12a,b) characterizing the modified Pigouvian Rule, easily generalize to models with heterogeneous individuals. Denote household types by $j=1,2,\ldots,H$. One can then show, in the context of a many consumer Ramsey tax problem, or an optimal tax problem à la Mirrlees (1971) as in Cremer and Gahvari (2001), that if output taxes are unrestricted,

$$-C_j'(e_j) = -\sum_{h=1}^{H} \rho^h V_E^h/\mu - A_t^j,$$
$$-C_j'(e_j) = -\sum_{h=1}^{H} \rho^h V_E^h/\mu - A_\beta^j,$$

where ρ^h is the weight assigned to an h-type individual in the social welfare function, and where $\Sigma_{h=1}^H y_j^h$ replaces y_j in the definitions of A_t^j and A_β^j, with y_j^h being the consumption of y_j by type h.

[13] Substitute for $\partial\omega_j/\partial t_j, \partial e_j/\partial t_j, \partial\omega_j/\partial\beta_j$ and $\partial e_j/\partial\beta_j$, from Eqs. (A.1)–(A.2) and (A.5)–(A.6) derived in Appendix A, into (11a,b).

costs, A_t^j's or A_β^j's, are equalized across firms (being set equal to the marginal social damage of emissions, $-V_E/\mu$). But since monitoring costs differ across firms, marginal private benefits of emissions exclusive of monitoring costs are not equalized. Note also that production efficiency is not restored even if all firms were to face identical $f(.)$ functions and if $d(.)$ were symmetric in terms of parameters pertaining to different industries. As long as different firms have different per unit emission cost functions, $C_j(e_j)$'s, they will end up with different emission levels, e_j's, and thus different marginal monitoring costs.

Conditions (12a,b) also allow a simple characterization of effective and statutory emission taxes. Recall from Eq. (8b) that firm j sets $-C_j'(e_j) = \theta_j$. This, along with the definition of θ in (5), then allow us to write, for all $j = 1,2,\ldots,m$,[14]

$$\theta_j = -V_E/\mu - A_t^j, \tag{14a}$$

$$t_j = \frac{1}{1-\omega_j}\left(-V_E/\mu - A_t^j - f^j\right). \tag{14b}$$

Eqs. (14a,b) are quite telling. The first important implication of these equations is that different industries should face different effective and statutory emission taxes. This follows because, as we noted earlier, A_t^j's will generally differ across industries. This result is in sharp contrast with the observable emission case where all industries will face identical (marginal) emission taxes.[15]

Secondly, because $A_t^j > 0$, Eq. (14a) indicates that every polluting good must face an effective emission tax that is less than the full marginal social damage of emissions. This is also in contrast with the observable emissions case where the emission tax is equal to the marginal social damage (see Cremer and Gahvari, 2001). It is apparent that because of the welfare cost associated with monitoring, the emissions taxes must be set at levels which fall short of the full social marginal damage of emissions. Put differently, the existence of monitoring costs implies that one should cut emissions by less than what one would otherwise do. Note, however, that the statutory emission taxes are not necessarily less than the marginal social damage and may in fact exceed it. This conclusion follows immediately from Eq. (14b).

Third, note that optimal tax terms do not appear in Eqs. (14a,b). Emission taxes in different industries (statutory or effective) are determined on the basis of the

[14] Again, in the context of heterogeneous individuals, we will have for all $j = 1,2,\ldots H$,

$$\theta_j = -\sum_{h=1}^{H} \rho^h V_E^h/\mu - A_t^j,$$

with the same definitions as in footnote 12.

[15] Smulders and Vollebergh (1999) also have shown that optimal emission taxes must be modified to take account of administrative and enforcement costs.

social marginal damage of emissions and monitoring costs. The usual optimal tax objectives play no role here. Simply put, emission taxes reflect Pigouvian considerations and monitoring costs, but not optimal tax objectives.[16]

Finally, we turn to the question of the tradeoff between the statutory emission tax and the regulatory control parameter in determining the effective emission tax. First note that from Eqs. (12a,b), and the definitions of A_t^j and A_β^j, we have

$$\frac{\partial d/\partial t_j + (\partial d/\partial \omega_j)(\partial \omega_j/\partial t_j)}{-\partial e_j/\partial t_j} = \frac{\partial d/\partial \beta_j + (\partial d/\partial \omega_j)(\partial \omega_j/\partial \beta_j)}{-\partial e_j/\partial \beta_j}. \tag{15}$$

The left-hand side of (15) indicates marginal monitoring costs induced by a change in t_j. The right-hand side indicates the corresponding marginal monitoring costs induced by β_j. Eq. (15) shows that the marginal costs from both sources must be the same at the optimum.

