Optimal Environmental Standards Under Asymmetric Information and Imperfect Enforcement*

Carmen Arguedas

Universidad Europea de Madrid

December 13, 2004

Abstract

We study optimal policies composed of pollution standards, probabilities of inspection and fines dependant on the degree of noncompliance with the standards, in a context where regulated firms own private information. In contrast with previous literature, we show that optimal policies, being either pooling or separating, can imply violations to strictly positive standards. This result crucially depends on the monitoring costs, the types of firms and the regulator's degree of uncertainty. Somewhat surprisingly, separating policies are not always socially preferred.

JEL classification: D82, K32, K42, L51.

Key words: standard-setting, costly inspections, convex fines, asymmetric information, noncompliance.

*Address for correspondence: Departamento de Economía, Universidad Europea de Madrid. Calle Tajo, s/n, 28670 Villaviciosa de Odón, Madrid, Spain. Tel. (+34) 91-211-5645. Fax. (+34) 91-616-8265. E-mail: mcarmen.arguedas@uem.es.

1 Introduction

Very often, environmental regulations require that firms comply with recommended pollution limits or *standards*. However, regulators do not normally have perfect information about the polluting firms, either *ex-ante* or *ex-post*. *Ex-ante* concerns standard setting. Regulators are less informed than firms about their technological characteristics, and they implement mechanisms to elicit private information.[1] *Ex-post* refers to the behavior of firms in response to the standards already in place. Here, regulators do not observe the performance of firms unless they engage in costly monitoring. Therefore, they design *enforcement policies* composed of inspection frequencies and sanctions in case firms are discovered exceeding the standards.[2] Depending on the monitoring costs, the standards to be enforced and the information authorities own about the regulated firms, enforcement may be imperfect, that is, some firms may find it profitable to violate environmental standards.

Recently, Arguedas and Hamoudi (2004) and Arguedas (2004) have studied the characteristics of optimal policies composed of pollution standards, probabilities of inspection and fines, under perfect information *ex-ante* and imperfect enforcement. There, it is shown that the optimal policy can induce noncompliance only to zero standards, which occurs when fines are continuous and strictly convex in the degree of violation. Thus, it is not possible to explain firms' violations to strictly positive standards, although they are rather frequent in practise.

[1] For example, under the US National Pollutant Discharge Elimination System (NPDES) Program, the Environmental Protection Agency (EPA) issues individual permits to facilities which discharge pollutants into waters of the US, based on reported information about their pollution control processes.

[2] The Civil Penalty Policy of the Clean Water Act establishes the factors that the EPA should consider when imposing sanctions for noncompliance. Among them, the degree of noncompliance is a key gravity factor.

In this paper, we show that the *ex-ante* informational constraint plays a key role in the results. We consider a firm that owns private information about its production process. The firm is first asked to report that information and, contingent on it, the regulator then sets the optimal policy considering the firm's pollution level in response to the policy. By the *revelation principle*, we can restrict attention to direct mechanisms which induce the firm to reveal its true characteristics. That is, we can concentrate on the subset of *incentive-compatible* policies.[3]

We consider a model of two types, namely *clean* and *dirty*, based on the firm's induced pollution costs in response to a given policy. In principle, the policy can induce either full compliance with the standards, partial compliance (where only the clean type complies) or full noncompliance. If the policy induces full compliance, we find that it can be only pooling, that is, the same policy for both types. However, if the policy induces some noncompliance, it can be either pooling or separating.

In the case of a pooling policy, we show that policies that induce some compliance are never optimal whenever the full noncompliance policy exists. The explanation of the result is similar to the one under perfect information. That is, considering a policy that induces compliance, it is always worth to infinitesimally decrease the probability of inspection because the savings in monitoring costs are larger than the decrease in welfare associated with both types' larger pollution levels. In the complete information case, a positive standard is never

[3] Our approach differs from that in which, given the standard, the firm reports its emission level with the possibility of under-reporting, such as in Sandmo (2002). In our case, we have an added *ex-ante* informational asymmetry and, since we analyze standards optimality, we can restrict ourselves to incentive compatible policies. Also, once emissions have been released, we assume that they are measured through costly monitoring.

optimal, since the standard and the probability of inspection can be reduced at the same time keeping the firm's induced pollution level unchanged and reducing monitoring costs.[4] However, a zero standard under incomplete information may result in over-enforcement of the clean type, with the corresponding negative effect on social welfare. This result is relevant and provides an additional explanation to the literature in favor of nonmaximal fines.[5]

In fact, we find violations to strictly positive standards under large regulator's uncertainty and intermediate values of clean type's profitability. Also, the result is more likely under low monitoring costs. The explanation is very intuitive, since there exists a trade-off between enforcement costs and clean type's overenforcement. Given clean type's profitability, the larger the monitoring costs, the larger the enforcement costs. By contrast, given a level of the monitoring costs, the larger clean's type profitability, the smaller the over-enforcement problem. However, if clean type's profitability is very small, an interior solution may not exist, since the full noncompliance region in this case may be very small. Finally, when uncertainty decreases (in favor of any of the types) , the solution approximates to the complete information solution, which implies a zero standard regardless of the type.

If the policy is separating, both the standard and the probability of inspection are smaller for the dirty type, to preserve incentive compatibility.[6] Also,

[4]This is similar to Becker (1968)'s well known result of imposing maximal fines to keep enforcement costs at the minimum. Given a pollution level and a structure of fines dependant on the degree of noncompliance, a lower standard increases the fine for noncompliance and, therefore, it is possible to decrease the probability of inspection, then saving monitoring costs.

[5]After Becker (1968), several papers in the crime context have explained the reasons why fines are not maximal, such as risk aversion (Polinsky and Shavell (1979)), imperfect information about the regulatory policy (Bebchuk and Kaplow (1991), Kaplow (1990)), differences in wealth (Polinsky and Shavell (1991)) or marginal deterrence (Andreoni (1991), Shavell (1992), Heyes (1996)). In all these papers, however, standards are exogenous.

[6]In the tax evasion literature, the optimal inspection probability is a decreasing function of reported income. For example, see Reinganum and Wilde (1985).

the dirty type always finds it profitable to violate the standard, which again can be positive under low monitoring costs. However, as opposed to the pooling case, we now find that it is more likely to find this result when both clean type's profitability and likelihood are large, since in these two cases, the solution approximates to the clean type's compliance solution.

Finally, we provide some numerical examples which suggest that a pooling policy might be socially preferred to a separating policy.

The literature on standards and enforcement issues started with Downing and Watson (1974) and it is vast nowadays (Heyes (2000) provides a comprehensive survey in the environmental context). However, the approach we consider here is novel, namely combining standard-setting, endogenous imperfect enforcement and asymmetric information. This allows us to obtain violations to strictly positive standards, a result that is not possible under alternative assumptions within the principal-agent framework. For example, Ellis (1992a) and Gottinger (2001) study standard-setting under *ex-ante* incomplete information, but they restrict attention to policies which induce compliance. There are papers which study incentive compatible optimal pollution taxes, such as Jebjerg and Lando (1997), which implicitly constrain to zero standards. Swierzsbinky (1994) consider optimal taxation also, relaxing the assumption of incentive compatibility, but they again restrict to zero standards. The only exception is Arguedas (2005), which considers a bargaining context under complete information and assumes that the firm can choose the environmental technology as well. It is shown that it may be beneficial for both the regulator and the firm to achieve a cooperative agreement where the firm chooses a cleaner technology in exchange for a relaxed regulation that may consist of a positive standard and a reduction of the fine for exceeding it.

The remainder of the paper is organized as follows. In the next section, we present the model. In Section 3, we study the optimal behavior of the firm. In Section 4, we analyze the characteristics of the optimal policy in each region. In Section 5, we discuss the likelihood of obtaining violations to optimal positive standards. We conclude in Section 6. All the proofs are in the Appendix.

2 The Model

We consider a firm that generates pollution as a by-product of its production activity. The firm obtains private profits which depend on the pollution level $e \geq 0$ and a parameter $\theta_i > 0$, $i = 1, 2$, $\theta_1 < \theta_2$, which refers to the firm's degree of profitability or the *dirtiness* of its technology.[7] Let $B(e, \theta_i) = \theta_i b(e)$ represent the firm's profits, where $b(e)$ is continuous and strictly concave in the pollution level with an interior maximum at $\widetilde{e} > 0$, and such that $b(0) = 0$ and $b'''(e) \geq 0$.[8] Given a pollution level, the clean type (θ_1) obtains lower profits than the dirty type (θ_2). The firm knows its type but the regulator only knows the probability distribution of the types. Let γ_i denote the probability that the firm is type θ_i, such that $\gamma_i \in [0, 1]$ and $\gamma_1 + \gamma_2 = 1$.

Pollution generates external damages measured by the function $d(e)$, which is continuous, strictly increasing and convex in the pollution level, and such that $d(0) = 0$.

Let e_i^w denote the efficient pollution level when the firm is type θ_i, that

[7] As we will see later on, θ_2 is generally associated with larger induced pollution and environmental damages than θ_1.

