Time-constrained Dynamic Mechanisms for College Admissions*

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Abstract

Recent literature shows that dynamic matching mechanisms may outperform standard mechanisms in delivering desirable results. We highlight an under-explored design dimension: the time constraints that students face under such a dynamic mechanism. First, we theoretically explore the effect of time constraints and show that the outcome can be worse than the outcome produced by the student-proposing deferred acceptance mechanism. Second, we present evidence from Inner Mongolian university admissions, which indicates that time constraints can prevent dynamic mechanisms from achieving stable outcomes, thereby creating losers and winners among students.

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1 Introduction

Research on market design has successfully helped to design or redesign many real-life applications in the past decades, including school choice and college admissions (Roth, 1982; Pathak, 2017). At the core of the design is the use of a matching mechanism that produces assignments based on students' submitted preferences, priorities, and available capacities. The mechanism is direct, as it asks students to submit their preferred choices, and it is static, as the submitted choices cannot be changed and are considered input to the mechanism. Strategyproof mechanisms such as the student-proposing deferred-acceptance (DA) mechanism, first proposed by Gale and Shapley (1962), are endorsed by the literature as students are ensured that they can do no better than submitting preferences truthfully. Recent studies point out that, however, strategypoof mechanisms may fail to convince all students to submit preferences truthfully in the lab and in the field. This arises due to, for instance, uncertainty (Pais and Pintér, 2008; Hassidim, Marciano, Romm, and Shorrer, 2017; Rees-Jones, 2018; Chen and Pereyra, 2019), complex choices (Shorrer and Sóvágó, 2021), or because agents have richer preferences - exhibiting loss aversion or having rank- dependent utilities- (Dreyfuss, Heffetz, and Rabin, 2022; Meisner and von Wangenheim, 2023; Meisner, 2023). While some non-truthful behaviors are consistent with equilibrium strategies, others could harm students and prevent the mechanisms from reaching desirable outcomes.

One explanation of non-truthful behaviors under uncertainty and complex choices is that students may not fully understand the mechanism. Therefore, providing them with more information might alleviate the problem because they can learn to be truthful over time. Indeed, recent experimental research reveals that dynamic mechanisms—in which matchings are implemented sequentially, and students can revise their choices after receiving information or feedback about their current allocations—can outperform the standard ones (Stephenson, 2016; Ding and Schotter, 2017; Klijn, Pais, and Vorsatz, 2019; Bó and Hakimov, 2020; Gong and Liang, 2024). While the experimental evidence supporting dynamic mechanisms often involves small markets, many markets in practice are considerably large, and the sequential process could be extremely time-consuming. As a result, clearinghouses often impose an ending time by which students have to finalize their submissions. The time given to students varies substantially across systems, ranging from hours to months (see Table 1), and the time allowed may not be enough for students to revise their choices. This leads to tremendous stress for students and sometimes failure to obtain desirable matches. A recent news article describes the stressful moments faced by students towards the end of their application in university admissions in the province of Inner Mongolia in China, which uses a dynamic mechanism:

> The final ten seconds [of the application] can decide the fate of students, especially

for those with lower scores. They must exercise extreme caution to avoid being kicked out by higher-scoring students in the last second . . . Her preferred choice is Beijing Normal University, which aimed to recruit ten students in Inner Mongolia. Her ranking at the university fluctuated between tenth and eleventh positions . . . She kept the choice despite her mother urging her to change. When the application stopped, she succeeded . . . Another student was seen in tears as she walked down [from an Internet cafe upstairs]. She failed to be accepted.[1]

Table 1: Dynamic mechanisms in practice

	Education system	Restrictions	Information	Sources
Brazil	University admissions	5 days	University cutoffs	Bo and Hakimov (2022)
Germany	University admissions	31 days	Ranking within each university applied to	Grenet, He, and Kübler (2022)
Inner Mongolia, China	University admissions	11 hours	University/program cutoffs, ranking within each university/program (first choice) applied to	Gong and Liang (2024) and own source
Wake County, US	School choice	2 weeks	Number of applicants in the first choice	Dur, Hammond, and Morrill (2017)

In this paper, we highlight this previously overlooked design dimension: the time constraint of dynamic mechanisms. We consider centralized college or university admissions in which stability—no student prefers a college assigned to a student with lower priority—is desirable.[2] Instead of submitting a full preferences list as in the standard mechanism, students submit one choice initially. These choices are then used to produce a tentative matching of students to colleges. In subsequent rounds, all students are allowed to revise their choices based on updated information about the tentative matching. During both the initial and revision rounds, students make choices simultaneously. The matching can end before students have enough time to revise, hence the time-constrained dynamic mechanism (TCDM). The final choices submitted determine the final matching.

We introduce a simple model where students have strict preferences over colleges, and their priorities at colleges are unique and determined by, for instance, their exam results.

[1]Published on July 1, 2022, available online (in Chinese) at: http://news.cyol.com/gb/articles/2022-07/01/content_ln6xVHWAQ.html

[2]Throughout the paper, we use colleges and universities interchangeably.

This framework facilitates clear comparisons with the DA mechanism. In this setting, DA finds the unique stable matching that is also Pareto efficient. Following the literature, we assume that students adopt *straightforward strategies*. Each student applies in the first round to her most preferred college and, whenever rejected, applies to the most preferred college where she has priority, or equivalently, with a cutoff score no higher than her score. Tentatively accepted students do not change applications. When students have enough time to revise their choices, following these strategies leads to a stable outcome equal to the one found by the DA mechanism (Bo and Hakimov, 2022).

In the first step, we show that, when the time constraint is binding, whenever a student receives a preferred outcome under TCDM, another student with higher priority is unassigned under TCDM while he would be assigned under DA. The intuition is simple: if a student prefers her assigned college under TCDM than under DA, it must be the case that under DA, a higher-ranked student occupies her place at that college. Consequently, either this high-ranked student is unassigned under TCDM, or he is assigned to a preferred college. In the latter case, the same reasoning applies to an even higher-ranked student.[3] This implies that TCDM is neither stable nor Pareto efficient. Moreover, we show that the outcome of DA can Pareto dominate that of TCDM, but never the other way around.

In the second step, we compare the distribution of the assignment probabilities over colleges under TCDM and DA from an ex-ante point of view. Suppose that students' preferences are independent and identically distributed (i.i.d.) according to a uniform distribution. TCDM gives students a (weakly) higher probability of being assigned to their first choice and of remaining unassigned but a lower probability of getting to the other options. We argue that the result still holds when preference may be correlated, as illustrated by simulations. This result implies, in a setting with cardinal utilities, that those students with a utility for their first option high enough may prefer TCDM. In addition, we study how the distribution over colleges under TCDM changes as we increase the number of steps. We show that the probability that every student is assigned to their first option (weakly) decreases. In contrast, the chance they are assigned to the rest of the options (weakly) increases.

Finally, using data from university admissions in the province of Inner Mongolia in China, we find evidence in line with our theoretical predictions of the time constraint effect. The Inner Mongolian procedure lasts for a day, starting in the early morning and finishing in the evening. Students initially apply to a university, and can revise their choices after receiving information about their tentative assignments. Every hour, the procedure provides students with information about the cutoffs of all universities and the applicants to each university. The hourly updated information allows us to construct a dataset to study students' applica-

[3] This exercise is equivalent to compare the outcome of DA with one of its intermediate outcomes when it stops before the final round.

tion behavior by hour and assignment outcomes. Students also observe privately real-time information regarding whether they are accepted at the university to which they applied and the rank among all currently accepted students. As a result, students' actual changes may be more frequent than what we observe in our data, which is constructed based on the hourly information.

Measuring the impact of time constraints is challenging, as we do not observe students' actual preferences in the data generated by an indirect mechanism. For this, we focus on the effect on final assignments. Over 10 percent of students in our data are unassigned in the end. In particular, we consider those unassigned students with a score higher than the final cutoff of a university where they applied at some point previously. This measure is very demanding and probably underestimates the effect of time constraints. However, these students would likely have applied to another university if they had more time. We observe in the data that more than 5 percent of the students are in this situation. Under the assumption that these students prefer a university they have previously chosen to being unassigned, this also indicates that the final matching is not stable as these students have justified envy over other students' assignments. Thus, the effect of time constraints is not only theoretically interesting, but also can negatively impact real-life cases.[4]

A policy change to alleviate the negative effects of time constraints could involve a final phase where students submit a rank ordered list of colleges, followed by the use of DA. This was originally proposed by Bo and Hakimov (2022). However, this proposed mechanism may undermine the transparency inherent in the current procedure, making its implementation less attractive. Indeed, transparency was the main driving force that policymakers cited when transitioning to the dynamic mechanism. Prior to the change, the clearinghouse experienced substantial pressure from parents and students actively seeking information related to the final match, including verification of their final assignments and investigation of potential alternative options. Some even exploited their connections to inquire information, raising concerns about unfairness. The current system could trace its origin back to the SARS outbreak in 2003. Restrictions on gatherings implied that parents and students could not visit the clearing house to verify information, resorting instead to online requests. This unintended transition underscored the value of the online platform for the clearinghouse. They realized that such a platform, which provides information for everyone, could reduce their unnecessary workload during the whole process while ensuring transparency.[5]

We contribute to the literature by pointing out that the existence of a binding time

[4]Kang, Chen, and Deng (2023) analyze the Inner Mongolian mechanism empirically and find that the Inner Mongolian mechanism performs worse than the Parallel mechanism due to justified envy, which is in line with our findings.