Condition (15) can be rewritten in another informative way as

$$\frac{\partial e_j/\partial \beta_j}{\partial e_j/\partial t_j} = \frac{\partial d/\partial \beta_j + (\partial d/\partial \omega_j)(\partial \omega_j/\partial \beta_j)}{\partial d/\partial t_j + (\partial d/\partial \omega_j)(\partial \omega_j/\partial t_j)}. \tag{16}$$

This condition characterizes the fundamental tradeoff between the emission tax and the regulatory control parameter in industry j. The left-hand side of (16) is the rate of substitution between β_j and t_j such that e_j, and hence the marginal (private) benefit of emissions (by way of reducing the unit cost of production) and the marginal social damage of emissions remain constant. The right-hand side, on the other hand, is the rate of substitution between β_j and t_j such that the resource cost of evasion (monitoring cost) remains constant. Eq. (16) states that at the optimum these two rates of substitution must be equal.

We summarize the results of this section as

Proposition 2. *Assume emissions are not perfectly observable. Then*:

(i) *The Pigouvian rule of equating the net marginal private benefit of a firm's emission to the marginal social damage of emissions is modified to take account of*

[16] This result depends on the assumption that output taxes are not evaded. If they are, emission taxes may include optimal tax terms. To see this most clearly, consider the extreme case where monitoring costs of output taxes are so high that no output taxes are levied. It is plain that in this case, emission taxes would have to assume the role of output taxes. Such added complications will then lead to an even more serious breakdown of the principle of targeting.

the resource costs of monitoring and enforcement. The adjustment term is given by A_t^j *or* A_β^j *as defined by* (11a,b).

(ii) *The production efficiency rule in the firms' usage of polluting inputs does not hold.*

(iii) *Emissions from different sectors must be taxed at different (effective and statutory) rates. The taxes are characterized by* (14a,b).

(iv) *Every polluting good must face an effective emission tax that is less than the full marginal social damage of emissions. Statutory emission taxes, on the other hand, may fall short of as well as exceed the marginal social damage of emissions.*

(v) *The emission tax formulas reflect Pigouvian considerations and monitoring costs; they do not include optimal tax terms.*

(vi) *A given effective emission tax can be achieved through different mixes of the statutory emission tax and the regulatory control parameter. The tradeoff between the two instruments is governed by* Eq. (16).

5. Optimal output taxes

Denote the marginal utility of lump-sum income by α, the compensated demand functions for nonpolluting and polluting goods by x_k^c $(k=1,2,\ldots,n)$ and y_s^c $(s=1,2,\ldots,m)$. In Appendix A, we show that optimal output taxes are characterized by

$$-\frac{1}{x_i}\left\{\sum_k \eta_k \frac{\partial x_k^c}{\partial p_i} + \sum_s \left[\left(\frac{V_E}{\mu}+\theta_s\right)e_s + \tau_s\right]\frac{\partial y_s^c}{\partial p_i}\right\} = \Delta, \tag{17a}$$

$$-\frac{1}{y_j}\left\{\sum_k \eta_k \frac{\partial x_k^c}{\partial q_j} + \sum_s \left[\left(\frac{V_E}{\mu}+\theta_s\right)e_s + \tau_s\right]\frac{\partial y_s^c}{\partial q_j}\right\} = \Delta, \tag{17b}$$

where

$$\Delta \equiv 1 - \frac{\alpha}{\mu} + \sum_k \eta_k \frac{\partial x_k}{\partial T} + \sum_s \left[\left(\frac{V_E}{\mu}+\theta_s\right)e_s + \tau_s\right]\frac{\partial y_s}{\partial T}. \tag{18}$$