[8] This specification of profits simplifies the algebra without affecting the qualitative nature of the results. However, we will point out the differences when needed.

is, $e_i^w = \arg\max_{e \geq 0} \{\theta_i b(e) - d(e)\}$. Observe that $e_2^w > e_1^w$. In the absence of regulation, the firm does not internalize external damages and pollution is $\widetilde{e} = \arg\max_{e \geq 0} \theta_i b(e) > e_i^w$, for all i.

We assume there exists a regulator who sets a standard $s \in [0, \widetilde{e}]$, that is, a maximum level of permitted pollution.[9] The regulator cannot observe the pollution level unless it monitors the firm, which is costly and perfectly accurate. The cost *per* inspection is $c > 0$. Therefore, the regulator does not generally inspect the firm in every instance but only with probability $p \in [0, 1]$. Once inspected, if the firm is discovered exceeding the standard, then it is forced to pay a penalty which depends on the degree of noncompliance, $e - s$. We assume that the sanction is represented by the function $F(e - s)$, which is quadratic, strictly increasing and convex in $e - s \geq 0$, and such that $F(e - s) = 0$ for all $e - s \leq 0$. Given these assumptions, we have that $(F')^2 - FF'' > 0$ for all $e - s \geq 0$, a property that plays a key role in the results, as we will see later on. We assume that the sanction is fixed by a government entity other than the regulator, for example, the judiciary.[10]

We consider a principal-agent framework where the regulator (principal) chooses the standard and the probability of inspection that maximizes social welfare, considering the optimal response of the firm (agent) to the policy.

Given $\{s, p\}$, a firm of type θ_i chooses the pollution level that maximizes its expected payoff, that is, private profits minus expected penalties, as follows:

[9] Obviously, the regulator is not interested in a standard larger than the pollution level chosen by the firm in the absence of regulation.

[10] This assumption is common in the literature in this context except, for example, in Heyes (1996) or Arguedas (2005). In other contexts, such as crime, there are several papers which endogenize fines together with probabilities of inspection, such as Becker (1968), Polinsky and Shavell (1979, 1990) or Bebchuck and Kaplow (1991), but there the standard is exogenous. In the context of tax evasion, few papers consider endogenous fines. See, for instance, Mookherjee and Png (1989).

$$P(s,p,\theta_i) = \max_{e\geq 0}\{\theta_i b(e) - pF(e-s)\} \tag{1}$$

Let $e(s,p,\theta_i)$ be the pollution level chosen by type θ_i given the policy $\{s,p\}$, i.e., $e(s,p,\theta_i) = \arg\max_{e\geq 0}\{\theta_i b(e) - pF(e-s)\} \leq \widetilde{e}$.

Considering the firm's best response, the regulator now chooses the policy that maximizes social welfare. Since the regulator does not know the true type of the firm, the policy cannot be based upon it. There are two kind of policies the regulator may choose. The first is a *pooling (or uniform) policy* $\{s,p\}$, that is, the same policy regardless of the type. In this case, the regulator does not need to elicit any information from the firm and social welfare is as follows:

$$SW(s,p) = \sum_{i=1}^{2}\gamma_i\left[P(s,p,\theta_i) - d(e(s,p,\theta_i)) + pF(e(s,p,\theta_i) - s)\right] - cp \tag{2}$$

The regulator is concerned about the firm's expected payoff, the generated damages, the expected collected fines and the expected monitoring costs. We assume that there are no social costs associated with collecting fines, and that fines are redistributed lump-sum. Also, we do not impose any budget requirement on the monitoring activity. Considering (1), (2) reduces to:

$$SW(s,p) = \sum_{i=1}^{2}\gamma_i\left[\theta_i b(e(s,p,\theta_i)) - d(e(s,p,\theta_i))\right] - cp \tag{3}$$

The second type of policy is *separating*, that is, a policy contingent on type. Here, the regulator has to design a mechanism to elicit the firm's private information. By the *revelation principle*, we can concentrate on direct mechanisms where the regulator asks the firm to report its type, $\widehat{\theta}_i$, and then, it sets the policy based on the report, $\left\{s\left(\widehat{\theta}_i\right), p\left(\widehat{\theta}_i\right)\right\}$, such that it induces the firm to reveal its true type, $\widehat{\theta}_i = \theta_i$. This is the well known *incentive compatibility condition*, represented as follows:

$$\theta_i \in \arg\max P\left(s\left(\widehat{\theta}_i\right), p\left(\widehat{\theta}_i\right), \theta_i\right) \tag{4}$$

For convenience, we assume that if the firm is indifferent between announcing any of the two types, then it announces the true type.

Denoting $s_i = s\left(\widehat{\theta}_i\right)$ and $p_i = p\left(\widehat{\theta}_i\right)$, $i = 1, 2$, social welfare is now:

$$SW\left(s_1, s_2, p_1, p_2\right) = \sum_{i=1}^{2} \gamma_i \left[\theta_i b\left(e\left(s_i, p_i, \theta_i\right)\right) - d\left(e\left(s_i, p_i, \theta_i\right)\right) - cp_i\right] \tag{5}$$

where (s_1, s_2, p_1, p_2) satisfy (4). Observe that a uniform policy is trivially incentive compatible.[11]

An important assumption of our model is that the regulator commits to the announced inspection probability. This assumption can be justified considering that the regulator has to build up a reputation, that is, policy announcements must be credible to induce the desired behavior.[12]

In the next section, we study the firm's induced behavior with respect to the announced policy.

[11] Besides incentive compatibility, the literature on economics of information considers participation constraints also, that is, feasible policies must be such that firms' payoffs are nonnegative. In our case, this additional requirement is trivially satisfied since $b(0) = 0$.

[12] A formal justification of this assumption would require to consider a dynamic model, which is beyond the scope of this paper. In static models such as ours, the assumption of commitment is common in the literature. Some exceptions in the environmental context are Ellis (1992b) and Grieson and Singh (1990).

3 The Optimal Behavior of the Firm

Consider a feasible policy $\{s, p\}$. As explained in the previous section, the corresponding type θ_i's expected payoff is given by (1).

If type θ_i complies with the standard $(e < s)$, it does not incur any penalty. Since $b(e)$ is strictly increasing in $e \leq \widetilde{e}$, the *optimal compliance decision* is s and its payoff is $\theta_i b(s)$.

If type θ_i exceeds the standard $(e > s)$, then there is a chance that is inspected and punished. Consequently, the *optimal noncompliance decision* is $n(s, p, \theta_i) = \arg\max_{e>s} \{\theta_i b(e) - pF(e - s)\} > s$ and the corresponding payoff is $\pi(s, p, \theta_i)$. Since the maximand is strictly concave in e, the first order condition characterizes the interior noncompliance decision:

$$\theta_i b'(e) = pF'(e - s) \tag{6}$$

Implicitly differentiating (6), we obtain $n_{ip} = n_p(s, p, \theta_i) = \frac{F'}{\theta_i b'' - pF''}$ and $n_{is} = n_s(s, p, \theta_i) = -\frac{pF''}{\theta_i b'' - pF''}$. Observe that $n_{ip} < 0$ and $0 \leq n_{is} < 1$. That is, type θ_i's pollution level increases when the probability of inspection decreases and the standard increases. However, since $n_{is} < 1$, the degree of violation decreases when the standard increases.[13]

Given $\{s, p\}$, type θ_i chooses whether to comply or not depending on the expected payoff of each possibility. Thus, its optimal response is:

$$e(s, p, \theta_i) = \begin{cases} s & \text{if } \theta_i b(s) \geq \pi(s, p, \theta_i) \\ n(s, p, \theta_i) & \text{if } \theta_i b(s) < \pi(s, p, \theta_i) \end{cases} \tag{7}$$

[13]Note that $n_{is} = 0$ when either $F'' = 0$ or $p = 0$.

and its expected payoff can be further expressed as:

$$P(s,p,\theta_i) = \max\ \{\theta_i b(s), \pi(s,p,\theta_i)\} \tag{8}$$

In the following lemma, we show the properties of the function $P(s,p,\theta_i)$:

Lemma 1 *The function $P(s,p,\theta_i)$ is nondecreasing and concave in s, nonincreasing and convex in p, it has a nonnegative cross partial, and it is such that $P(s,p,\theta_2) > P(s,p,\theta_1)$. Moreover, $P(s,0,\theta_i) = \theta_i b(\widetilde{e})$ for all i.*

We now characterize the set of policies for which each type is indifferent between complying and noncomplying with the standard. Since sanctions are continuous at $e = s$, the maximand in (1) is continuous in s. Therefore, considering (6), type θ_i complies with the standard only if $\theta_i b'(s) \leq pF'(0)$. Thus, the minimum probability that induces type θ_i to comply with s is:

$$p^c(s,\theta_i) = \frac{\theta_i b'(s)}{F'(0)} \tag{9}$$

which is strictly decreasing and convex in s, and such that $p^c(s,\theta_2) > p^c(s,\theta_1)$.[14] Since $p \leq 1$, there may exist a subset of nonenforceable standards for each θ_i.[15]

In Figure 1, we represent the functions $p^c(s,\theta_i)$ in the space of feasible policies. In the horizontal axis we measure the standard and in the vertical axis, we measure the probability of inspection. These functions divide the set of feasible policies into three regions, namely the *compliance* (C), *partial compliance* (PC) and *noncompliance (NC) regions*. Therefore, all the policies on or above the function $p^c(s,\theta_2)$ induce both types to comply with the standard. The set of

[14] The assumptions on the penalty function ensure that $F'(0)$ is finite and strictly positive.