[5]See news report published on September 4, 2014, available online (in Chinese) at https://www.whb.cn/zhuzhan/xue/20140904/14043.html.

constraint may dampen the good properties of dynamic implementations of standard mechanisms. Starting from the seminal paper by Gale and Shapley (1962) on college admission problems, the Gale-Shapley student-proposing DA mechanism has been the workhorse in school choice (Abdulkadiroğlu and Sönmez, 2003) and college admissions (Balinski and Sönmez, 1999). This mechanism is strategyproof for students and produces a stable outcome. Notably, it is direct and static. Recently, non-direct dynamic mechanisms have received attention. The literature has focused on the comparison between the standard static version of DA and its dynamic counterpart (Bó and Hakimov, 2020; Gong and Liang, 2024; Klijn, Pais, and Vorsatz, 2019; Stephenson, 2016; Dur, Hammond, and Kesten, 2021). Their results highlight the advantages of the dynamic versions of DA over the static implementation through laboratory experiments. In particular, they find that dynamic mechanisms can produce more stable outcomes, lead to higher efficiency, and induce more truth-telling behaviors.

Bo and Hakimov (2022) introduce a family of iterative deferred acceptance mechanisms, where they identify a simple strategy, the straightforward strategy, which consists in choosing the most preferred college in each round of the mechanism. The authors show that straightforward strategy can be a robust equilibrium that yields the student optimal stable matching, whenever students are not restricted in the number of changes they can make. The main difference between our paper and theirs is that they assume students have enough time to revise and change their applications. To put differently, the mechanism does not stop because it reaches the maximum number of rounds, but because it reaches a stable matching where no student wants to change.

In another related paper, Gong and Liang (2024) model the dynamic game induced by the TCDM as a Poisson process with parameter λ. In particular, each student has an individual Poisson clock and can change their application every time the clock rings. Thus, if the mechanism runs up to time T, the average number of rings is λT. Gong and Liang (2024) assume that λ is large enough, so the students are not constrained in the number of changes they can do.

There is a growing literature evaluating the impacts of the sequential implementation and mechanism-generated information using field data. Dur, Hammond, and Morrill (2017) also study a dynamic mechanism where students can change their preferences before the final assignment. In their case, students do not observe the tentative outcome, only the number of students that have ranked each school first, in the coordination phase. The Boston mechanism (also known as the Immediate Acceptance mechanism) is used afterwards for the matching. They find that sophisticated students are assigned to higher performing schools. Grenet, He, and Kübler (2022) study a multi-offer dynamic mechanism in the German university admission systems and find that the combination of a sequential mechanism and a direct mechanism allows students to better discover their preferences. Luflade (2018) finds that in the Tunisian university admissions, offering information about vacancies to students under DA with restricted number of choices could yield similar outcome as DA without restriction.

The TCDM when students do not have enough time to revise their choices resembles the DA in constrained school choice (Calsamiglia, Haeringer, and Klijn, 2010; Haeringer and Klijn, 2009). Indeed, the final round of the TCDM is equivalent to constrained DA when students rank only one option. However, the outcomes of these two mechanisms are in general different as TCDM allows students to skip choices where they will not be accepted because of their priority. In other words, under TCDM, students do not waste places in their submitted preferences by ranking unreachable colleges.

The organization of the paper is as follows. We introduce the model setup and related mechanisms in Section 2. Central to our analysis is the assumption that students adopt a straightforward strategy. Section 3 explains when such a strategy can arise at equilibrium. Section 4 presents the main theoretical results of the paper. Section 5 describes the Inner Mongolian procedure, details the data construction, and presents the evidence supporting the impact of time constraint. Finally, we conclude in Section 6. Omitted proofs and information can be found in the Appendix.

2 Model setup and mechanisms

There is a set of **students** I to be be assigned to a set of **colleges** $C \cup \{\emptyset\}$, where $\emptyset$ denotes a student's outside option. Each student $i \in I$ has a strict preference order $\succ_i$ defined over the set of colleges, including the outside option. Each college $c \in C$ has capacity q_c. Students' priorities at colleges are assumed to be strict and based on (non-negative) students' scores from a centralized exam. Then, there is a common priority order of students, and those with a higher score have priority over those with lower scores.[6]

A **matching** is a function $\mu : I \to C \cup \{\emptyset\}$ that matches students to colleges as well to the empty set (which means that they are unassigned), satisfying:

1. $\mu(i) \in C \cup \{\emptyset\}$ for every $i \in I$, and
2. $|\mu^{-1}(c)| \leq q_c$ for each college c.

A student i blocks a matching μ if $\emptyset \succ_i \mu(i)$. A student-college pair (i, c) blocks a matching μ if student i prefers c to her current match at μ, and either college c does not fill its capacity, or it is matched to another student who has a lower priority than i. A matching

[6] All our results hold for the case where priorities may differ across colleges. The only difference is in Proposition 3 where the student who is not assigned, may not have higher priority at the corresponding college. We keep the assumption of a unique priority in order to make the arguments clearer.

is **stable** if it is not blocked by a student or by a student-college pair. A matching is **efficient** if there is no other matching such that all students are weakly better off, and some student is strictly better off. The fact that priorities are common across colleges, implies that there is a unique stable matching. In addition, the unique stable matching is efficient.

Many real-life applications release information on their admission thresholds. Given a matching μ and a college c, the **cutoff** of c under μ is the minimum score among all the students assigned to c if the college reaches its capacity, and 0 if there is remaining capacity. The cutoffs here serve the role of prices for standard exchange or production economies that clear supply and demand (Azevedo and Leshno, 2016).

A **mechanism** is a function that selects a matching for each set of students' preferences and colleges' priorities and capacities. We describe below the two mechanisms studied in our paper.[7]

The Student-proposing Deferred Acceptance mechanism (DA) is a direct mechanism. It asks students to submit a rank-ordered list of preferences, which together with the priorities and capacities of colleges will be used to determine the final allocation in the following way:

- Round 1, consider all students' most preferred submitted choice. Each college accepts tentatively students with the highest priority up to its capacity, and rejects the rest of students.
- Round r, for those students who were rejected in the previous round consider their next submitted choice. Each college accepts tentatively students that apply to it or that were tentatively accepted in the previous round, following priority order up to its capacity. The rest of students are rejected.
- The process continues in this way until every student has either been tentatively accepted to a college or rejected by all colleges in her list, at which point all acceptances become definitive.

DA therefore runs the algorithm on behalf of students according to their submitted preferences, while respecting colleges' priority and capacity constraints. It induces a static preferences-revelation game where truth-telling is a weakly dominant strategy for students (Dubins and Freedman, 1981; Roth, 1982). To allow for comparison, we will assume in the rest of the paper that students submit preferences truthfully under DA.

[7] In the language of matching with contracts (Hatfield and Milgrom, 2005; Aygün and Sönmez, 2013), note that colleges' choice functions are such that each college chooses, among the students with an exam's score higher than its cutoff, those with the highest scores up to its capacity. This implies that the choice function verifies (among others) the condition of irrelevance of rejected contract (Bo and Hakimov, 2022).

The Time-constrained Dynamic Mechanism (TCDM) is an indirect mechanism, departing from the direct mechanism studied in classic matching theory. It consists of $T \geq 1$ rounds, and operates as follows:

- Round 1: Students apply to a college. Each college considers all the students that have applied to it, and accepts tentatively students with the highest priority up to its capacity. The rest of students are rejected. At the end of this round, information including the tentative matching outcome (or the acceptance cutoff of each college) is updated.

- Round $1 < t \leq T$: Given the information revealed by the mechanism, students can keep or revise their previous choices. Each college accepts tentatively those with the highest priority up to its capacity, and rejects the lowest-ranked students in excess of its capacity. At the end of this round, information including the tentative matching outcome (or the acceptance cutoff of each college) is updated.

- The match terminates when T rounds are reached. The tentative matching is then finalized.

The game induced by the TCDM is a dynamic game with T rounds. At each period the set of actions of student i is $A_i = C \cup \{\emptyset\}$. A history is a sequence of applications where each student may have applied more than once. A strategy for a student indicates an action for each history where the student has to play (that is, for each round). In particular, when a student applies in a given round, she does not know the colleges to which other students apply in the same round. She knows to which they applied in the previous rounds, and the colleges' cutoffs after the last round. The choices in the last round determine the final outcome of the game. A formal description of the extensive–form game induced by TCDM is presented in Appendix A.

Like the sequential mechanisms studied in the literature (Klijn, Pais, and Vorsatz, 2019; Bo and Hakimov, 2022), TCDM induces a dynamic game where students do not have dominant strategies, in contrast to DA. Unlike the sequential mechanisms studied in the literature though, TCDM allows students to change a choice without having been rejected.[8] These characteristics make the strategies of each student under TCDM more complex than under DA. We consider the following straightforward strategy, first introduced by Bo and Hakimov (2022), which is a natural extension of truthful reporting under DA to our setting. Below, we provide an intuitive definition of this strategy to facilitate a clearer understanding (see Appendix A for a formal definition).

[8]An adaptation of DA in the sequential context would require only students rejected by a college to change their choices, as studied in the literature. However, the TCDM we describe here is more general, and it can be compared to the literature by restricting students application behavior as we describe below.