Eqs. (17a,b) are, with one exception, identical to Ramsey equations in the traditional optimal tax problem without the externality. The difference is that the tax term for the polluting good in the traditional problem is replaced here by the expression $(V_E/\mu + \theta_s)e_s + \tau_s$. Consequently, if θ_s were to be "set" equal to

$-V_E/\mu$, then the commodity taxes would have precisely the same properties as the traditional Ramsey taxes.[17]

When emissions are perfectly observable, $\theta_s = -V_E/\mu$ and we are back to the Ramsey taxes. With imperfect observability of emissions, on the other hand, this will not be the case. As we showed in the previous section, optimality will now call for $\theta_s < -V_E/\mu$ so that output taxes will no longer be purely ''Ramsian''. They will also include ''Pigouvian'' terms despite the existence of emissions taxes. The so-called targeting principle, the dichotomy of setting emissions taxes to correct for externalities and commodity taxes for optimal tax considerations, thus breaks down.

At this level of generality, it will be impossible to derive general lessons regarding the direction of the Pigouvian adjustment in output taxes. Consequently, to shed light on this problem, we will consider two special cases.

5.1. Lump-sum taxation

The first special case we consider is when lump sum taxation is feasible. In the traditional Ramsey problem, in the presence of lump sum taxes, commodity taxes are not needed. With externalities, but full observability of emissions, first-best taxation requires that the government levies only an emission tax equal to marginal social damage of emissions, $-V_E/\mu$, and a lump-sum tax. Output taxes are again not needed. This picture changes drastically when observability of emissions is imperfect, as we will show next.

Appendix A shows that with lump-sum taxation Δ, defined by (18), will be reduced to zero. In turn, Eqs. (17a,b) would then imply that optimal commodity taxes are equal to

$$\eta_i = 0, i = 1,2,\ldots,n, \tag{19}$$

[17] The generalization of Eqs. (17a,b) to the many-consumer Ramsey tax problem is straightforward and follows the same pattern as the one-consumer case. Equations characterizing the optimum commodity taxes will in this case be identical to the traditional many-consumer Ramsey tax equations except that the tax term for the polluting good is now replaced by the expression $(\Sigma_{h=1}^{H}\rho^h V_E^h/\mu + \theta_s)e_s + \tau_s$. Hence if one sets θ_s equal to $-\Sigma_{h=1}^{H}\rho^h V_E^h/\mu$, then τ_s will have an identical characterization to the traditional many-consumer Ramsey tax. Note also that if we allow for a uniform lump-sum tax, and define γ^h to denote the marginal social utility of income for an h-type person (as is customarily done in that literature), we will have the traditional result that at the optimum $\mu = \Sigma_{h=1}^{H}\gamma^h/H$. However, γ^h will now also include the marginal social damage of emissions. That is,

$$\gamma^h \equiv \rho^h\alpha^h - \mu\left\{\sum_k \eta_k \frac{\partial x_k^h}{\partial T} + \sum_s\left[\left(\sum_{h=1}^{H}\rho^h V_E^h/\mu + \theta_s\right)e_s + \tau_s\right]\frac{\partial y_s^h}{\partial T}\right\}, h = 1,2,\ldots,H.$$

To sum, output tax formulas will now have two components: the traditional many-consumer Ramsey tax formulas and a Pigouvian-cum-monitoring adjustment term. The redistributive properties of polluting good taxes are then captured by covariance terms in the usual way.

$$\tau_j = (-V_E/\mu - \theta_j)e_j > 0, j = 1,2,\ldots,m. \tag{20}$$

Consequently, even in the presence of lump-sum taxes, emission taxes are no longer sufficient to correct for externalities. They must be supplemented with *positive* output taxes. To see the intuition, note that when emissions are observable, the only source of distortion is the externalities. An emission tax is then sufficient to correct this. (Lump-sum taxes can then be utilized to raise any revenue requirement). With imperfect observability, we will have two sources of distortion: externalities and the resource costs associated with monitoring. Emission taxes alone are not sufficient to correct for both. Moreover, as we observed earlier, it is not optimal to set the emissions tax equal to the full marginal social damage of emissions. Because of these, part of the Pigouvian-cum-monitoring correction is passed on to output taxes (on polluting goods).