[15] If there exists $\widehat{s}_i > 0$ such that $p^c(\widehat{s}_i,\theta_i) = 1$, then $s \in [0,\widehat{s}_i)$ cannot be enforced for θ_i.

policies between $p^c(s, \theta_1)$ and $p^c(s, \theta_2)$ induce the clean type to comply only. Finally, the policies below the function $p^c(s, \theta_1)$ induce both types to violate the standard. Thus, θ_2's noncompliance region is larger than that of θ_1.

In the figure, we also include each type's indifference map, where each indifference curve is composed of the set of policies such that type θ_i´s expected payoff is constant. By Lemma 1, type θ_i's payoff increases to the southeast, i.e., whenever the standard is larger and the probability of inspection is smaller. And it obtains the maximum expected payoff at $s = \widetilde{e}$, $p \in [0, 1]$ and $s \geq 0$, $p = 0$. The shape of the indifference curves is now presented in the following:

Lemma 2 *If a policy $\{s, p\}$ induces type θ_i to comply with the standard, the indifference curve at that policy is vertical. If it induces noncompliance, the indifference curve at that policy is strictly increasing and convex. At any $\{s, p\}$, the slope of θ_1's indifference curve is no smaller than that of θ_2.*

In fact, in both the full noncompliance and the partial compliance regions, indifference curves satisfy the *single crossing property*. However, in the full compliance region, indifference curves do not cross.

The revelation principle allows us to restrict attention to incentive compatible policies. For example, a policy $\{s_1, p_1\}$ for $\widehat{\theta_1}$ and a policy $\{s_2, p_2\}$ in the shaded area of Figure 1 for $\widehat{\theta_2}$ is incentive compatible, i.e., no type has an incentive to lie. Note that $s_2 \leq s_1$ and $p_2 \leq p_1$.

Having studied the types' optimal responses, we now analyze the features of the optimal policy in each region.

4 The Optimal Policy in Each Region

We begin with the full compliance region. Here, both types' indifference curves are vertical, which implies that there does not exist any incentive compatible separating policy. The following result summarizes the characteristics of the (interior) optimal policy in this region.

Proposition 3 *The optimal full compliance policy is pooling and such that*

$$\sum_{i=1}^{2} \gamma_i \left(\theta_i b'(s^*) - d'(s^*) - c \frac{dp^c(s^*, \theta_2)}{ds} \right) = 0 \tag{10}$$
$$p^* = p^c(s^*, \theta_2)$$

The intuition of this result is simple. Given a standard s, both types are indifferent between any policy (either pooling or separating) that implies compliance with that standard. From a social welfare viewpoint, the lower the probability, the better. Thus, the optimal probability is the minimum that induces full compliance, i.e., $p = p^c(s, \theta_2)$. Any policy that assigns different standards induces the type with the lowest assigned standard to misreport its type. Therefore, the optimal policy in this region is pooling.[16]

Since $\theta_1 < \theta_2$, (10) implies that $\theta_1 b'(s^*) - d'(s^*) - c\frac{dp^c(s,\theta_2)}{ds} < 0$ and $\theta_2 b'(s^*) - d'(s^*) - c\frac{dp^c(s,\theta_2)}{ds} > 0$. In words, the clean type is under-enforced and the dirty type is over-enforced with respect to the complete information case, where the first order condition would read $\theta_i b'(s) - d'(s) = c\frac{dp^c(s,\theta_i)}{ds}$.

[16] If we assumed that types' private profits had different interior maxima, namely $\widetilde{e}_1$ and $\widetilde{e}_2$, $\widetilde{e}_1 < \widetilde{e}_2$, the result would vary if the optimal pooling policy implied $s^* > \widetilde{e}_1$. Since type θ_1 is indifferent between any policy with a standard equal to $\widetilde{e}_1$ or larger, it would be socially preferred to announce, for type θ_1, $s_1 = \widetilde{e}_1$ and the probability such that type θ_2 is indifferent between the former pooling policy and this new policy announced for type θ_1. Therefore, in this case we would have $s_1 < s_2$ and $p_1 < p_2$. For all the other cases, the results under this alternative assumption remain the same.

We now consider the partial compliance region. Here, policies lie between $p^c(s,\theta_1)$ and $p^c(s,\theta_2)$, where only type θ_1 complies with the standard. In principle, both a uniform and a separating policy are possible in this case, since indifference curves in this region satisfy the single crossing property. The following proposition shows the features of each possible (interior) policy.

Proposition 4 *The optimal partial compliance policy can be either pooling or separating.*

(i) If it is pooling, then

$$\frac{\gamma_1\left(\theta_1 b'\left(s^*\right)-d'\left(s^*\right)\right)+\gamma_2\left(\theta_2 b'\left(n_2\right)-d'\left(n_2\right)\right)n_{2s}}{\gamma_2\left(\theta_2 b'\left(n_2\right)-d'\left(n_2\right)\right)n_{2p}-c}=-\frac{dp^c\left(s^*,\theta_1\right)}{ds} \tag{11}$$

$$p^*=p^c\left(s^*,\theta_1\right)$$

(ii) If it is separating, then:

$$\frac{\gamma_1\left(\theta_1 b'\left(s_1^*\right)-d'\left(s_1^*\right)-c\frac{dp^c\left(s_1^*,\theta_1\right)}{ds_1}\right)}{\gamma_2\left(\left(\theta_2 b'\left(n_2\right)-d'\left(n_2\right)\right)n_{2p}-c\right)}=-\frac{\frac{\partial P\left(s_1^*,p_1^*,\theta_2\right)}{\partial s_1}}{\frac{\partial P\left(s_2^*,p_2^*,\theta_2\right)}{\partial p_2}} \tag{12}$$

$$\frac{\gamma_1\left(\theta_1 b'\left(s_1^*\right)-d'\left(s_1^*\right)-c\frac{dp^c\left(s_1^*,\theta_1\right)}{ds_1}\right)}{\gamma_2\left(\theta_2 b'\left(n_2\right)-d'\left(n_2\right)\right)n_{2s}+\eta_2}=-\frac{\frac{\partial P\left(s_1^*,p_1^*,\theta_2\right)}{\partial s_1}}{\frac{\partial P\left(s_2^*,p_2^*,\theta_2\right)}{\partial s_2}} \tag{13}$$

$$s_2^*\geq 0,\ \eta_2\geq 0,\ s_2^*\eta_2=0$$

$$p_1^*=p^c\left(s_1^*,\theta_1\right)$$

$$P\left(s_2^*,p_2^*,\theta_2\right)=P\left(s_1^*,p_1^*,\theta_2\right) \tag{14}$$

where $\eta_2\geq 0$ is the Lagrange multiplier associated with $s_2^*\geq 0$.

Observe that a pooling policy in this region means that the standard is generally positive (the contrary requires type θ_1 to be enforced to comply with a zero standard, see footnote 15). Therefore, the dirty type violates a positive standard, a result that is not possible under complete information. Here, (11) implies that the optimal standard and probability are such that the marginal rate of substitution in terms of efficiency of the induced pollution levels must equal the marginal rate of substitution to ensure type θ_1's compliance.

If the policy is separating, (14) implies $s_1^* > s_2^* \geq 0$ and $p_1^* > p_2^*$. Thus, type θ_1 faces both a larger standard and a larger probability of inspection in order to preserve incentive compatibility. Here, the standard for type θ_2 could be zero but not necessarily, since $\eta_2 \geq 0$. By (14), type θ_2 is indifferent between (s_1^*, p_1^*) and (s_2^*, p_2^*). By Lemma 2, this means that type θ_1 strictly prefers (s_1^*, p_1^*). At the optimal separating policy, type θ_1 is over-enforced and type θ_2 is under-enforced with respect to the complete information case, since $\theta_1 b'(s_1^*) - d'(s_1^*) - c\frac{dp^c(s_1^*,\theta_1)}{ds_1} = \mu_2 \frac{\partial P(s_1^*,p_1^*,\theta_2)}{\partial s_1} > 0$ and $(\theta_2 b'(n_2) - d'(n_2))\, n_{2p_2} - c = -\mu_2 \frac{\partial P(s_2^*,p_2^*,\theta_2)}{\partial p_2} > 0$, by Lemma 1 and $\mu_2 > 0$, the Lagrange multiplier associated with type θ_2's binding incentive compatibility condition. That is, the standard for type θ_1 is smaller than the one under complete information, and the inspection probability for type θ_2 is also smaller than the one under complete information.

Conditions (12) and (13) mean that, at the optimum, (s_1, s_2, p_2) are such that the marginal rate of substitution between each pair of variables in terms of efficiency of the induced pollution levels equals the marginal rate of substitution between thar pair of variables to induce type θ_2's truthful revelation. Also, at the optimum, type θ_1 is indifferent between the compliant and noncompliant decisions.