Definition 1. (*Straightforward strategy*) A student follows a straightforward strategy if at the first round she applies to her most preferred college, and whenever rejected she applies to the most preferred college among those where her score in the exam is higher than the college's cutoff.

An inherent implication of the straightforward strategy is that a student will only change her choice if she is rejected. *We assume in the paper that students follow straightforward strategies under TCDM.* When the TCDM stops at T before reaching the stable matching, a student may be incentivized to deviate by playing a different strategy. Thus, in the presence of a time constraint, a profile of straightforward strategies may not be an equilibrium.[9] Nevertheless, we will focus on this strategy for several reasons. First, as mentioned earlier, we aim to analyze the effects of imposing a limit on the number of rounds in the sequential mechanism. All previous papers have assumed that the constraint in time is not binding, and found experimental evidence that a high proportion of subjects play this strategy (Bo and Hakimov, 2022; Gong and Liang, 2024). This motivates us to keep all other assumptions, and change only the one related to the maximum number of allowed rounds. Second, as we will show in the empirical section, there is evidence suggesting that the strategies by the majority of students are consistent with the straightforward strategy.

3 Equilibrium Analysis

The equilibria of the game induced by the TCDM are complex. In this section, we explore possible equilibria under different assumptions about the information available to the students. Starting with complete information, we then examine the case of incomplete information. In both scenarios, we analyze when straightforward strategies may be an equilibrium.

3.1 Complete information.

When the time constraint is set at $T = 1$, the game induced by the TCDM is static. In this case, all students who are assigned to a college in the stable matching have a unique equilibrium strategy: apply to this choice. Indeed, suppose a student is assigned to college c in the stable matching. When all students apply to the college where they are assigned in the stable matching, the student is assigned c. Suppose she deviates by applying to $c' \neq c$. If c is preferred to c', she will obtain a worse outcome (either being assigned to c' or the outside

[9] For example, suppose there are two students, two schools (each with a capacity of one seat), and the time constraint is set at $T = 1$. Moreover, assume the students have the same most preferred college, and both colleges are preferred by the students to the outside option. If both follow a straightforward strategy, the student with the lower priority will end up unassigned. However, if she applies to her second option in the first step, she will get it.

option). Conversely, if c' is preferred to c, she will be rejected and receive the outside option which is less preferred than c. Those students who are not assigned in the stable matching, in equilibrium, may apply to any college (among those that are preferred to the outside option). In this case, a profile of straightforward strategies is an equilibrium only when every student is either assigned to her *top* choice or not assigned in the stable matching.

When the time constraint is set at $T > 1$ rounds, consider the last round of the game. Suppose a profile of applications such that the outcome of the game, μ, is different from the unique stable matching, denoted by μ^S. Let i_1 be the first student in the priority ranking. If i_1 is not assigned to the same college in μ than in μ^S (which is her most preferred college), she can deviate by applying to her college under μ^S. By doing so, she will be better off. Take the second student in the priority ranking, i_2. If this student is not assigned to the same college in μ as in μ^S, she can deviate by applying to her assignment in μ^S. By doing so, she will be assigned and, consequently, better off. Continuing this logic, we can show that at equilibrium, every student is assigned the same college in both matchings μ and μ^S. *Thus, when students have complete information, every equilibrium outcome of the game induced by the TCDM coincides with the unique stable matching.* However, many strategy profiles sustain this matching in equilibrium because the outcome of the game is determined only by the choices in the last round.

Also, as T increases, the set of equilibrium strategies always enlarges. Formally, consider an equilibrium strategy for a certain T. This implies that the stable matching is reached at time T or before. For $T + 1$, consider for each $t \leq T$, the same strategy as before, and for $T + 1$, let the action be such that students do not change the action played in the previous step. This strategy is an equilibrium for the game with $T + 1$ steps.

Next, we discuss when straightforward strategies can be sustained at equilibrium.

The case of $T \geq |I|$. When the maximum number of rounds is no less than the number of students, time constraint is not binding and a profile of straightforward strategies is an equilibrium which yields the unique stable matching. Intuitively, the history defined by a profile of straightforward strategies is very similar to the steps of the DA. Students apply to their most preferred college at the first round, and then, after being rejected, they apply to the most preferred college among those *where their scores are higher than the respective college's cutoff.* The difference with DA is that students do not apply to those colleges where they have a score lower than the cutoff of the college. The maximum number of steps needed to reach the stable matching is attained when all students have the same preferences, all colleges are preferred over outside option for every student, all colleges have unit capacity, and $|I| = |C|$. In the case, the stable matching is reached after $|I|$ steps.

We establish formally these results as benchmarks for comparison. The proofs are detailed in Appendix B.

Proposition 1. *Suppose the time constraint is not binding and students follow straightforward strategies. Then, the mechanism stops if and only if it reaches the (unique) stable matching.*

Proposition 2. *Suppose the time constraint is not binding. Then a profile where students follow straightforward strategies is a Nash equilibrium.*

The first proposition aligns with the findings from Bo and Hakimov (2022) (Corollary 1), which are established in a more general context of matching with contracts. We give a simple proof of the second proposition which is a special case of Corollary 2 of Bo and Hakimov (2022). The proof helps to illustrate the dynamics of TCDM.

The case of $T < |I|$. For every $T < |I|$, there are students' preferences such that a profile of straightforward strategies does not constitute an equilibrium. This is the case, for example, that we discussed above with $T = 1$. Intuitively, the likelihood of straightforward strategy being an equilibrium diminishes as students' preferences become more correlated.

3.2 Incomplete information.

In practice, students often make decisions with incomplete information. Suppose they know their own preferences, they typically lack information on the preferences of others. In a dynamic mechanism where everyone submits one choice and can revise their choices at any time, they do not observe the full preferences of others. Under incomplete information, when time constraint is not binding, a profile where students follow straightforward strategies is an ordinal Bayesian Nash equilibrium as shown in Bó and Hakimov (2021). When time constraint is binding, one natural way to study the equilibrium strategies is to assume that students play myopic strategies by maximizing their expected utility at each round. For each student, the expected utility of applying to a college depends on her (cardinal) utility for the college, and the probability of being assigned to it. Formally, let u_{ic} be the utility of student i for college c. We use the normalization $u_{i\emptyset} = 0$ for every $i \in I$. We denote as p^t_{ic} the probability of student i being assigned to college c at round t. This probability hinges on the preferences and strategies of other students, the priority order, and the capacity of the colleges. Student i maximizes her expected utility at each round by applying to c' such that:

$$c' = \underset{c \in C \cup \{\emptyset\}}{\operatorname{argmax}} \; p^t_{ic} u_{ic}.$$

Consider the case where every college has *unit* capacity and preferences are independent and identically distributed according to a uniform distribution.[10] Take a student i in the first round of the mechanism, and assume all other students follow straightforward strategies. If i is the first student in the priority order, she is guaranteed assignment to the college that she applies to. Therefore, i should apply to her most preferred college to maximize her expected utility. Now, suppose i is the second student, with c being her most preferred college. If she applies to c, the only scenario where she would not be assigned to it is when the first student's most preferred college is also c. However, as preferences are uniformly distributed and independent across students, this probability does not depend on the identity of college c. Then, conditional on applying, the probability of being assigned to her most preferred college is the same as the probability of being assigned to her second most preferred college, and the same for all other colleges. Thus, the student maximizes her expected utility in the first round by applying to her most preferred college. The logic extends to i being third or lower in the priority.

In subsequent rounds, a similar rationale holds. The distinction now is that students receive updated information regarding the matching from the previous round that allows them to see which choices are "feasible", that is, colleges whose cutoffs are below their scores. For student i, if all higher priority students are assigned in this round, i is certain to be assigned to any college with a cutoff below her score. Therefore, she maximizes her expected utility by applying to her most preferred college among these feasible ones. If some higher priority students remain unassigned, then the probability of being assigned to a college is either zero (if the cutoff is higher that her score, because cutoffs do not decrease when students follow straightforward strategies) or is the same for any other feasible college. Indeed, with uniformly distributed and independent preferences across students, the probability such that a higher priority student will apply to a feasible college does not depend on which is the college. By applying to the most preferred college with her score range, the student maximizes her expected utility. Consequently, by maximizing expected utility each round, every student follows a straightforward strategy.

This result does not hold when colleges have larger capacities. In such cases, students must consider the remaining capacity of each college at each round.[11] However, one could recover this result when, for example, students' cardinal utilities for each college are different enough, so probabilities are of second order of importance when computing the expected

[10] The assumption on preference could be strong, but it is reasonable if we restrict to students applying for a college within a batch, like the case in our empirical exercise.

[11] Solving the equilibrium strategies for the general problem of heterogeneous preferences and a finite number of students is challenging, even in the static case. For example, Chade, Lewis, and Smith (2014) analyze a related problem with two colleges, and a continuum of students with homogeneous preferences. Aykol, Hafalir, and Miralles (2023) consider a model with two colleges and three students with heterogeneous preferences, but without priorities.

utilities.

4 Comparisons with DA

In this section, we discuss the effects of a binding time constraint on the outcome of the TCDM, and we compare it with the outcome of DA.