5.2. Uniform taxation

Rather than extending the set of tax instruments to include lump-sum taxes, let us now restrict the consumers' preference structure. Specifically, assume preferences are weakly separable in labor supply (L) and the consumption goods ($\underline{x}$, $\underline{y}$), with the consumption good subutility being homothetic. In the context of the traditional Ramsey tax problem without externality, Sandmo (1974) has shown that these assumptions imply commodity taxes are uniform. In Appendix A, we show that the same assumption in our setting implies the following structure for optimal commodity taxes

$$\frac{\eta_i}{p_i} = \frac{\tau_j - (-V_E/\mu - \theta_j)e_j}{q_j} = \eta, \quad i = 1,2,\ldots,n, \text{ and } j = 1,2,\ldots,m, \tag{21}$$

where η is some constant independently of i and j.

This result is very interesting. It tells us that all nonpolluting goods must be taxed at some uniform rate η. On the other hand, the commodity tax rates on polluting goods are non-uniform. Moreover, the tax rates deviate from the uniform rate of η according to

$$\frac{\tau_j}{q_j} - \eta = \frac{(-V_E/\mu - \theta_j)e_j}{q_j}. \tag{22}$$

Now because $-V_E/\mu - \theta_j > 0$ (for all j), we must have

$$\frac{\tau_j}{q_j} > \eta. \tag{23}$$

Inequality (23) tells us that the commodity tax rate on polluting goods must be adjusted upwards, in relation to the tax rate on private goods, to correct for pollution. The intuition for this is similar to the case with lump-sum taxation. With imperfect observability of emissions, we have an additional source of distortion due to monitoring. Consequently, emission taxes cannot correct both types of distortions at once. Part of the Pigouvian-cum-monitoring correction is passed on to output taxes (on polluting goods).

Finally, note that it also follows from (21) that the tax rate differential between two polluting goods y_j and y_s is governed by two factors: the polluting characteristic of y_j relative to y_s (i.e., e_j versus e_s), and the "uncorrected" portion of the marginal social damage of emissions in the two industries (i.e., $-V_E/\mu - \theta_j$ versus $-V_E/\mu - \theta_s$). The higher is either of these factors, everything else being equal, the higher should be the commodity tax rate on j relative to s.

We summarize the results of this section as

Proposition 3. *Assume emissions are not perfectly observable and that emission taxes are set optimally. Then*:

(i) *Output taxes must supplement emission taxes to correct for pollution*; *the principle of targeting of tax instruments breaks down.*

(ii) *Output taxes are required even in the presence of lump-sum taxes. In this case, output taxes are necessarily positive.*

(iii) *If preferences are weakly separable in labor supply and consumption goods, with the consumption goods subutility being homothetic, we will have*:

(a) *All nonpolluting goods must be taxed at a uniform proportionate rate*; *while polluting goods are taxed non-uniformly at rates that exceed the uniform tax rate on nonpolluting goods.*

(b) *Everything else being equal, one has to levy a higher commodity tax on a polluting good that entails a high per unit emission rate, or has a low emission tax levied on it.*

6. Concluding remarks

Cremer and Gahvari (2001) had recently shown that when emissions are publicly observable the Pigouvian rule of equating the marginal private benefit of emissions in production to the marginal social damage of emissions holds in the second-best as long as there are no complementarity and/or substitutability relationships between emissions and consumption goods. They had also shown that under this circumstance, tax instruments must be targeted to achieve different objectives: A proportional emission tax corrects the impact of externalities, while output (and income) tax instruments are determined solely on the basis of optimal tax objectives. The current paper has reexamined these conclusions in a framework where emissions are publicly unobservable, but can be discovered through costly inspection and monitoring. It has shown that the unobservability problem has far reaching implications for these results. Specifically, the Pigouvian rule must be modified to take account of the resource costs of monitoring and enforcement; emissions from different sources must be taxed at different (statutory and effective) rates depending on their emissions and the associated monitoring costs; the presence of monitoring costs imply that effective emission taxes are less than the full marginal social damage of emissions; emission taxes are set solely on the basis of Pigouvian considerations and monitoring costs; the principle of targeting of instruments fails and output taxes must supplement emission taxes to correct for pollution (even in the presence of lump-sum taxes).