Finally, we consider the full noncompliance region. In the following proposition, we show the characteristics of the optimal (interior) policy in this case.[17]

Proposition 5 *The optimal full noncompliance policy can be either pooling or separating.*

(i) If it is pooling, then:

$$\sum_{i=1}^{2} \gamma_i \left(\theta_i b'\left(n_i\right) - d'\left(n_i\right)\right) n_{ip} = c \tag{15}$$

$$\sum_{i=1}^{2} \gamma_i \left(\theta_i b'\left(n_i\right) - d'\left(n_i\right)\right) n_{is} + \lambda = 0 \tag{16}$$

$$s^* \geq 0, \;\; \lambda \geq 0, \; \lambda s^* = 0$$

where $\lambda \geq 0$ is the Lagrange multiplier associated with $s^* \geq 0$.

(ii) If it is separating, then

$$\frac{\left(\theta_1 b'\left(n_1\right) - d'\left(n_1\right)\right) n_{1s}}{\left(\theta_1 b'\left(n_1\right) - d'\left(n_1\right)\right) n_{1p} - c} = \frac{\frac{\partial P\left(s_1^*, p_1^*, \theta_2\right)}{\partial s_1}}{\frac{\partial P\left(s_1^*, p_1^*, \theta_2\right)}{\partial p_1}} \tag{17}$$

$$\frac{\gamma_1 \left(\theta_1 b'\left(n_1\right) - d'\left(n_1\right)\right) n_{1s}}{\gamma_2 \left(\left(\theta_2 b'\left(n_2\right) - d'\left(n_2\right)\right) n_{2p} - c\right)} = -\frac{\frac{\partial P\left(s_1^*, p_1^*, \theta_2\right)}{\partial s_1}}{\frac{\partial P\left(s_2^*, p_2^*, \theta_2\right)}{\partial p_2}} \tag{18}$$

$$\frac{\gamma_1 \left(\theta_1 b'\left(n_1\right) - d'\left(n_1\right)\right) n_{1s}}{\gamma_2 \left(\theta_2 b'\left(n_2\right) - d'\left(n_2\right)\right) n_{2s} + \eta_2} = -\frac{\frac{\partial P\left(s_1^*, p_1^*, \theta_2\right)}{\partial s_1}}{\frac{\partial P\left(s_2^*, p_2^*, \theta_2\right)}{\partial s_2}} \tag{19}$$

$$\eta_2 \geq 0, \;\; s_2^* \geq 0, \;\; \eta_2 s_2^* = 0$$

$$P\left(s_1^*, p_1^*, \theta_2\right) = P\left(s_2^*, p_2^*, \theta_2\right)$$

where $\eta_2 \geq 0$ is the Lagrange multiplier associated with $s_2^* \geq 0$.

[17] We omit the proof as it is similar to that of Proposition 4.

If the policy is pooling, the optimal standard need not be zero. Therefore, it is possible that both types violate positive standards. Observe that even a nonzero standard implies that $\theta_1 b'(n_1) - d'(n_1) > 0$, which means that type θ_1 pollutes below its efficient level.[18] Therefore, a zero standard could restrict type θ_1's pollution even more, with the corresponding welfare decrease. By contrast, type θ_2 is under-enforced.

Alternatively, if the policy is separating, we again have $s_1^* > s_2^* \geq 0$ and $p_1^* > p_2^*$, due to the incentive compatibility constraint. In this case, we also have that type θ_2 is under-enforced and type θ_1 is over-enforced with respect to the complete information case. If the regulator were to naively impose the complete information solution, type θ_2 would find it profitable to misreport its type, and this is why type θ_2's incentive compatibility constraint is binding. Conditions (17), (18) and (19) have an identical meaning as those of partial compliance, except that now both types find it profitable to violate the standards.

The results of this section would not substantially vary if we allowed for a continuum of types. Following the same reasoning as that of Proposition 4, if the optimal policy induced some types to comply and others to violate the standards, the latter ones would be the dirtiest. Moreover, the optimal policy would imply partial pooling: the compliant types would be confronted to the same policy to avoid misreporting, as in Proposition 3.

In general, we can obtain violations to positive standards under incomplete information. An interesting question to ask now is under what conditions is this the case? We provide the answer in the following section.

[18] If $s^* > 0$, then $\lambda = 0$. By (16), we have $\gamma_1 A_1 n_{1s} = -\gamma_2 A_2 n_{2s}$, where $A_i = \theta_i b'(n_i) - d'(n_i)$, which implies that $c = \gamma_2 A_2 \left(n_{2p} - \frac{n_{2s}}{n_{1s}} n_{1p} \right)$. Since $n_{2p} - \frac{n_{2s}}{n_{1s}} n_{1p} = \frac{F'(n_2 - s) - F'(n_1 - s)}{\theta_2 b'' - pF''} < 0$ and $c > 0$, we then have $A_2 < 0$ and $A_1 > 0$.

5 Discussion

In this section, we analyze the influence of the parameters of the model in the different types of policies. The following result applies generally and shows the type of policy that we obtain if we restrict attention to pooling policies.

Proposition 6 *Whenever it exists, the optimal pooling policy induces full noncompliance.*

Full compliance is too expensive from a social viewpoint, and welfare can always be increased if we decrease the inspection probability, since clean type's incentives remain unchanged, and the savings in monitoring costs are larger than the decrease in efficiency due to the larger dirty type's induced pollution level. However, partial compliance is also too costly from a social welfare perspective. Thus, whenever possible, it is worth to set the pooling policy in the noncompliance region. However, we may have nonexistence of the full noncompliance solution when monitoring costs are small or when the full noncompliance region is small. This is illustrated in Example 1, below.

Also, when the full noncompliance policy exists, it is optimal to set a positive standard for some values of the parameters. Example 1 illustrates that this is more likely under low monitoring costs, large θ_1 and large uncertainty, that is, when γ_1 takes intermediate values. Also, the larger θ_1, the smaller the interval of the monitoring costs for which the standard is positive.

5.1 Example 1

Consider the specific case where:

$$b(e) = \begin{cases} e, & e \leq 1 \\ 1-e, & e > 1 \end{cases}$$

$$d(e) = e^2$$

$$F(e-s) = (e-s) + (e-s)^2$$

Observe that $\widetilde{e} = 1$.[19] For simplicity, we fix $\theta_2 = 1$. Therefore, $\theta_1 < 1$. Now, we compute the corresponding pooling policies in each one of the regions.

Starting with the full compliance pooling policy (FCP), observe that $p^c(s, \theta_2) = \theta_2 = 1$. Applying Proposition 3, we obtain:

$$p^{FCP} = 1; \; s^{FCP} = \frac{1 - \gamma_1 (1 - \theta_1)}{2}$$

Note that $0 < s^{FCP} < \widetilde{e} = 1$ for all $c \geq 0$, $\gamma_1 \in [0, 1]$ and $\theta_1 < 1$. Therefore, there always exists an interior FCP solution.

To obtain the partial compliance pooling policy (PCP), we now have $p^c(s, \theta_1) = \theta_1 < 1$. Also, from (6), we have:

$$n_i(s, p, \theta_i) = \frac{\theta_i - p(1 - 2s)}{2p}$$

and, consequently, $n_{ip}(s, p, \theta_i) = -\frac{\theta_i}{2p^2}$ and $n_{is}(s, p, \theta_i) = 1$. Applying part (i)

[19] Note that $b(e)$ is linear, which considerably simplifies the algebra without affecting the results. The main difference in this case is that $p^c(s, \theta_i) = \frac{\theta_i}{F'(0)}$, i.e., θ_i's threshold probability does not depend on s.

of Proposition 4, we have:

$$p^{PCP} = \theta_1; \; s^{PCP} = \frac{\gamma_1 \theta_1^2 + (1-\gamma_1)(2\theta_1 - 1)}{2\theta_1}$$

In this case, we obtain an interior standard when $\gamma_1 \in \left[\frac{1-2\theta_1}{(1-\theta_1)^2}, \frac{1}{(1-\theta_1)^2}\right]$. Since $\gamma_1 \in [0,1]$ and $\theta_1 < 1$, there exists an interior solution when either $\theta_1 \in \left(\frac{1}{2}, 1\right)$ or $\theta_1 \leq \frac{1}{2}$ and $\gamma_1 \in \left[\frac{1-2\theta_1}{(1-\theta_1)^2}, 1\right]$.

Finally, we obtain the full noncompliance pooling policy (FNCP) applying part (i) of Proposition 5. Now, $p < \theta_1$ is needed to induce both types to exceed the standard. The optimality conditions to obtain $\left(s^{FNCP}, p^{FNCP}\right)$ reduce to the following system of equations:

$$\begin{aligned}
&\gamma_1 \theta_1 \frac{p(\theta_1 + 1 - 2s) - \theta_1}{2p^3} + (1-\gamma_1)\frac{2p(1-s) - 1}{2p^3} + c = 0 \\
&\gamma_1 \frac{p(\theta_1 + 1 - 2s) - \theta_1}{p} + (1-\gamma_1)\frac{2p(1-s) - 1}{p} + \lambda = 0 \\
&\lambda s = 0, \; s \geq 0, \; \lambda \geq 0
\end{aligned}$$

In Table 1, we present some numerical solutions for different values of the parameters as well as the corresponding social welfare evaluations. Observe that full compliance is always dominated by partial compliance, since the probability of inspection in the latter case is much smaller ($p = 1$ under full compliance and $p = \theta_1$ under partial compliance). But, whenever it exists, the full noncompliance solution is always socially preferred to partial compliance. Under full noncompliance, the probability of inspection increases when monitoring costs decrease, when θ_1 increases or when γ_1 decreases. These three relationships are quite intuitive. Also, we obtain a strictly positive standard when $c = 0.2$,

$\theta_1 = 0.5$, $\gamma_1 = 0.5$, a result that is not possible under complete information. In fact, this suggests that a positive standard may be obtained when monitoring costs are sufficiently low. We now explore this result in detail.