4.1 Ex-post comparisons

At individual level, there are two main effects when the time constraint is binding. A first effect is what we call the *direct time constraint effect*. A student is affected by it when she is rejected in the last round from a college where she was held in the $T-1$ round, and did not have enough time to apply to her next preferred college. The effect is (weakly) negative as the student is unassigned under TCDM. We illustrate it in the example below.

Example 1 (**The direct time constraint effect**). Consider 4 students and 4 colleges, each with one seat. Suppose that TCDM has a maximum number of rounds of 2 ($T = 2$). The priorities are such that i_1 has the highest priority, followed by i_2, i_3, and i_4. Students' preferences are as follows:

$$\begin{array}{lllll}
\succ_1: & c_1 & c_2 & c_3 & c_4 \\
\succ_2: & c_1 & c_2 & c_4 & c_3 \\
\succ_3: & c_2 & c_3 & c_1 & c_4 \\
\succ_4: & c_3 & c_4 & c_1 & c_2
\end{array}$$

We first look at the matching process under TCDM (see panel (a) of Figure 1). At $t = 1$, students apply to their most preferred colleges, and all students except i_2 are temporarily assigned to their first choices. At $t = 2$, students observe the matchings from the previous round. All students that are temporarily assigned to their most preferred colleges do not need to revise their choices (which is indicated in the figure by letting them applying to the same choice this round). The only rejected student, i_2, observes that her second choice, c_2, holds i_3 who has a lower priority than her. Then she applies to c_2. Since this is the closing round, c_2 accepts i_2 and rejects i_3, and i_3 is unassigned because he does not have time anymore to revise his choice. Under DA (see panel (b) of Figure 1), i_3 is assigned to his second choice, c_3. Therefore, student i_3 is harmed by the direct time-constraint effect when comparing to her outcome under DA. □

Remark 1. In our framework DA is efficient because there is a unique priority order. Therefore, it is never the case that TCDM dominates the outcome of DA. However, it is easy to

Figure 1: The direct time constraint effect

	$t=1$	$t=2$
i_1 :	c_1	$\boxed{c_1}$
i_2 :	c_1	$\boxed{c_2}$
i_3 :	c_2	c_2
i_4 :	c_3	$\boxed{c_3}$

(a) TCDM with $T = 2$

	$r=1$	$r=2$	$r=3$
i_1 :	c_1	c_1	$\boxed{c_1}$
i_2 :	c_1	c_2	$\boxed{c_2}$
i_3 :	c_2	c_2	$\boxed{c_3}$
i_4 :	c_3	c_3	$\boxed{c_4}$

(b) DA

Notes: The dashed box indicates tentative assignment, whereas the solid box indicates final assignment.

construct examples where DA dominates TCDM.[12]

The second effect, the *indirect time constraint effect*, is a consequence that, for a given student, higher ranked students can also be rejected in the last minute, without enough time to apply to all their preferred options. This indirect time-constraint effect for this given student is thus positive since the college assigned under TCDM is preferred to the one assigned under DA.

Example 2 (**The indirect time constraint effect**). Consider the same setting as in Example 1. Student i_4 is assigned to her first choice, c_3. This is because, i_3, who has higher priority, is rejected in the last round, and she does not have enough time to apply to her next choice which is the first choice of i_4. If however we use DA, i_4 will not be assigned to her first choice, as DA resumes to the third round, and i_3 is able to out-compete i_4 at c_3, and i_4 is assigned to her second choice in round 3. $\square$

Remark 2. The previous examples show that:

1. TCDM is neither stable, nor efficient.

2. There may be losers and winners when changing from DA to TCDM.

From an ex-post perspective, it could be that a student receives a preferred college under DA than under TCDM (i_3 in Example 1), or the other way around (i_4 in Example 2). It also could be that a student is assigned under one mechanism but not under the other. This makes it difficult to obtain general results comparing the two mechanisms. However, as we show in Proposition 3, if a student receives a preferred assignment under TCDM than under DA, there is a higher-ranked student who is not assigned under TCDM.

[12] In Example 1, if the preferences of i_4 are changed to $\succ_4$: c_4 c_3 c_1 c_2. Then, DA dominates TCDM as every student but i_3 receives the same assignment under both mechanisms, while i_3 is not assigned under TCDM and she is assigned to c_3 under DA.

Proposition 3. *Suppose students play straightforward strategies. Consider a student who prefers the college assigned by TCDM over the college assigned under DA. Then, there is a student with higher priority who is not assigned under TCDM.*

Proof. Let college c be the college where student i is rejected under DA but assigned to under TCDM. Then, there is a higher-ranked student who does not apply to c under TCDM, and does apply under DA. If this last student is not assigned under TCDM the proof is completed. If she is assigned under TCDM to a preferred option, denoted by c', hence, there is another higher-ranked student who does not apply to c' under TCDM, but applies to that option under DA. If this new student is not assigned under TCDM the proof ends, otherwise, we repeat the reasoning and as there is a finite number of students, we can always find an unassigned student under TCDM. □

It is worth noting that the assumption of straightforward strategies is necessary for Proposition 3. In the proof, we consider a student who prefers her outcome under TCDM than under DA. Without the assumption, we can only affirm that there is a higher-ranked student who prefers her assignment under DA than under TCDM, but she may be assigned. When students follow straightforward strategies, we can do a more precise statement: this higher-ranked student must be unassigned under TCDM.

4.2 Ex-ante comparisons

We now analyze the difference between DA and TCDM from an ex-ante perspective. We assume that students' preferences are independent and identically distributed (i.i.d.) according to a uniform distribution. We think that the assumption makes sense in a situation where student preferences are partitioned into tiers such that they agree the colleges from a higher tier are better than the next tier, but their preferences within the tier are not highly correlated. We first present an example of i.i.d. and uniform preferences to establish intuitions for our theoretical results. Of course, student preferences could be correlated, so we present some simulations in Appendix C in order to explore the extension of our results in this framework.

Example 3. Consider the case of 4 students to be assigned to 4 colleges, each with only one seat. Assume as before that TCDM consists of 2 steps ($T = 2$). To compute the probability of a given student of being assigned to each of her choices as well as being unassigned, we first fix some preferences for the student. Then, we compute for each of the preference profiles of the rest of the students, the assignment outcome under TCDM when all students follow straightforward strategies. Finally, we compute the probability of being assigned to each of her choices and of being unassigned, as the number of profiles where the student is assigned to each option (or she is not assigned) over the total number of profiles. In the same way we compute the probability under DA. Table 2 presents the results.

Table 2: Probabilities of assignment under the two mechanisms

	TCDM (T=2)					DA				
Choice	1st	2nd	3rd	4th	Unassigned	1st	2nd	3rd	4th	Unassigned
i_1	1	0	0	0	0	1	0	0	0	0
i_2	0.75	0.25	0	0	0	0.75	0.25	0	0	0
i_3	0.5	0.29	0.12	0	0.09	0.5	0.33	0.17	0	0
i_4	0.27	0.20	0.15	0.09	0.29	0.25	0.25	0.25	0.25	0

First of all, neither i_1 nor i_2 are constrained by time. For i_1, since she has the top priority, she is always assigned to her top choice under both mechanisms. For i_2, as she has the second highest priority, the probabilities to be assigned to her l-th choice under both mechanisms are also identical.

The remaining students can be however pressed by the time constraint. For i_3, the probabilities with which she is assigned to her first choice under both mechanisms are identical. This is because the two students with higher priority have an equal probability to be assigned to any choice under both mechanisms. However, she has a lower probability to be assigned to the rest of the options under TCDM. To see why, recall that there are both positive and negative effects when comparing TCDM to DA. The direct (and negative) time-constraint effect (Example 1) comes into play only for the $l \geq 2$ choice, which results into a lower probability to be assigned to her $l \geq 2$ option. For this student then, the distribution over her choices under DA first order stochastic dominates the distribution under TCDM.

For student i_4, she has a higher probability to be assigned to her top choice under TCDM, but a lower probability to be assigned to the rest of options. To see why she has a higher chance of getting her top choice under TCDM, first note that she can benefit from the fact i_3 may not have enough time to revise her choice, and therefore not getting her next choice which may happen to be student i_4's top choice. On the other hand, i_4 can also be hurt by the time constraint, that is, not having sufficient time to revise herself. Although, whenever this occurs, i_4 is not able to be assigned to her top choice neither under DA. Therefore, when it comes to her top choice, only the indirect time-constraint effect binds. This is no longer the case when we move to her other options, where both of the direct and indirect time-constraint effects bind, and given that she has the lowest priority among all students, time constraint hurts her more. $\square$

In the next proposition, we compare the distribution over ranked options of each student under TCDM and DA. For this, we need to differentiate those students that are not constrained from those that are potentially constrained under TCDM. Relabel colleges such that college c_1 is the one with the lowest capacity, c_2 the one with the second lowest capacity, and so on and so forth (two colleges with the same capacity are ordered in an arbitrary way). Let $\boldsymbol{\kappa}$ be the vector of colleges' capacities after reordering. Then, for each $k = 1, \ldots, |C|$, let $\Sigma(k)$ be the sum of first k elements of $\boldsymbol{\kappa}$.

Clearly, the student with the highest priority is not constrained under TCDM as she always gets her first choice. The other students may be constrained depending on the number of steps of TCDM (T), and the vector of capacities. For example, even when $T = 1$, i_2 is not constrained if $\Sigma(1) \geq 2$. Indeed, although i_2 may have the same top choice as i_1, because $\Sigma(1) \geq 2$, there is enough space for both students at that college. This motivates the following definition.