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Appendix A

Proof of Proposition 1: Differentiate Eqs. (8a), (8b), (5) and (9) with respect to t, and β, and simplify. The resulting equations are reported below; they have the claimed signs.

$$\frac{\partial \omega}{\partial t} = \frac{1 - f_{\omega t}}{f_{\omega\omega}}, \tag{A.1}$$

$$\frac{\partial e}{\partial t} = -\frac{(1-\omega) + f_t}{C''}, \tag{A.2}$$

$$\frac{\partial \theta}{\partial t} = (1-\omega) + f_t, \tag{A.3}$$

$$\frac{\partial q}{\partial t} = [(1-\omega) + f_t]\, e, \tag{A.4}$$

$$\frac{\partial \omega}{\partial \beta} = -\frac{f_{\omega\beta}}{f_{\omega\omega}}, \tag{A.5}$$

$$\frac{\partial e}{\partial \beta} = -\frac{f_\beta}{C''}, \tag{A.6}$$

$$\frac{\partial \theta}{\partial \beta} = f_\beta, \tag{A.7}$$

$$\frac{\partial q}{\partial \beta} = f_\beta e. \tag{A.8}$$

The optimal tax problem: Form the Lagrangian

$$\Lambda = \boldsymbol{V}(\boldsymbol{\Phi}(\underline{p},\underline{q},w,T), E) + \mu\Bigg[\sum_{k=1}^{n}\eta_k x_k(\underline{p},\underline{q},w,T)$$
$$+\sum_{s=1}^{m}(\tau_s + \theta_s e_s)y_s(\underline{p},\underline{q},w,T) + T - d(\underline{\omega},\underline{t},\underline{\beta}) - \bar{R}\Bigg]. \tag{A.9}$$

The first-order conditions for this problem are, for all $i = 1,2,\ldots,n$ and $j = 1,2,\ldots,m$,

$$\frac{\partial \Lambda}{\partial \eta_i} = \frac{\partial V}{\partial p_i} + \left(\sum_s e_s \frac{\partial y_s}{\partial p_i}\right)V_E + \mu\left[x_i + \sum_k \eta_k \frac{\partial x_k}{\partial p_i} + \sum_s (\tau_s + \theta_s e_s)\frac{\partial y_s}{\partial p_i}\right]$$
$$= 0, \tag{A.10}$$

$$\frac{\partial \Lambda}{\partial \tau_j} = \frac{\partial V}{\partial q_j} + \left(\sum_s e_s \frac{\partial y_s}{\partial q_j}\right)V_E + \mu\left[y_j + \sum_k \eta_k \frac{\partial x_k}{\partial q_j} + \sum_s (\tau_s + \theta_s e_s)\frac{\partial y_s}{\partial q_j}\right]$$
$$= 0, \tag{A.11}$$

$$\frac{\partial \Lambda}{\partial t_j} = \left\{\frac{\partial V}{\partial q_j} + \left(\sum_s e_s \frac{\partial y_s}{\partial q_j}\right)V_E + \mu\left[\sum_k \eta_k \frac{\partial x_k}{\partial q_j} + \sum_s (\tau_s + \theta_s e_s)\frac{\partial y_s}{\partial q_j}\right]\right\}\frac{\partial q_j}{\partial t_j}$$
$$+\left(\sum_s y_s \frac{\partial e_s}{\partial t_j}\right)V_E + \mu\left[\sum_s\left(e_s\frac{\partial \theta_s}{\partial t_j} + \theta_s\frac{\partial e_s}{\partial t_j}\right)y_s - \left(\frac{\partial d}{\partial t_j} + \frac{\partial d}{\partial \omega_j}\frac{\partial \omega_j}{\partial t_j}\right)\right]$$
$$= 0, \tag{A.12}$$