In Figures 2 and 3, we illustrate the relationship between the optimal standards and the monitoring costs under full noncompliance for different values of θ_1, in the two extreme cases of large uncertainty ($\gamma_1 = 0.5$) and no uncertainty at all ($\gamma_1 = 1$), respectively. In Figure 2, if $\theta_1 = 0.8$ for example, there does not exists a full noncompliance solution when $c \in [0, 0.002]$ and, therefore, the optimal pooling solution in this interval induces partial compliance to $s = 0.3875$. If $c \in [0.002, 0.0024]$, the optimal pooling policy induces full noncompliance to a positive standard, which decreases as the monitoring cost increase. Finally, if $c > 0.0024$, the optimal standard is zero. If θ_1 is lower, the full noncompliance solution does not exist for a larger interval of the monitoring costs. This is intuitive since the lower θ_1, the lower the full noncompliance region, and therefore, the larger the restriction for the full noncompliance solution to exist. If $\theta_1 = 0.5$, we now obtain a larger interval of the monitoring costs for which the optimal standard is positive, $c \in [0.125, 0.227]$. For θ_1 sufficiently small, we do not obtain full noncompliance to positive standards.

A graph similar to Figure 2 can be obtained under alternative values of γ_1, that is, under different degrees of uncertainty. For example, if $\gamma_1 = 0.1$, the range of monitoring costs for which we obtain full noncompliance to a strictly positive standard is $c \in [0.0007, 0.007]$ if $\theta_1 = 0.8$ and $c \in [0.045, 0.049]$ if $\theta_1 = 0.5$. Alternatively, if $\gamma_1 = 0.9$, we have $c \in [0.0007, 0.01]$ and $c \in [0.045, 0.16]$ for $\theta_1 = 0.8$ and $\theta_1 = 0.5$, respectively. In these two cases, uncertainty has decreased with respect to the case in which $\gamma_1 = 0.5$ and, consequently, the interval of the monitoring costs for which we obtain violations to strictly positive standards is smaller. Figure 3 shows the limit case of no uncertainty, where the solution jumps from partial compliance to full noncompliance to a zero standard, with no possible violations to positive standards.

In the next example, we analyze the separating policies.

5.2 Example 2

We consider the same functions of the previous example. Regarding the partial compliance separating policy (PCS), by part (ii) of Proposition 4, we have $p_1 = \theta_1$. Also, $p_2 < \theta_1$ to ensure incentive compatibility. Considering (8), expected profits are:

$$\begin{aligned} P(s, p, \theta_1) &= \theta_1 s_1 \\ P(s, p, 1) &= \frac{1 - 2p(1 - 2s) + p^2}{4p} \end{aligned} \tag{20}$$

since the clean type complies with the standard and the dirty type does not.

Therefore, since $p_1 = \theta_1$, the incentive compatibility constraint presented in part (ii) of Proposition 4 reads:

$$\frac{1 - 2\theta_1(1 - 2s_1) + \theta_1^2}{4\theta_1} = \frac{1 - 2p_2(1 - 2s_2) + p_2^2}{4p_2} \tag{21}$$

From (20), we obtain $\frac{\partial P(s_i, p_i, \theta_2)}{\partial s_i} = 1$ and $\frac{\partial P(s_i, p_i, \theta_2)}{\partial p_2} = -\frac{1-p^2}{4p^2}$. Thus, the conditions to obtain the optimal PCS policy of part (ii) of Proposition 4 reduce to (21) and the three following equations:

$$\frac{\gamma_1(\theta_1 - 2s_1)}{(1-\gamma_1)(1-2p_2(1-s_2)-2cp_2^3)} = \frac{2}{p_2(1-p_2^2)}$$
$$\gamma_1(\theta_1 - 2s_1) = (\gamma_1 - 1)\frac{2p_2(1-s_2)-1}{p_2} - \eta_2$$
$$s_2 \geq 0,\ \eta_2 \geq 0,\ \eta_2 s_2 = 0$$

Finally, we apply the conditions of part (ii) of Proposition 5 to obtain the full noncompliance separating (FNCS) policy. Now, $p_1 < \theta_1$, since both types violate the standard. The following equations characterize the optimal solution in this case:

$$\frac{1-2p_1(1-2s_1)+p_1^2}{4p_1} = \frac{1-2p_2(1-2s_2)+p_2^2}{4p_2}$$
$$\frac{p_1(\theta_1+1-2s_1)-\theta_1}{\theta_1 p_1(\theta_1+1-2s_1)-\theta_1^2+2cp_1^3} = \frac{2}{1-p_1^2}$$
$$\left(\frac{\gamma_1}{\gamma_1-1}\right)\left(\frac{p_2}{p_1}\right)\left(\frac{p_1(\theta_1+1-2s_1)-\theta_1}{2p_2(1-s_2)-1+2cp_2^3}\right) = \frac{2}{1-p_2^2}$$
$$\gamma_1\frac{p_1(\theta_1+1-2s_1)-\theta_1}{p_1} = (\gamma_1-1)\frac{2p_2(1-s_2)-1}{p_2} - \eta_2$$
$$s_2 \geq 0,\ \eta_2 \geq 0,\ \eta_2 s_2 = 0$$

We have computed the results for different values of the parameters. While we can find interior solutions for the PCS policy, however we cannot find an interior solution in the case of a FNCS policy, for any feasible values of the parameters. Therefore, if the policy were to be separating, in this case it would induce partial compliance.

In Figure 4, we present the relationship between the optimal standards and the monitoring costs, for different values of θ_1 and $\gamma_1 = 0.5$, that is, when there is large uncertainty. If $\theta_1 = 0.5$, there does not exist a PCS solution when

$c \in [0, 0.3125]$. While type θ_1 always complies with s_1, type θ_2 violates a strictly positive standard s_2 when $c \in [0.3125, 0.58853]$. Finally, $s_2 = 0$ if $c > 0.58853$. Note that both types' standards decrease when monitoring costs increase. Also, both inspection probabilities decrease when monitoring costs increase. When $\theta_1 = 0.8$, we observe the same pattern, but here, the interval where type θ_2 violates a strictly positive standard is larger, i,e., $c \in [0.016, 0.70177]$. Therefore, it is more likely that we find noncompliance to strictly positive standards when θ_1 is large, since this enlarges the full noncompliance region.

We find an analogous structure of the solution under different values of γ_1. However, it is interesting to see that, the smaller γ_1, the smaller the intervals of the monitoring costs where type θ_2 violates a positive standard. Thus, if $\gamma_1 = 0.1$, we find that type θ_2 violates a positive standard when $c \in [0.0625, 0.069524]$ if $\theta_1 = 0.5$ and when $c \in [0.0032, 0.0804]$ if $\theta_1 = 0.8$. Conversely, if $\gamma_1 = 0.9$, these intervals are, respectively, $c \in [0.5625, 2.304]$ and $c \in [0.028828, 2.8518]$.

Finally, the interval of the monitoring costs for which we obtain violations to strictly positive standards under FNCP always contains lower values than the interval under PCS. For example, Figure 5 shows the comparison between these intervals for the case of large uncertainty and $\theta_1 = 0.5$. Regarding social welfare, we have made some computations which show that a pooling policy may be preferred to a separating policy. For example, if $c = 0.58853$, $\theta_1 = 0.5$ and $\gamma_1 = 0.5$, we obtain $sw^{FNCP} = -0.11822 > sw^{PCS} = -0.1478$. Alternatively, if $c = 0.069524$, $\theta_1 = 0.5$ and $\gamma_1 = 0.1$, we have $sw^{FNCP} = 0.191169 > sw^{PCS} = 0.1909$. Therefore, this example suggests that separating policies may not always be preferred to pooling policies.

6 Conclusions

In this paper, we have studied the characteristics of an optimal regulatory policy composed of pollution standards, probabilities of inspection and fines for non-compliance in a context of asymmetric information and imperfect enforcement, an approach different from that which has been done in the literature. Our model is able to explain a salient feature of environmental regulation, namely violations to strictly positive standards, a result that is not possible under either complete information or incomplete information subject to perfect enforcement, the two approaches studied until now within the principal-agent approach.

Also, our results suggest that restricting attention to incentive compatible environmental taxation (where all the pollution levels are punishable) may not always be correct, since under some circumstances, it may be socially preferred to leave a certain amount of pollution uncharged to avoid over-enforcement.

In fact, we have shown that violations to positive standards are more likely when monitoring costs are low, when uncertainty is large, and also when the full noncompliance region is large. Since a positive standard means that the fine for noncompliance is not maximum, this result is more likely when enforcement costs are less important than the costs associated with the over-enforcement of the clean type. Also, when uncertainty decreases, the results converge to the complete information case, where it is not possible to obtain violations to positive standards. Finally, the smaller the full noncompliance region (or the smaller the parameter showing the dirtiness of the clean type), the smaller the likelihood of obtaining violations to strictly positive standards.