Definition 2. A student is *unconstrained* under TCDM if her priority is between 1 and $\Sigma(T)$. Otherwise, we say the the student is *constrained*.[13]

Proposition 4. *Suppose students play straightforward strategies, and students' preferences are i.i.d. distributed according to a uniform distribution. Consider any student, and let p_l^{TCDM} and p_l^{DA} denote the probability with which the student is assigned to her l-th option (where $l = \emptyset$ represents the option of not being assigned) under TCDM and DA, respectively. Then,*

1. *$p_l^{TCDM} = p_l^{DA}$, for every $l \in \{1, \ldots, |C|, \emptyset\}$ if the student is unconstrained.*
2. *$p_1^{TCDM} \geq p_1^{DA}$, for every constrained student.*
3. *$p_l^{TCDM} \leq p_l^{DA}$, with $l \geq 2$, for every constrained student.*
4. *$p_\emptyset^{TCDM} \geq p_\emptyset^{DA}$, for every constrained student.*

Proof. The result for unconstrained students follows from the fact that as they are not constrained by the number of rounds, then playing straightforward strategies leads to the same outcome as under DA.

For the rest of the students, first note that students cannot be harmed by the time constraint for their first option. That is, if they are assigned to their first option under DA, they are also assigned to it under TCDM. However, the converse does not hold because a student can benefit from the indirect time constraint for her first option. Then we have that $p_1^{\mathrm{TCDM}} \geq p_1^{\mathrm{DA}}$.

For those options different from the first one, the idea of the proof is to show that for each preference profile such that the student benefits from the time constraint, there is another

[13] For example, when each college has unit capacity, $\Sigma(T) = T$.

preference profile where she is harmed by the time constraint, and these relation is one-to-one. Then, given our assumption that preferences profiles are distributed uniformly and i.i.d., this completes the proof.

Consider a profile of preferences P_{-i} for students $1, \ldots, i-1$ such that student i benefits from the time constraint (we include the case where the student is assigned under TCDM but she is not under DA). Then, there is a student i' with higher priority than i, $i' P_c i$, such that she is rejected (because of the application of some student, say $i'' P_c i'$) in the last round from the college she applied to, c', and she cannot apply to her next option (see illustration below).

$$\begin{array}{lllllll}
i'': & \ldots & c' & \ldots & c & \ldots \\
\ldots & & & & & \\
i': & \ldots & c' & \ldots & c & \ldots \\
\ldots & & & & & \\
i: & \ldots & c & \ldots & &
\end{array}$$

First note that c should be ranked below c' by student i', if not she should have applied and got rejected by a high ranked student, and then i cannot be tentative assigned to c. So define $\hat{P}$ such that the preferences of all students different from i' remain the same, and at the preferences of student i' include c in the same place as c', and vice versa. Thus, at $\hat{P}$, i is rejected from c at the last minute (she is harmed by the time constraint). As we only change the preferences of one student, and in particular we swap only the position of two options of the student, the relation is one-to-one. □

In the last proposition, we assume that preferences are independent and uniformly distributed. As we will argue, we expect the same result to hold when preferences are correlated. Indeed, under correlated preferences it is less likely that students benefit from the positive indirect time constraint effect. Moreover, they will be more frequently impacted by the negative direct time constraint effect. Therefore, we expect that under correlated preferences the difference $p_1^{TCDM} - p_1^{DA}$, will be lower while $p_l^{DA} - p_l^{TCDM}$ $(l \geq 2)$ and $p_\emptyset^{TCDM}$ will be higher. Simulations confirm these patterns (see Appendix C for details).

Based on the last proposition, we can find cardinal utilities of a student such that she prefers, from an ex-ante perspective, the outcome under TCDM than under DA.

Corollary 1. *Suppose students play straightforward strategies, and students' preferences are i.i.d. distributed according to a uniform distribution. There exists a vector of cardinal utilities for any student such that she weakly prefers the outcome under TCDM over her outcome under DA.*

Proof. Denote by u_j the cardinal utility a student gets from being assigned to her j-th choice (where the cardinal utility of being unassigned is normalized to zero). Then, if u_1 is such that:

$$u_1 \geq \frac{\sum_{j=2}^{n}(p_j^{DA} - p_j^{TCDM})u_j}{p_1^{TCDM} - p_1^{DA}},$$

the student prefers her assignment under TCDM than under DA. In words, if a student has a high enough cardinal intensity for her top choice, her expected utility under TCDM is higher than under DA.

□

In the last part, we study how the probability distribution over ranked colleges under TCDM changes as we increase the number of rounds. In particular, we show that as T increases, the probability with which every constrained student is assigned to her first option decreases, while the probability with which she is assigned to the rest of the options increases.

Proposition 5. *Suppose students play straightforward strategies. Consider any student, and let p_l^T denote the probability with which a student is assigned to her l-th option under TCDM with $T \geq 1$ rounds. Then,*

1. $p_l^T = p_l^{T+1}$*, for every* $l \in \{1, \ldots, |C|, \emptyset\}$ *if the student is unconstrained;*
2. $p_1^T \geq p_1^{T+1}$*, for every constrained student;*
3. $p_l^T \leq p_l^{T+1}$*, where* $l \geq 2$*, for every constrained student.*

Proof. We prove by induction. By Proposition 4 we know that for any T, p_1^T is always higher than the probability of getting first choice under DA. When we increase T, a higher ranked student now applies to a first option of another student where she previously did not apply to, and causes the rejection of the previously assigned student. Thus, $p_1^T \geq p_1^{T+1}$.

Consider the l-th option ($l \geq 2$) of a student. When we increase from T to $T+1$, there are two effects of opposite sign. On the one hand, it may be that the student is not assigned under TCDM with T steps because she is rejected at the last step, and she applies and is assigned to her l-th option when we increase one step. This effect is positive, and it emerges when the student is harmed by the time constraint when there are T steps. On the other hand, it may be that the student is assigned with T steps to her l-th option, and when the number of steps increases by one, she is rejected from her l-th option (thus she is unassigned). In this case, the effect is negative, and implies that the student benefits from the time constraint with T steps. By Proposition 4 and for a given T, the probability that a student is harmed by the time constraint is higher than the probability that the student benefits. Thus, the probability of the first positive effect is higher than the probability of the second negative effect, which implies that $p_l^T \leq p_l^{T+1}$. $\square$

It is worth mentioning that this proposition implies that as we increase the number of rounds T, the probability distribution under TCDM is closer to the probability distribution under DA.

5 Empirical evidence

University admissions in China are coordinated regionally, with each region managing its own matching of students to universities. Universities allocate their quota to each region. Students compete within their region based on priorities determined by their exam scores from the national entrance exams. Before 2003, all regions used the Immediate Acceptance (IA) mechanism, but many regions began experimenting with alternative mechanisms. Among them, many adopted the Parallel mechanism, which is an intermediate mechanism between IA and DA (Chen and Kesten, 2017). Inner Mongolia introduced a dynamic mechanism in 2006, with the objective to increase students' satisfaction, fairness, and reduce the no-show rate.

The Inner Mongolian procedure The Inner Mongolian (IM) procedure asks students to apply to one university and rank up to six programs within that university.[14] Our empirical analysis focuses on the assignment of students applying for the four-year general education programs offered by tier-one universities, which are considered as the top group universities.[15] The matching usually lasts one day, starting in the early morning and finishing in the evening. Before finalization, students have the opportunity to revise their choices, even if their scores are above the cutoffs.

Information plays a crucial role in the IM procedure. Before the application phase begins, students receive relevant information such as each university's and program's quota,[16] tuition fee, and past year's final cutoffs. A distinct feature of IM is that it generates new information

[14] Students can indicate if they accept programs outside their ranked programs in case all their ranked programs are unavailable.

[15] Universities are divided into tiers, and matchings are processed sequentially. Tier-one universities are considered better than tier-two ones, and tier-two ones are better than tier-three ones. Tier-one universities are typically administered and financed wholly or partially by the various central ministries, while local regions support universities in the rest of the tiers. Lower-tier universities consider students who failed to attend tier-one universities in the subsequent matches

[16] There are two types of quotas, planned and final quotas. The difference is that universities can reserve 120% of their planned quota when reviewing applicants. The exact ratio depends on their estimated demand according to simulations (see more details in the appendix of Chen and Kesten (2017)). Most universities commit to their planned quota, while some accept more than their planned quota, especially to account for students with identical scores around the admission cutoffs. Final assignments use the adjusted final quota.

during the application phase. There are two types of information provided. The first type of information is provided in real-time, allowing students to *privately* observe, whether they are among the most preferred up to the allowed quota at the university they applied to, their priority rankings within that university, the number of students with the same score, and additionally, priority rankings in their first-choice program. The second type of information is provided every hour, with everyone *publicly* observing, for each university, cutoffs of the university as well as all its programs,[17] all applicants to the university, their characteristics, and their choices of programs.

The IM procedure finalizes the submissions using a staggered closing rule. Students are divided into multiple batches, with the first batch containing students with the highest scores, the second batch contains students with scores lower than the first batch but higher than the rest of the students, and so on. Those from the first batch have to finish their applications first, followed by students in the next batch who have an additional hour to adjust their applications. All the subsequent batches have an extra hour to re-consider the applications, following the ending time of the students from the previous batch. When everyone stops their applications, their final choices are used to determine the matching to universities.