$$\frac{\partial \Lambda}{\partial \beta_j} = \left\{\frac{\partial V}{\partial q_j} + \left(\sum_s e_s \frac{\partial y_s}{\partial q_j}\right)V_E + \mu\left[\sum_k \eta_k \frac{\partial x_k}{\partial q_j} + \sum_s (\tau_s + \theta_s e_s)\frac{\partial y_s}{\partial q_j}\right]\right\}\frac{\partial q_j}{\partial \beta_j}$$
$$+\left(\sum_s y_s \frac{\partial e_s}{\partial \beta_j}\right)V_E + \mu\left[\sum_s\left(e_s\frac{\partial \theta_s}{\partial \beta_j} + \theta_s\frac{\partial e_s}{\partial \beta_j}\right)y_s - \left(\frac{\partial d}{\partial \beta_j} + \frac{\partial d}{\partial \omega_j}\frac{\partial \omega_j}{\partial \beta_j}\right)\right]$$
$$= 0. \tag{A.13}$$

In addition, whenever T is a feasible instrument, the first-order conditions will be supplemented with

$$\frac{\partial \Lambda}{\partial T} = \frac{\partial V}{\partial T} + \left(\sum_s e_s \frac{\partial y_s}{\partial T}\right)V_E + \mu\left[1 + \sum_k \eta_k \frac{\partial x_k}{\partial T} + \sum_s (\tau_s + \theta_s e_s)\frac{\partial y_s}{\partial T}\right]$$
$$= 0. \tag{A.14}$$

Derivation of (12a) and (12b): Using (A.11), thus assuming the commodity taxes on $\underline{y}$ are unrestricted, and Lemma 1, Eqs. (A.12)–(A.13) for determining t_j and β_j will be simplified to, for all $j = 1,2,\ldots,m$,

$$\left(\frac{V_E}{\mu} + \theta_j\right)\frac{\partial e_j}{\partial t_j} + e_j\frac{\partial \theta_j}{\partial t_j} - \frac{\partial q_j}{\partial t_j} - \frac{1}{y_j}\left(\frac{\partial d}{\partial t_j} + \frac{\partial d}{\partial \omega_j}\frac{\partial \omega_j}{\partial t_j}\right) = 0, \tag{A.15}$$

$$\left(\frac{V_E}{\mu}+\theta_j\right)\frac{\partial e_j}{\partial \beta_j}+e_j\frac{\partial \theta_j}{\partial \beta_j}-\frac{\partial q_j}{\partial \beta_j}-\frac{1}{y_j}\left(\frac{\partial d}{\partial \beta_j}+\frac{\partial d}{\partial \omega_j}\frac{\partial \omega_j}{\partial \beta_j}\right)=0. \tag{A.16}$$

Next differentiate Eq. (9) partially with respect to t_j and β_j. We will have

$$\frac{\partial q_j}{\partial t_j}=C_j'\frac{\partial e_j}{\partial t_j}+e_j\frac{\partial \theta_j}{\partial t_j}+\theta_j\frac{\partial e_j}{\partial t_j}, \tag{A.17}$$

$$\frac{\partial q_j}{\partial \beta_j}=C_j'\frac{\partial e_j}{\partial \beta_j}+e_j\frac{\partial \theta_j}{\partial \beta_j}+\theta_j\frac{\partial e_j}{\partial \beta_j}. \tag{A.18}$$

Substituting for $\partial q_j/\partial t_j$ from (A.17) into (A.15) and simplifying yield (12a). Similarly, substituting for $\partial q_j/\partial \beta_j$ from (A.18) into (A.16) and simplifying yield (12b).

Derivation of (17a,b): Using Roy's identity, one may rewrite Eqs. (A.10)–(A.11) for determining η_i and τ_j, for all $i=1,2,\ldots,n$ and $j=1,2,\ldots,m$, as

$$\left(1-\frac{\alpha}{\mu}\right)x_i+\sum_k \eta_k\frac{\partial x_k}{\partial p_i}+\sum_s\left[\left(\frac{V_E}{\mu}+\theta_s\right)e_s+\tau_s\right]\frac{\partial y_s}{\partial p_i}=0, \tag{A.19}$$

$$\left(1-\frac{\alpha}{\mu}\right)y_j+\sum_k \eta_k\frac{\partial x_k}{\partial q_j}+\sum_s\left[\left(\frac{V_E}{\mu}+\theta_s\right)e_s+\tau_s\right]\frac{\partial y_s}{\partial q_j}=0. \tag{A.20}$$

Applying Slutsky decompositions to Eqs. (A.19)–(A.20), one will get Eqs. (17a,b) in the text.