Some computations have shown that separating policies may not always be preferred from a social viewpoint and the best the regulator could do in those cases is to set the same policy for all the possible types. This result is surprising and suggests that it may not always be worth to collect information about firms prior to standard setting, even if this information collection were costless.

However, we have had to rely on numerical examples to analyze the likelihood of violations to positive standards. Also, we do not have a definite answer to the question of what is the best policy from a social point of view yet. Our results are intuitive enough to think that they must hold more generally, but this needs more investigation.

7 Appendix

Proof of Lemma 1.

When $P(s,p,\theta_i) = \theta_i b(s)$, the function is strictly increasing and concave in s, but it does not depend on p. Also, $\theta_2 b(s) > \theta_1 b(s)$. Conversely, when $P(s,p,\theta_i) = \pi(s,p,\theta_i)$, we have:

$$\pi_s(s,p,\theta_i) = pF'(n_i - s) \geq 0 \tag{22}$$

$$\pi_{ss}(s,p,\theta_i) = pF''(n_i - s)(n_{is} - 1) \leq 0 \tag{23}$$

$$\pi_p(s,p,\theta_i) = -F(n_i - s) < 0 \tag{24}$$

$$\pi_{pp}(s,p,\theta_i) = -F'(n_i - s)\, n_{ip} > 0 \tag{25}$$

$$\pi_{sp}(s,p,\theta_i) = F'(n_i - s) + pF''(n_i - s)\, n_{ip} > 0 \tag{26}$$

where $n_i = n(s,p,\theta_i)$. Also, we trivially obtain that $\pi(s,p,\theta_2) > \pi(s,p,\theta_1)$. Summing up both possibilities we obtain the desired result.

Finally, $\pi(s,0,\theta_i) = \max_{e>0} \theta_i b(e) = \theta_i b(\widetilde{e})$, for all i. Thus, $P(s,0,\theta_i) =$

$\theta_i b(\widetilde{e})$, as desired. ■

Proof of Lemma 2.

In θ_i's compliance region, the expected payoff is $\theta_i b(s)$, that is, it does not depend on the probability of inspection. Therefore, indifference curves have an infinite slope. In the noncompliance region, the expected payoff is $\pi(s, p, \theta_i) = b(n(s, p, \theta_i)) - pF(n(s, p, \theta_i) - s)$. Implicitly differentiating $\pi(s, p, \theta_i) = k$, we obtain:

$$\frac{dp}{ds} |_{\pi=k} = \frac{pF'(n(s, p, \theta_i) - s)}{F(n(s, p, \theta_i) - s)} > 0 \tag{27}$$

Now, differentiating (27) with respect to s we have:

$$\frac{d^2p}{ds^2} |_{\pi=k} = \frac{F'}{F}\frac{dp}{ds} |_{\pi=k} + \frac{p}{F^2}\left(F''F - (F')^2\right)(n_{is} - 1) > 0 \tag{28}$$

since $n_{is} < 1$ and $F''F - (F')^2 < 0$.

(For analytical convenience, we prove the last part considering a continuum of types. The result is easily adapted to the case in which θ takes discrete values.)

In the compliance region, both types' indifference curves are vertical. In the partial complance region, θ_1's are vertical and θ_2's are strictly increasing In the full noncompliance region, we differentiate (27) with respect to θ to obtain:

$$\frac{d^2p}{dsd\theta} |_{\pi=k} = \frac{F''F - (F')^2}{(F)^2} pn_\theta(s, p, \theta) \tag{29}$$

Since $F''F-(F')^2<0$, $\frac{d^2p}{dsd\theta}\mid_{\pi=k}$ and $n_\theta(s,p,\theta)$ have the opposite sign. Differentiating (6) with respect to θ, we obtain $n_\theta(s,p,\theta)=-\frac{b'}{\theta b''-pF''}>0$. Therefore, $\frac{d^2p}{dsd\theta}\mid_{\pi=k}<0$, as desired. ■

Proof of Proposition 3.

The problem the regulator faces in the full compliance region is:

$$\begin{aligned}&Max_{s,p}\sum_{i=1}^{2}\gamma_i\left[\theta_i b\left(e\left(s,p,\theta_i\right)\right)-d\left(e\left(s,p,\theta_i\right)\right)\right]-cp\\&s.t.\ p\geq p^c\left(s,\theta_2\right)\end{aligned}\tag{30}$$

Since $p\geq p^c(s,\theta_2)$, we then have $e(s,p,\theta_i)=s$ for all i. The Lagrangian of problem (30) is the following:

$$L(s,p,\lambda)=\sum_{i=1}^{2}\gamma_i\left[\theta_i b(s)-d(s)\right]-cp-\lambda\left(p^c(s,\theta_2)-p\right)$$

where $\lambda\geq 0$ is the corresponding Lagrange multiplier. The solution is given by the following Kuhn-Tucker conditions:[20]

$$\gamma_1\left(\theta_1 b'(s)-d'(s)\right)+\gamma_2\left(\theta_2 b'(s)-d'(s)\right)-\lambda\frac{dp^c(s,\theta_2)}{ds}=0$$

$$c-\lambda=0$$

$$\lambda\left(p^c(s,\theta_2)-p\right)=0$$

Since $c>0$, we have $\lambda>0$ and $p^c(s,\theta_2)=p$, which lead us to the result. ■

Proof of Proposition 4.

[20] The assumptions of the model ensure that these conditions are necessary an sufficient for an interior optimum. This continues to hold for the remaining optimality results, below.

If the policy is pooling, the problem is:

$$\begin{aligned} & Max_{s,p} && \sum_{i=1}^{2} \gamma_i \left[\theta_i b\left(e\left(s,p,\theta_i\right)\right) - d\left(e\left(s,p,\theta_i\right)\right)\right] - cp \\ & s.t. && p \geq p^c\left(s,\theta_1\right) \end{aligned} \tag{31}$$

Now, we have $s = e\left(s,p,\theta_1\right)$ and $n_2 = e\left(s,p,\theta_2\right)$. The Kuhn-Tucker conditions are the following:

$$\gamma_1\left(\theta_1 b'\left(s\right) - d'\left(s\right)\right) + \gamma_2\left(\theta_2 b'\left(n_2\right) - d'\left(n_2\right)\right) n_{2s} - \lambda \frac{dp^c\left(s,\theta_1\right)}{ds} = 0$$

$$\gamma_2\left(\theta_2 b'\left(n_2\right) - d'\left(n_2\right)\right) n_{2p} - c + \lambda = 0$$

$$\lambda\left(p^c\left(s,\theta_1\right) - p\right) = 0$$

where $\lambda \geq 0$ is the corresponding Lagrange multiplier of the Lagrangian associated with problem (31).

Observe that $\lambda = c - \gamma_2\left(\theta_2 b'\left(n_2\right) - d'\left(n_2\right)\right) n_{2p} \geq 0$. If $\lambda = 0$, we have $c = \gamma_2\left(\theta_2 b'\left(n_2\right) - d'\left(n_2\right)\right) n_{2p}$, which implies that $c > \left(\theta^2 b'\left(n^2\right) - d'\left(n^2\right)\right) n_{2p}$, since $\gamma_2 < 1$. Therefore, since $p \geq p^c\left(s,\theta_1\right)$, welfare can increase if p decreases infinitesimally, since type θ_1 continues to comply with s. Therefore, $\lambda \geq 0$ and $p = p^c\left(s,\theta_1\right)$. Rearranging terms, we obtain the desired result.

If the policy is separating, the problem is:

$$
\begin{aligned}
& Max_{s_1,s_2,p_1,p_2}\left\{\gamma_1\left(\theta_1 b\left(s_1\right)-d\left(s_1\right)-p_1 c\right)+\gamma_2\left(\theta_2 b\left(n_2\right)-d\left(n_2\right)-p_2 c\right)\right\} \\
\text{s.t.} \quad & p_1 \geq p^c\left(s_1,\theta_1\right) \\
& P\left(s_1,p_1,\theta_1\right) \geq P\left(s_2,p_2,\theta_1\right) \\
& P\left(s_2,p_2,\theta_2\right) \geq P\left(s_1,p_1,\theta_2\right) \\
& s_1 \geq 0,\ s_2 \geq 0
\end{aligned} \tag{32}
$$