In addition to these features, students must submit a choice within two hours of the application procedure's start to be eligible for the matching. When changing choices, students are required to enter a verification code received via mobile phones.

5.1 Data

The public information generated by the Inner Mongolian procedure provides a unique dataset of students' initial submissions and snapshots of their subsequent changes by hour. Our data come from the hourly application information published by the clearinghouse in 2018. It covers the universe of universities and students applying for tier-one universities in the province of Inner Mongolia that year. For each university at every hour, it contains information on the planned and the final quota, cutoffs using both the planned and final quota, the number of current applicants, choices of programs, and additional characteristics for each applicant. The additional characteristics include students' exam scores, exam scores with bonus points,[18] gender, and ethnicity. It is worth noting that students receive real-time feedback privately, so our analysis based on the publicly observed data may overlook their behaviors within the hour between updates.

The university-level data in the last hour contains 21,107 from the science and technology

[17]University cutoffs are computed using both planned quota and final quota after adjustment. Program cutoffs are computed according to the first choices of students who applied to the university and with a score above the university cutoff.

[18]Students could receive bonus points if they meet specific criteria, for instance, if they are ethnic minorities. The bonus point for minority students is 10 points.

track and 6,089 students from the social science and humanities track, both processed using the same mechanism[19] We summarize the main observations here (details on the summary statistics can be found in Appendix D.1.) First, in terms of students' characteristics, the two tracks exhibit similar patterns, except that there is a higher share of female students in the social science and humanities track. In addition, despite similar means in exam scores, including and excluding bonus points, there is higher variance in exam scores for students from the science and technology track. The scores (including bonus points) used to divide students into different batches are evenly spaced by 30 points. This results in unevenly divided batches by the number of students within each batch. Second, we use two measures for the final assignments to capture the potential difference caused by tie-breaking rules.[20] The first measure considers whether a student is ranked among the most preferred ones up to the allowed final quota—the one used by the clearinghouse for final assignments. The second measure considers whether a student scores above a university's final cutoff according to the final quota. These two measures give similar results, assuring us that the measurement error due to tie-breaking is minimal. We only consider the matching to universities, which is also the main task for the clearinghouse, even though that university may eventually reject students assigned to a university due to issues with program allocations.[21]

Student-level data is helpful in exploring application behavior over time and how time constraints play into their choices and affect their final assignments. One challenge with the data is that it does not provide a unique identifier to track students over time. We applied a linking method based on students' characteristics and the staggered closing rule to backtrack students from the last hour to the first hour. The main idea is that starting from the application data in the last hour, we link all applicants found in all universities at a later hour to all applicants found in all universities earlier, using their four characteristics. Backtracking students is more effective than tracking from an earlier hour to a later one as it leverages the staggered closing rule. In the last hour, only those from the last batch can revise their choices, and in the previous hour, these students, together with those of an earlier batch, can change their choices while the rest are inactive. We can link precisely the students who are inactive during these hours. With a bit of abuse of terminology, when a student is linked to a student from a previous hour, we call the linked pair a match as well. However, for

[19] In the first hour, there are 20,426 and 5,864 students, respectively. The majority of students have already entered a choice from the beginning, and only a small fraction of students entered a choice later but before the required hour.

[20] As universities still need to approve the allocations sent by the clearinghouse, the hourly information does not include this information.

[21] For instance, when students indicate that they do not want any program outside the ranked ones, they may be rejected eventually despite being assigned to a university by the clearinghouse. The clearinghouse is mainly responsible for placing students in universities and communicating the necessary information to each university, while universities have the final decision power in allocating programs. Unassigned students can apply to next-tier universities.

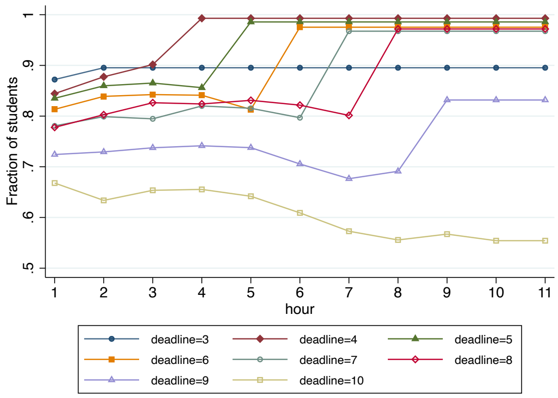
students who are active in the last hour and the previous hour, using common characteristics may not uniquely identify all of them. Therefore, we use two natural heuristics (Appendix D.2 presents details on the procedures), which treat two students observed at two different hours with the same characteristics and university or program choices as the same student. There is, however, a caveat with our linking method. We might overlook the possibility that another student with the same characteristics changed to this choice while the original student applied elsewhere. This implies that our linking method potentially provides a lower bound of student changes.

5.2 Results

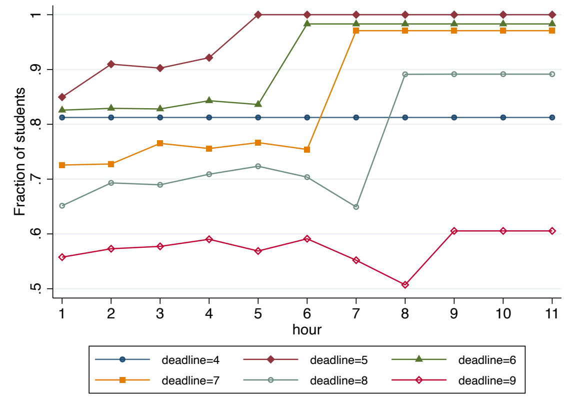
Welfare analysis in the literature often tries to recover the rank-ordered preferences for students using discrete choice models built on equilibrium concept (He, 2012; Luflade, 2018; Calsamiglia, Fu, and Güell, 2020). The following concerns kept us from doing so, and instead looking at direct evidence of time constraints in a parsimonious way. First, it is not clear if students have strictly ranked choices. While they often have a list of preferred universities before applying, students only need to ensure their final choices are feasible and desirable. Second, the game of TCDM is complicated, and as we have discussed earlier, even a straightforward strategy may not necessarily be an equilibrium strategy. Moreover, as students can adjust their applications even if they are not rejected, it is unclear if a choice observed at an earlier hour is more preferred than a choice observed at a later hour. Thus, it is challenging to recover students' preferences using a structural approach built on the equilibrium concept. Furthermore, as Pathak and Shi (2021) pointed out recently, the structural approach based on the discrete choice model may not outperform the ad-hoc approach if there is a prediction error in applicants' characteristics.

We first look at students' application status every hour. Application status at a given hour indicates if a student has a score above or equal to the current cutoff of the university to which she applied. Figure 2 presents a pattern that after students from a higher score batch finished their applications, the assignment rate for students from a lower score batch increases. Students from the top score batch finalized their choices at hour 3. There is an increase in assignment rate for students of the next score batch who finished at hour 4 compared to hour 3. Similarly, for students in the third score batch, who finished at hour 5, there is a sharp increase in assignment rate when moving from hour 4 to hour 5. The assignment rate before hour 4 sometimes decreases because students did not adjust their choices when rejected. This pattern is similar for students from both tracks. The sharp increase in the current assignment rate could result from better information after the finalization of higher-priority students or the fact that the deadline was approaching. This suggests that students' behaviors were consistent with our assumption of straightforward

Figure 2: Fraction of tentative assigned students by hour



(a) Science and technology track



(b) Social science and humanities

strategy, in the sense that following rejections, they applied to a choice where their scores exceeded the cutoff.

To gain deeper insights into the application strategies and behaviors of students, we conducted a survey in 2022, on the day of the application and shortly after the application process had ended. When asked about their intended application strategy, over 50 percent of students reported using straightforward strategies. Moreover, more than 85 percent indicated that they apply to the most preferred choice among those that can grant admission. This is consistent with the application that a straightforward strategy would indicate at the end of the application process. In another question regarding their behavior towards the end of the application process, 31 percent of students were satisfied with their choices and made no further changes, while 53 percent continued to revise their choices. Additionally, 14 percent of students indicated that they ultimately chose a university where they were confident they would be accepted (details are available from the authors upon request).

Second, Table 3 reports descriptive evidence on time constraints. To begin with, 10,003 (47.4% of total) students from the science and technology track and 3,303 (54.2% of total) students from the social science and humanities track changed their choices in the final round. Among these students, 1,208 (5.7% of total) students from the science and technology track, and 577 (9.5% of total) students from the social science and humanities track, are nonetheless rejected despite their last-hour changes. These students are potentially harmed by the time constraints, as they could have been assigned if there had been additional time. If we look further into these students, 1,161 of the unassigned students from the science and technology track (5.5%) had a score higher than or equal to the final cutoff of a university to which they applied in previous hours. This figure is 325 for unassigned students from the social science and humanities track (5.3%). These students have indicated an acceptable choice at some point during their application. However, they were not assigned when the application ended. This indicates the existence of justified envy, where some students prefer a university to be unassigned and have a score higher than the cutoff of that preferred university. This justified envy aligns with Proposition 3, as it highlights a scenario where a lower-scoring student is accepted to a university that is preferred by a higher-scoring student who ultimately remains unassigned.