Lump-sum taxation: Simplifying (A.14) yields,

$$\Delta\equiv(1-\frac{\alpha}{\mu})+\sum_k \eta_k\frac{\partial x_k}{\partial T}+\sum_s\left[\left(\frac{V_E}{\mu}+\theta_s\right)e_s+\tau_s\right]\frac{\partial y_s}{\partial T}=0. \tag{A.21}$$

Uniform taxation: Differentiate the individual's budget constraint (3) partially with respect to p_i and q_j:

$$x_i+\sum_k p_k\frac{\partial x_k}{\partial p_i}+\sum_s q_s\frac{\partial y_s}{\partial p_i}=w\frac{\partial L}{\partial p_i}, i=1,2,\ldots,n, \tag{A.22}$$

$$y_j+\sum_k p_k\frac{\partial x_k}{\partial q_j}+\sum_s q_s\frac{\partial y_s}{\partial q_j}=w\frac{\partial L}{\partial q_j}, j=1,2,\ldots,m. \tag{A.23}$$

Assume $\boldsymbol{U}$ is weakly separable in L and the consumption goods $(\underline{x},\underline{y})$, with the subutility in $(\underline{x},\underline{y})$ being homothetic. Sandmo (1974) has shown that these assumptions imply

$$\frac{\partial L}{\partial p_i}=\gamma x_i, i=1,2,\ldots,n, \tag{A.24}$$

$$\frac{\partial L}{\partial q_j} = \gamma y_j, j = 1,2, \ldots ,m. \tag{A.25}$$

where γ is some function of the arguments in the utility function $\boldsymbol{U}$. Substituting from (A.24)–(A.25) into (A.22)–(A.23), we will then obtain

$$\sum_k p_k \frac{\partial x_k}{\partial p_i} + \sum_s q_s \frac{\partial y_s}{\partial p_i} = (w\gamma - 1)x_i, i = 1,2, \ldots ,n, \tag{A.26}$$

$$\sum_k p_k \frac{\partial x_k}{\partial q_j} + \sum_s q_s \frac{\partial y_s}{\partial q_j} = (w\gamma - 1)y_j, j = 1,2, \ldots ,m. \tag{A.27}$$

Next substitute for x_i from (A.26) into (A.19) and for y_j from (A.27) into (A.20) and simplify. We will have, for all $i = 1,2, \ldots ,n$, and $j = 1,2, \ldots ,m$,

$$\sum_k \left[\frac{1 - \alpha/\mu}{1 - w\gamma} p_k - \eta_k \right] \frac{\partial x_k}{\partial p_i} + \sum_s \left[\frac{1 - \alpha/\mu}{1 - w\gamma} q_s - (\frac{V_E}{\mu} + \theta_s) e_s - \tau_s \right] \frac{\partial y_s}{\partial p_i} = 0, \tag{A.28}$$

$$\sum_k \left[\frac{1 - \alpha/\mu}{1 - w\gamma} p_k - \eta_k \right] \frac{\partial x_k}{\partial q_j} + \sum_s \left[\frac{1 - \alpha/\mu}{1 - w\gamma} q_s - (\frac{V_E}{\mu} + \theta_s) e_s - \tau_s \right] \frac{\partial y_s}{\partial q_j} = 0. \tag{A.29}$$

Assuming the matrix associated with (A.28)–(A.29) is non-singular, the solution to Eqs. (A.28)–(A.29) is given by, for all $i = 1,2, \ldots ,n$, and $j = 1,2, \ldots ,m$,

$$\frac{\eta_i}{p_i} = \frac{(V_E/\mu + \theta_j)e_j + \tau_j}{q_s} = \frac{1 - \alpha/\mu}{1 - w\gamma}. \tag{A.30}$$

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