Considering $\lambda \geq 0$, $\mu_1 \geq 0$, $\mu_2 \geq 0$, $\eta_1 \geq 0$ and $\eta_1 \geq 0$ to be the corresponding Lagrange multipliers associated with each respective restriction in problem (32), and $P\left(s_1,p_1,\theta_1\right)=\theta_1 b\left(s\right)$, the Kuhn-Tucker conditions are the following:[21]

$$
\begin{aligned}
& \gamma_1\left(\theta_1 b'\left(s_1\right)-d'\left(s_1\right)\right)-\lambda\frac{dp^c\left(\theta_1\right)}{ds_1}=-\mu_1\frac{\partial P\left(s_1,p_1,\theta_1\right)}{\partial s_1}+\mu_2\frac{\partial P\left(s_1,p_1,\theta_2\right)}{\partial s_1} \\
& \lambda=\gamma_1 c \\
& \gamma_2\left(\theta_2 b'\left(n_2\right)-d'\left(n_2\right)\right)n_{2s}=\mu_1\frac{\partial P\left(s_2,p_2,\theta_1\right)}{\partial s_2}-\mu_2\frac{\partial P\left(s_2,p_2,\theta_2\right)}{\partial s_2}-\eta_2 \\
& \gamma_2\left(\left(\theta_2 b'\left(n_2\right)-d'\left(n_2\right)\right)n_{2p}-c\right)=\mu_1\frac{\partial P\left(s_2,p_2,\theta_1\right)}{\partial p_2}-\mu_2\frac{\partial P\left(s_2,p_2,\theta_2\right)}{\partial p_2} \\
& \lambda\left(p^c\left(s_1,\theta_1\right)-p_1\right)=0 \\
& \mu_1\left(P\left(s_2,p_2,\theta_1\right)-P\left(s_1,p_1,\theta_1\right)\right)=0 \\
& \mu_2\left(P\left(s_1,p_1,\theta_2\right)-P\left(s_2,p_2,\theta_2\right)\right)=0 \\
& \eta_1 s_1=\eta_2 s_2=0
\end{aligned}
$$

Assume first that $\mu_1=\mu_2=\eta_2=0$. This implies that $\theta_2 b'\left(n_2\right)-d'\left(n_2\right)=0$

[21] Trivially, $\eta_1=0$ since a separating policy implies that $s_1>s_2\geq 0$, as seen in Section 3.

and $c = 0$, since $n_{2p} < 0$. Since $c > 0$, either one of the incentive compatibility constraints must be binding or $\eta_2 \geq 0$.[22] Assume first that $\mu_1 \geq 0$ and $\mu_2 = \eta_2 = 0$. However, n_2 can be kept constant decreasing both (s_2, p_2) through expression (6) without distorting the incentive compatibility constraints.[23] Therefore, $\mu_1 \geq 0$, $\mu_2 = \eta_2 = 0$ is not possible.

Now, consider $\mu_1 = 0$, $\mu_2 = 0, \eta_2 \geq 0$. In this case, first order conditions would reduce to $\theta_1 b'(s_1) - d'(s_1) = c\frac{dp^c(\theta_1)}{ds_1}$ and $(\theta_2 b'(n_2) - d'(n_2))\, n_{2p} = c$, respectively, the optimal compliance solution for type θ_1 and the optimal noncompliance solution for type θ_2 if information were complete. But, in this case, type θ_2 would prefer to misreport its type. Therefore, $\mu_1 = 0$, $\mu_2 = 0, \eta_2 \geq 0$ is not possible. For the same reason, $\mu_1 \geq 0$, $\mu_2 = 0, \eta_2 \geq 0$ is not possible either.

Therefore, $\mu_1 = 0$ and $\mu_2 > 0$. As for η_2, both $\eta_2 = 0$ and $\eta_2 \geq 0$ are compatible with the solution, thus obtaining the desired result. ■

Proof of Proposition 6.

If the pooling policy induces full compliance, we have $\theta_1 b'(s^*) - d'(s^*) - c\frac{dp^c(s,\theta_2)}{ds} < 0$ and $\theta_2 b'(s^*) - d'(s^*) - c\frac{dp^c(s,\theta_2)}{ds} > 0$, by Proposition 3. This last expression can be written as $(\theta_2 b'(s^*) - d'(s^*))\frac{ds}{dp^c} - c < 0$, since $\frac{dp^c(s,\theta_2)}{ds} < 0$. By the continuity of the sanction at $e - s = 0$, we can infinitesimally decrease p to increase social welfare, without affecting θ_1's behavior. Therefore, a pooling policy which induces full compliance is never optimal.

Under partial compliance, we have $c \geq \gamma_2 \left(\theta^2 b'(n_2) - d'(n_2)\right) n_{2p}$ and $\theta^1 b'(s^*) -$

[22] It is easy to see that both incentive compatibility constraints cannot be binding except in the case of a pooling policy. Thus, $\mu_1 \geq 0$, $\mu_2 \geq 0$ is not possible if the policy is separating.

[23] To see this, consider (27) and $\frac{dp}{ds}\mid_{n_2} = -\frac{n_{2s}}{n_{2p}} = \frac{pF''}{F'}$ to conclude that $\frac{dp}{ds}\mid_{n_2} < \frac{dp}{ds}\mid_{P_2}$, since $(F')^2 - FF'' > 0$. By Lemma 2, we then have $\frac{dp}{ds}\mid_{n_2} < \frac{dp}{ds}\mid_{P_1}$.

$d'(s^*) > 0$, by part (i) of Proposition 4. Therefore, by the continuity of the sanction at $e - s = 0$ and $n_{ip} < 0$, we then have that an infinitesimal decrease in the probability increases social welfare since:

$$\gamma_1 \left(\theta^1 b'(n_1) - d'(n_1)\right) n_{1p} + \gamma_2 \left(\theta^2 b'(n_2) - d'(n_2)\right) n_{2p} - c < 0$$

Consequently, if the optimal policy is pooling, it can only induce full non-compliance. ■

References

[1] Andreoni, J. (1991), "Reasonable Doubt and the Optimal Magnitude of the Fines: Should the Penalty Fit the Crime?", The RAND Journal of Economics 22, 385-395.

[2] Arguedas, C. (2004), "Pollution Standards, Costly Monitoring and Fines", mimeo, Universidad Europea de Madrid.

[3] Arguedas, C. (2005), "Bargaining in Environmental Regulation Revisited", forthcoming, Journal of Environmental Economics and Management.

[4] Arguedas, C. and H. Hamoudi (2004), "Controlling Pollution with Relaxed Regulations", Journal of Regulatory Economics 26, 85-104.

[5] Bebchuk, L.A. and L. Kaplow (1991), "Optimal Sanctions when Individuals are Imperfectly Informed about the Probability of Apprehension", Discussion Paper 4/91, Harvard Law School.

[6] Becker, G.S. (1968), "Crime and Punishment: an Economic Approach", Journal of Political Economy 76, 169-217.

[7] Downing, P.B. and W.D. Watson (1974), "The Economics of Enforcing Air Pollution Controls", Journal of Environmental Economics and Management 1, 219-236.

[8] Ellis, G.M. (1992a), "Incentive Compatible Environmental Regulations", Natural Resource Modelling 6, 225-255.

[9] Ellis, G.M. (1992b), "Environmental Regulation with Incomplete Information and Imperfect Monitoring", Ph. D Dissertation, University of California, Berkeley.

[10] Gottinger, H.W. (2001), "Incentive Compatible Environmental Regulation", Journal of Environmental Management 63, 163-180.

[11] Grieson, R.E. and N. Singh (1990), "Regulating Externalities Through Testing", Journal of Public Economics 41, 369-387.

[12] Heyes, A.G. (1996), "Cutting Environmental Penalties to Protect the Environment", Journal of Public Economics 60, 251-265.

[13] Heyes, A.G. (2000), "Implementing Environmental Regulation: Enforcement and Compliance", Journal of Regulatory Economics 17, 107-129.

[14] Jebjerg, L. and H. Lando (1997), "Regulating a Polluting Firm Under Asymmetric Information", Environmental and Resource Economics 10, 267-284.

[15] Kaplow, L. (1990), "Optimal Deterrence, Uninformed Individuals and Acquiring Information about whether Acts Are Subject to Sanctions", Journal of Law and Economics Organization 6, 93-128.

[16] Mookherjee, D. and I. Png (1989), "Optimal Auditing, Insurance and Redistribution", Quarterly Journal of Economics.

[17] Polinsky, A.M. and S. Shavell (1979), "The Optimal Tradeoff Between the Probability and the Magnitude of the Fines", American Economic Review 69, 880-891.

[18] Polinsky, A.M. and S. Shavell (1990), "Enforcement Costs and the Optimal Magnitude and Probability of Fines", Discussion Paper 67, Stanford Law School.

[19] Polinsky, A.M. and S. Shavell (1991), "A Note on Optimal Fines when Wealth Varies Among Individuals", American Economic Review 81, 618-621.

[20] Reinganum, J.F. and L.L. Wilde (1985), "Income Tax Compliance in a Principal-Agent Framework", Journal of Public Economics 26, 1-18.

[21] Sandmo, A. (2002), "Efficient Environmental Policy with Imperfect Compliance", Environmental and Resource Economics 23, 85-103.

[22] Shavell, S. (1992), "A Note on Marginal Deterrence", International Review of Law and Economics 12, 345-355.

[23] Swierzbinsky, J.E. (1994), "Guilty Until Proven Innocent: Regulation with Costly and Limited Enforcement", Journal of Environmental Economics and Management 27, 127-146.