It is worth mentioning that, students could apply in the ending round to a safe choice, where their scores are way above the cutoff, increasing their chance of assignment. Therefore, focusing on the final assignments, we might underestimate the magnitude of justified envy.

Table 3: Application behavior and assignment outcome

	Science and technology	Social science and humanities
# changed at final round	10,003	3,303
# changed at final round and are rejected	1,208	577
# unassigned but with a score $\geq$ final cutoff of a university applied previously	1,161	325

One concern is that students may submit their choices towards the very end, leaving low-ranked students being unassigned. This behavior, often known as "sniping", has been observed in online auctions with fixed ending times (Ockenfels and Roth, 2006). However, such manipulation is less likely in our data. One form of unilateral manipulation involves postponing rejection cycles: by applying late, students can avoid being in part of a rejection cycle that eventually leads to a worse outcome (Bo and Hakimov, 2022). This type of rejection is only possible when student priorities vary across universities, and not possible in our case of common priorities. Gong and Liang (2024) experimentally examine the IM procedure and did not find evidence of late applications, supporting our argument. Additionally, design features such as submitting choices by a specific hour and additional verification cost associated with changing choices, which increase the risk of failing to submit on time, further discourages the late applications.

6 Conclusions

With the advance in information technology, more school choice and college admission systems have started to adopt dynamic mechanisms, allowing students to interact and receive feedback during the application, unlike the traditional environment of static applications. This paper points out that time constraints might undermine these improvements.

In the context of college admissions, we first analyze theoretically the effects of time constraint on assignment outcomes, and we compare it with the standard DA mechanism. From an ex-post perspective it is not possible to rank these mechanisms. However, we prove that if a student receives a preferred outcome under TCDM, there is another student with higher priority who is not assigned under TCDM while he is assigned under DA. From an ex-ante perspective, we show that TCDM gives students a (weakly) higher probability of being assigned to their first choice and of remaining unassigned, but a lower probability of being assigned to the other options. We then illustrate our results in the real-life case of the dynamic mechanism used in the province of Inner Mongolian in China. We describe how students behave in data and provide evidence of the effects of time constraints on the final assignments. In particular, we show that the procedure's outcome may not be stable due to time constraints.

We assume in the analysis that students play straightforward strategies. When students do not play these strategies, an alternative strategy could be to play safe at the last period. That is, to apply to a college where the student's score exceeds the cutoff *by a large margin*. We argue that the use of these strategies has negative and positive effects on the performance of the TCDM. On the one hand, when students play safe strategies, a new type of instabilities arises. Under straightforward strategies, the instabilities always involve an unassigned student rejected at the last period. When some students play safe strategies, it is possible that a student assigned to a college may prefer to go to another college where there is available capacity, or a student with lower priority. Such a scenario would not be possible under straightforward strategies. On the other hand, playing safe strategies may imply that some students, who are not assigned under straightforward strategies due to rejection at the last period, are now assigned to a college. The overall effect is ambiguous, depending on the particular preferences profile. Finding restrictions on the domain of preferences such that one effect dominates the other is an interesting direction for future research.

One challenge in utilizing field data generated by the dynamic mechanism is that we do not observe the students' actual preferences. Lab experiments can address this issue by controlling for students' preferences. Therefore, future research may think of running experiments to quantify the trade-offs of time constraints in dynamic environments more precisely. Furthermore, while it is crucial for students to have enough time to revise their options, one may wonder how the design of information could speed up the lengthy revision process. In the case of the IM procedure, every student receives the same type of information, whereas some need more information than others. Improving information design under such dynamic mechanisms could potentially alleviate the time constraints.

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Appendix

A The extensive-form game induced by TCDM.

We describe the game induced by TCDM in this section. Most of the definitions are based on Klijn, Pais, and Vorsatz (2019). The game has the following elements.

- A finite set of players I, which corresponds to the set of students.
- A set of actions $A_i = C \cup \{\emptyset\}$ for each student i. This set includes all possible applications that a student might potentially make at each point of the the game, including the "no college" option $\{\emptyset\}$. Let $A \equiv \cup_{i \in I} A_i$.

  We make the following convention. If in a given round a student does not change her application, we let her apply to the same college.

- A set of nodes or histories H, where
  - the initial node or empty history h_0 is an element of H,
  - each $h \in H \setminus \{h_0\}$ takes the form $h = (a_1, a_2, \ldots, , a_k)$ for some finitely many actions $a_i \in A$, and
  - if $h = (a_1, a_2, \ldots, , a_k) \in H \setminus \{h_0\}$ for some $k > 1$, then $(a_1, a_2, \ldots, , a_{k-1}) \in H \setminus \{h_0\}$.

  A history is a sequence of application, one for each student, but each student may have applied more than once. For example, a history $(a_1, a_2, \ldots, , a_k)$ means that student 1 have applied to college a_1 in the first round, student 2 to college a_2 in the first round, and so on for the first $|I|$ elements. The element $|I| + 1$ is the application of student 1 in the second round, element $|I| + 2$ is the application of student 2 in the second round, and so on, and so forth for the rest of the elements.

  The set of actions available to a student i whose turn it is to move after history $h \in H$, is $C \cup \{\emptyset\}$.

- A set of end histories $E \equiv \{h \in H : (h, a) \notin H \text{ for all } a \in A\}$. In particular, the set of end node is the set of all histories of length $T \cdot |I|$.
- A function $\iota : H \setminus E \to I$ that indicates whose turn it is to move at each decision node in $H \setminus E$.

  We assume, without loss of generality, that students move sequentially at each round. Student 1 moves first, 2 moves second, and so on. Thus, for every non-terminal history

h, the player function is

$$\iota(h) = i_{|h|(\bmod |I|)+1},$$

where $|h|$ is the length of the sequence h.[22]

- An information set $\mathcal{I}_i$, which is a partition of the set $\{h : \iota(h) = i\}$, where $h, h' \in I_i \in \mathcal{I}_i$ if the first $|h| - (i-1)$ elements of the two histories are the same. That is, when a student applies in a given round, she does not know the colleges to which other students applied in the same round, but she knows to which they applied in the previous rounds.
- An outcome function $g : E \to \mathcal{M}$ that associates each terminal history with a matching, where $\mathcal{M}$ is the set of all matchings.

For any history h reached at the end of a round, that is, $|h| = k|I|$ for some $k = 1, \ldots, T-1$, let μ^h be the matching found by considering the applications of students as defined by the last N elements of the sequence. Said differently, it is the matching found by the last applications, where student i_n applies to college $c_{N(k-1)+n}$. Thus, μ^h is the tentative matching at history h.

For any two *terminal* histories h, h', a student prefers h over h' if and only if $\mu^h(i) P_i \mu^{h'}(i)$, where $\mu^h(i) \in C \cup \{\emptyset\}$ is the assignment of student i at μ^h.

Definition 3. A (pure) strategy for student i in this extensive–form game is a function $\sigma_i : \mathcal{I}_i \to A$, such that $\sigma(I_i) \in C \cup \{\emptyset\}$, for all $I_i \in \mathcal{I}_i$.

Given a matching μ and a college c, a **cutoff** $f_c^\mu \in \mathbb{R}$ is the minimum score among all the admitted students if the colleges reaches its capacity, and 0 if there is remaining capacity.

Definition 4. (*Straightforward strategies*) A student i, with priority s_i (that is, with a score in the exam equal to s_i) is said to follow a straightforward strategy if at every history h such that $\iota(h) = i$, her strategy $\sigma_i(h)$ is such that

$$\sigma_i(h) \coloneqq \max_{\succ_i} \{c \in C \cup \{\emptyset\} : s_i \geq f_c^\mu\}, \tag{1}$$

where μ is the matching induced by h, μ^h.

That is, at every history after which the student has to move, she applies to the most preferred college among those that would accept her. It may be that she chooses the college where she is tentatively assigned at that round.

[22] For example, a history of length $|I|+3$ means that all the students have applied in the first round, and it is the turn of the fourth student to apply because the first three students have already applied in the second round.

B Omitted proofs

Proof of Proposition 1.

Proof. When students follow straightforward strategies, the process of applications is equivalent to the DA algorithm. Thus, TCDM stops when it reaches the stable matching. Next, suppose that the mechanism stops at an unstable matching. Then, there is a student i who prefers a college c to her current assignment, and has a score higher than that college's cutoff. As students only apply to colleges after being rejected, this means that the student did not apply to c and then she skipped c in a previous step. But this contradicts the assumption that students follow straightforward strategies. □

Proof of Proposition 2.

Proof. Let μ denote the matching resulting when all students play straightforward strategies. From Proposition 1, we know that μ is the unique stable matching and is efficient. Suppose now there is another matching μ', which is obtained when all students but i play straightforward strategies, and i plays a deviating strategy. i prefers the college c that he is assigned to at μ' to the one he receives at μ. Then, there must be a student j who has higher priority than i who is assigned to c at μ (otherwise μ is not stable). Clearly, student j does not apply to c in the process where μ' is reached, so he is assigned to a preferred college c'. This implies that there is a student l who is assigned to c' at μ but not at μ'. Again, it must be the case that l does not apply c' when the matching μ' is reached. Continuing with this reasoning, as the number of student is finite, we will eventually reach student i. Then we construct an exchange cycle where, departing from μ all students students in the cycle are made better off, this contradicts to the fact that μ is efficient. Therefore, student i can not do better than following straightforward strategy, and a profile where students follow straightforward strategies is indeed an equilibrium in absence of time constraint. □

C Ex-ante comparisons with correlated preferences

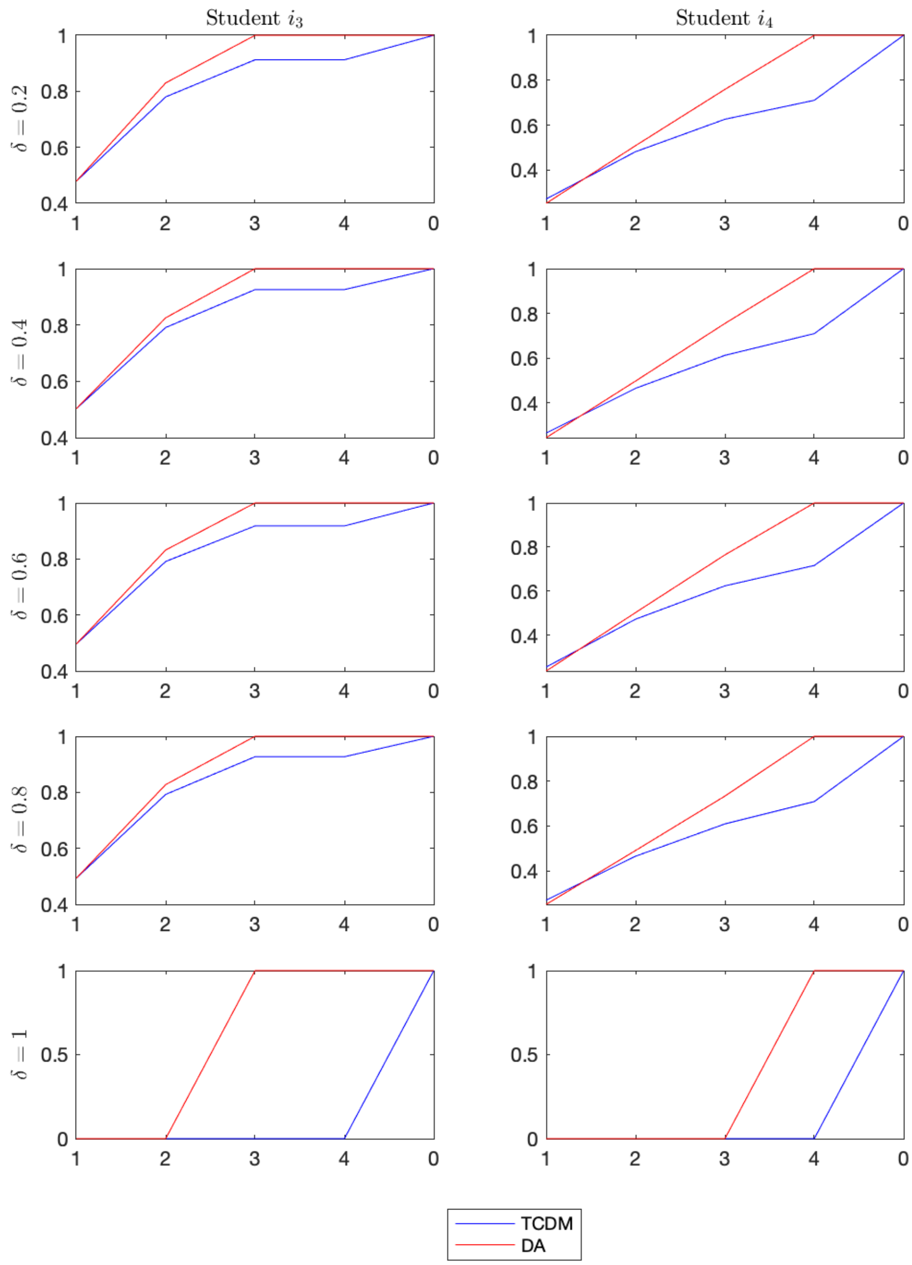
We consider 4 students to be assigned to 4 colleges, each with only one seat. As before, students priorities are such that student i_1 has the highest priority, followed by i_2, i_3 and i_4. Student's ordinal preference is derived from the utility that takes the following form:

$$u_c(i) = \delta v_c + (1 - \delta)\epsilon_{ic}.$$

The utility of individual i for attending college c consists two parts. The first part is a common utility component, denoted by v_c, and the second part is an idiosyncratic utility component that is unique to a student-college pair, denoted by ϵ_{ic}. Both v_c and ϵ_{ic} are drawn independently and identically on $[0, 1]$ according to uniform distribution. The parameter δ measures the correlation of preferences, when $\delta = 0$, we are back to the uniform and i.i.d. preferences studied in Example 3, and when $\delta = 1$, students' preferences are identical. Students follow straightforward strategies under TCDM where $T = 2$, and truth-telling strategies under DA.

Figure C.1 presents the simulated probabilities under both TCDM and DA for the two time-constrained students, namely i_3 and i_4. The results suggest patterns similar to what was shown in Example 3. When preferences are correlated but not identical ($\delta \neq 1$), then for i_3, DA yields a higher or equal probability to TCDM for being assigned to the first to the fourth choice. Similarly, for student i_4, except for the first choice, DA also yields a higher probability of being assigned to the second to the fourth choice. When preferences are identical, both of them are unassigned under TCDM; in this case, they are the most constrained with time.

Figure C.1: CDF of assignment probabilities for students who are constrained with time



Notes: We ran 2000 simulations and take the average probabilities. Each row takes a different correlation parameter, and we look at the correlation parameter from 0.2 to 1 by a grid of 0.2. Each column indicates a different student. In each subplot, x-axis indicates the CDF of being assigned to the first choice, the second choice, the third choice, the fourth choice and being unassigned.

D Additional information on empirical results

D.1 Descriptive statistics

Table D.1: Summary statistics from application data of final hour in 2018

	Science and technology track (1)	Social science and humanities track (2)
Panel A: Student characteristics		
Female	0.49	0.79
Ethnicity		
- non-Han	0.24	0.25
Exam score (incl. bonus)	547.66	542.35
	(48.60)	(29.86)
Exam score (excl. bonus)	545.54	540.21
	(48.62)	(29.78)
Fraction of students in[a]		
- Batch 1 (≥ 670)	0.01	
- Batch 2 ($640 - 669$)	0.04	0.003
- Batch 3 ($610 - 639$)	0.08	0.03
- Batch 4 ($580 - 609$)	0.13	0.10
- Batch 5 ($550 - 579$)	0.18	0.237
- Batch 6 ($520 - 549$)	0.21	0.36
- Batch 7 ($490 - 519$)	0.24	0.27
- Batch 8 ($460 - 489$)	0.11	
Panel B: Admission outcomes		
- Admitted to a university based on final choices according to rank up to quota[b]	0.89	0.83
- Admitted to a university based on final choices according to cutoff	0.89	0.84
Number of students	21,107	6,089
Panel C: Universities		
Planned quota	67.97	23.64
	(209.28)	(84.15)
Final quota	69.85	24.41
	(219.14)	(88.39)
Number of universities	291	211

Notes: [a]The score cutoffs to divide groups are indicated in brackets. In both tracks, there are no students who have score less than 460, the last batch according to the definition of the clearinghouse. In social science and humanities track, there are no students in Batch 1 nor Batch 8. [b]The first measure is based on whether a students is among the top most preferred students up to the allowed quota in their last-hour choices. The second measure is based on whether a student has a cutoff higher than or equal to the final cutoff at their last-hour choices. The two measures lead to a difference of 4 students in the science and technology track, and 58 students in the social science and humanities track.

D.2 Data linking

In order to solve the issue of multiple observations having the same characteristics, we use the two rules below.

Rule 1. Program priority: in a given university, if there is one observation who has the same characteristics and program choices as another observation at a previous hour, then we consider these two observations are the same student, and this student has not changed program choice nor university choice.

Rule 2. University priority: in a given university, if there is one observation who has the same characteristics as another student at a previous hour, then we consider these two observations are the same student, in addition we consider this student has not changed university choice.

The first rule essentially considers two students at two different hours that have the same characteristics together with university and program choices the same student. After first linking students using program priority, we use the second rule to link those who might have changed their program choices but not their university choices. When we cannot find a match, we search for another student with the same characteristics in another university at an earlier hour. We implemented the following procedure in Java.

1. For students observed in the last hour T, assign each of them an ID.
   (a) At $T-1$, for those who are inactive in both hours, assign them the IDs from T.
   (b) For those who are active in one of these two hours, for each university j, consider all students applied to it:
      i. If there exist some students with the same four characteristics that are in university j at $T-1$ as well as in university j at T, and have the same choices for programs, then, by Rule 1, these students are matched, and are assigned with IDs from those at T.
      ii. Among the rest of students, if there are some students with the same four characteristics that are in university j at $T-1$ as well as in university j at T, then by Rule 2, these students are matched and are assigned with IDs from those at T.
      iii. Among the rest of unmatched students, i.e. those who applied to j at T but not at $T-1$,
         A. If there are students, in other universities excluding j, that have the same four characteristics as those from T, then assign these students the IDs of students from T, following the order in the original data.
         B. If there are students still unmatched, assign them new IDs.

2. Repeat the procedure for $T - k$ until the hour 1, which gives us the student-level application data over time.