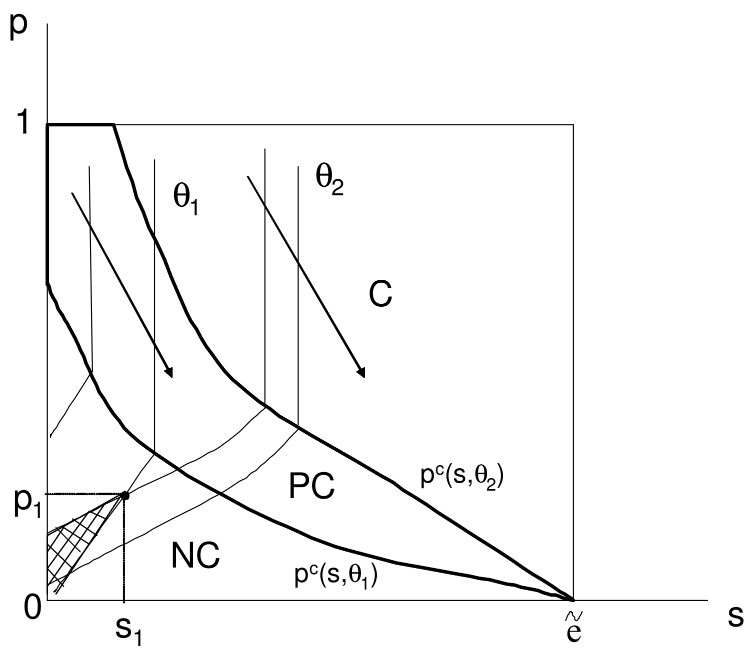

Figure 1: The compliance and noncompliance regions

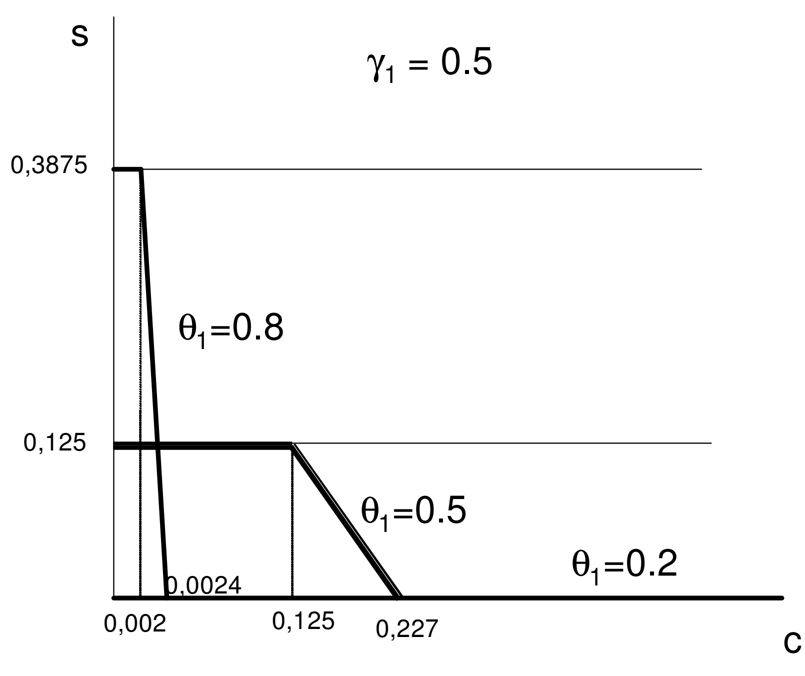

Figure 2: The optimal standard for pooling policies under large uncertainty

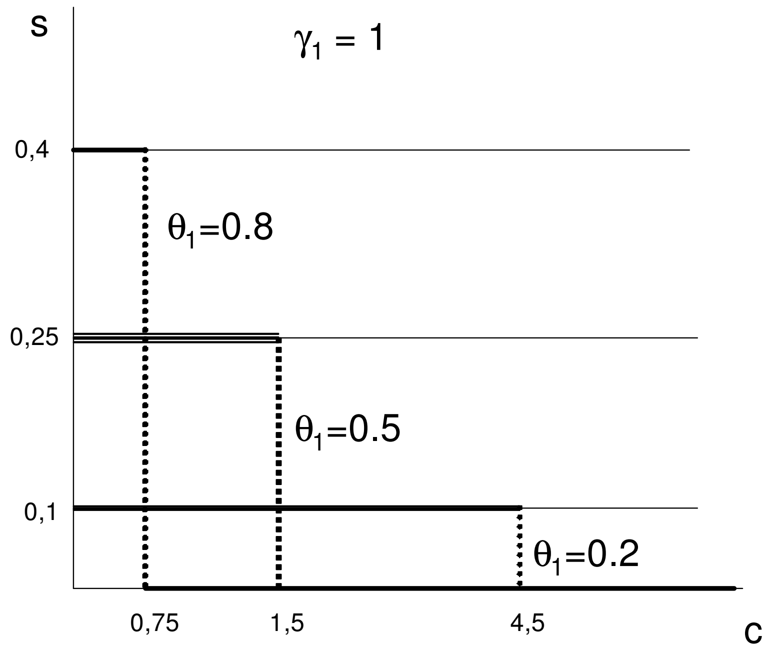

Figure 3: The optimal standard for pooling policies under no uncertainty

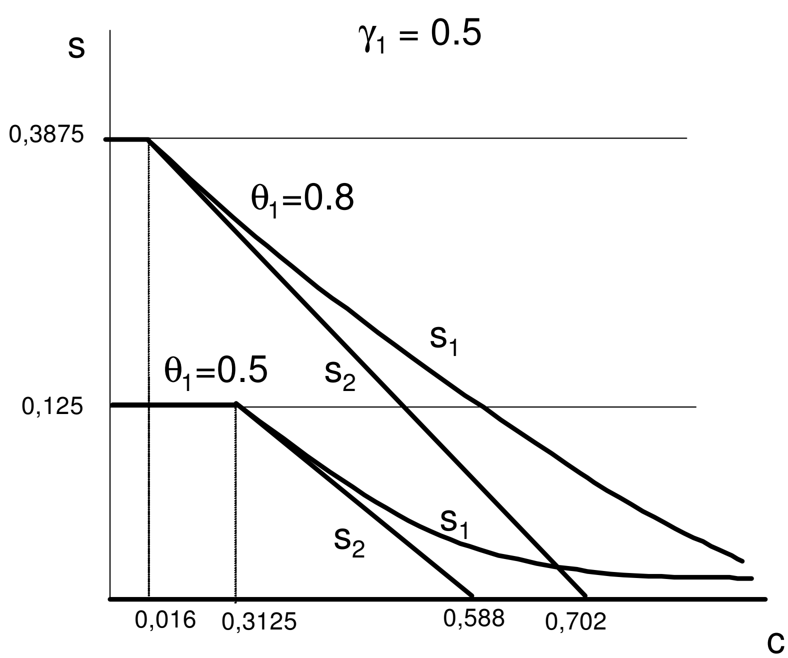

Figure 4: The optimal standards for partial compliance separating policy

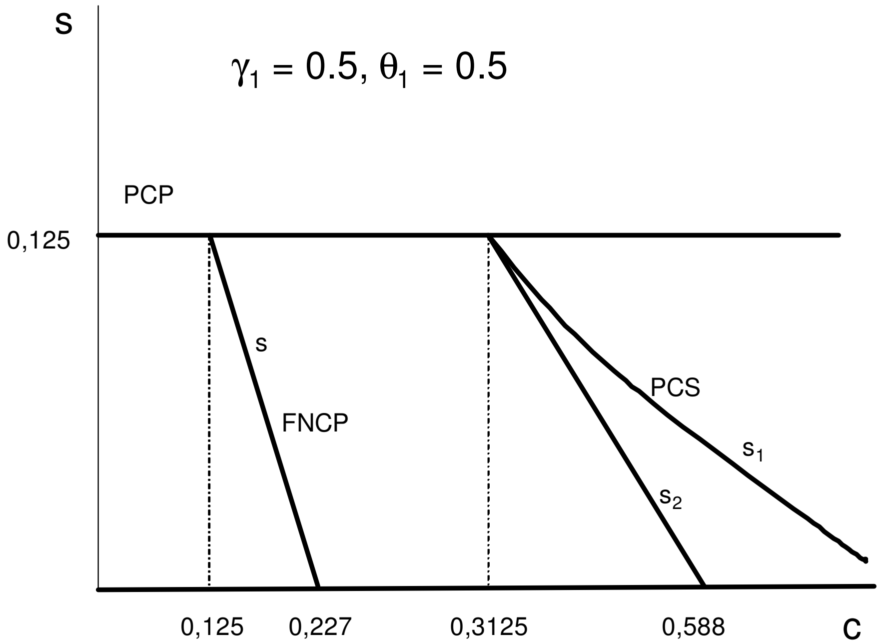

Figure 5: Comparison between pooling and separating policies

			FCP		PCP		FNCP		
c	θ_1	γ_1	s	sw	s	sw	s	p	sw
0,2	0,5	0,5	0,375	-0,06	0,125	0,041	0,029	0,443	0,043
0,5	0,5	0,5	0,375	-0,36	0,125	-0,11	0	0,406	-0,08
0,2	0,2	0,5	0,3	-0,11	0	-0,04	n	n	n
0,5	0,2	0,5	0,3	-0,41	0	-0,1	n	n	n
0,2	0,5	0,1	0,475	0,026	0,025	0,126	0	0,471	0,129
0,5	0,5	0,1	0,475	-0,27	0,025	-0,02	0	0,446	-0,01
0,2	0,8	0,1	0,49	0,04	0,378	0,081	0	0,474	0,143
0,5	0,8	0,1	0,49	-0,26	0,378	-0,16	0	0,449	0,005

Figure 6: Table 1. The pooling policies for various values of the parameters